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An introduction to low-cycle fatigue phenomena

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AN INTRODUCTION TO LOW-CYCLE FATIGUE PHENOMENA

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AN INTRODUCTION TO LOW-CYCLE FATIGUE PHENOMENA

1. INTRODUCTION

Fatigue is a process of repeated loading that could lead to failure of a component. Repeated loading, either thermal or mechanical, causes fluctuating stresses that initiate damage to the material, often on a microscopic scale. Fatigue analysis and testing have long been regarded as vital elements for the development and maintenance of a wide variety of engineering structures that are, or will be, subject to loading that might give rise to fatigue.

The fatigue behaviour of metals can be separated into two types: high-cycle fatigue (HCF), more than approximately 10 000 cycles to failure, and low-cycle fatigue (LCF), fewer than 10 000 cycles to failure. Sometimes, however, the transition from low- to high-cycle fatigue can occur at as few as 500 cycles. During HCF, the applied load is such as not to cause appreciable plastic deformation, even at a stress concentration. However, the recurring application of this load causes microscopic damage from which the material eventually fails by the propagation of cracks. In LCF, a larger load is applied, normally beyond the elastic range of the material, and so the material is able to withstand only relatively few of these cyclic applications.

Low-cycle fatigue failures have been reported for many types of metals in industry sectors such as: aerospace, automotive, oil, gas and chemical processing, nuclear and non-nuclear power generation, defence and construction. Low-cycle mechanical fatigue may affect pressure vessels, vehicles, cranes and other components and structures where plastic strains have not been, or cannot be, avoided.

Low-cycle thermal fatigue can have an effect on components where rapid heating and cooling occur in response to fluctuating power demands and start-up/shut-down procedures. Power-producing equipment, such as gas turbines, engines and high-temperature steam pipelines, are susceptible to high thermal transients. For short durances, most metals can endure some repeated plastic deformation; therefore conditions that subsequently lead to thermal fatigue failures are usually quite severe.

Section 2 of this Item introduces the basic equations of low-cycle fatigue analysis, and some of the limitations of their usage. Many of the data required for the application of these methods are provided in ESDU Data Items that may be located via the current ESDU Subject Index. The cyclic hardening and softening behaviour of materials and the response of materials at different test temperatures are discussed in Section 3. Section 4 outlines low-cycle fatigue testing procedures and discusses the uncertainties that may arise during testing. Suitable estimates of low-cycle fatigue properties that can be used when limited data are available are considered in Section 5. Worked examples are given in Section 7.

1.1 Notation

A_0	initial cross-sectional area	m^2	in^2
A	instantaneous cross-sectional area	m^2	in^2
A_f	cross-sectional area at fracture	m^2	in^2
b	fatigue strength exponent		
C	constant related to fatigue ductility		

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c	fatigue ductility exponent		
E	Young's modulus	N/m ²	lbf/in ²
e	total engineering strain		
e_e	elastic engineering strain		
e_p	plastic engineering strain		
e_f	engineering strain at fracture		
K_{sc}	strength coefficient	N/m ²	lbf/in ²
K'_{sc}	cyclic strength coefficient	N/m ²	lbf/in ²
k	exponent used to correlate N_f and $\Delta\epsilon_p$		
l	instantaneous length	m	in
l_0	initial length	m	in
M_d	maximum temperature at which martensite transformation is induced during monotonic tensile straining	°C	°F
N_f	number of cycles to failure [†]	cycles	cycles
N_t	transition fatigue life	cycles	cycles
n	strain-hardening exponent, see Equation (2.7)		
n'	cyclic strain-hardening exponent		
P	applied load	N	lbf
P_f	load at fracture	N	lbf
R	stress or strain ratio		
RA	reduction in area, a ratio that is often expressed as a percentage		
T_m	melting temperature	K	K
$\Delta\epsilon$	total strain range		
$\Delta\epsilon_e$	elastic strain range		
$\Delta\epsilon_p$	plastic strain range		

[†] Note that 'failure' may be defined in several different ways, see Section 4.3.

$\Delta\sigma$	total stress range	N/m ²	lbf/in ²
ε	total logarithmic strain		
ε_e	elastic logarithmic strain		
ε_f	logarithmic fracture ductility		
ε'_f	fatigue ductility coefficient		
ε_{max}	maximum strain		
ε_p	plastic logarithmic strain		
σ	true stress	N/m ²	lbf/in ²
σ_a	true stress amplitude	N/m ²	lbf/in ²
σ_E	engineering stress	N/m ²	lbf/in ²
σ_f	true fracture strength	N/m ²	lbf/in ²
σ'_f	true fatigue strength coefficient	N/m ²	lbf/in ²
σ_m	mean true stress	N/m ²	lbf/in ²
σ_{max}	maximum true stress	N/m ²	lbf/in ²
σ_{min}	minimum true stress	N/m ²	lbf/in ²
σ_r	stress amplitude at zero mean	N/m ²	lbf/in ²
σ_{ts}	tensile strength	N/m ²	lbf/in ²
σ_y	yield strength (for materials which do not show an obvious yield point, a proof stress, which produces a permanent plastic extension of, say, 0.2 per cent in the test piece gauge length, may be used instead)	N/m ²	lbf/in ²
σ^*	reference stress	N/m ²	lbf/in ²

2. LOW-CYCLE FATIGUE RELATIONSHIPS

The low-cycle fatigue properties of metals are generally studied using low-endurance, high-strain fatigue tests carried out under conditions of uninterrupted cycling. As well as resistance to mechanical loading, the resistance of materials to thermally-induced stresses and strains and to creep-fatigue are also of interest. To replicate the in-service conditions of components subject to high-temperature fatigue, hold times at the maximum stress are introduced. Under these conditions, the LCF life of components is reduced because of creep effects.

Although there have been advances in the understanding of the fatigue behaviour of metals since the 1950s, there still exists a need to verify component design and modifications to ensure that there remains adequate fatigue strength and fatigue life. This can involve reporting prototype or actual component testing, but experimental work can be complex, time-consuming and costly if service conditions are to be replicated accurately. A contemporary method is to apply computational modelling techniques, which tend to use basic test data, to replicate the environmental and working conditions of the component. The reliability and accuracy of these methods vary, depending on their complexity, and on the data that are available.

This Data Item introduces the quantitative relationships between plastic strain and fatigue life that form the basis of an understanding of the LCF behaviour of materials under inelastic cyclic deformation. Although most engineering structures are designed such that nominal stresses remain elastic, stress concentrations cause inelastic strains to develop around notches. The strain-life method, which takes into account plasticity around the notch root, is based on the observation that, in many components, the response of the material in critical locations becomes strain-, or deformation-dependent. Therefore, at high load levels in the LCF regime, the cyclic stress-strain response of the material is best considered under strain-controlled conditions.

For the strain-life approach, the plastic strain, or deformation, is measured directly and quantified. Crack growth is not explicitly accounted for in the strain-life method, but the component is said to have failed when the 'equally-stressed volume of material' fails. Therefore, strain-life methods are often considered to give estimates of life to crack initiation.

The effects of LCF deformation at high temperatures, the Bauschinger effect, mean stress, and other types of inelastic deformation are also discussed in this Item.

2.1 Monotonic Stress-Strain Curves

A basic understanding of the LCF behaviour of metals may be drawn from their monotonic stress-strain properties, see Section 5. Monotonic stress-strain properties of metals are determined by testing smooth polished test pieces with uniform cross-section under axial loading to measure the load, diameter and/or strain to obtain a stress-strain curve.

For isotropic materials, engineering stress and engineering strain are defined as follows:

$$\text{engineering stress, } \sigma_E = \frac{P}{A_0}, \quad (2.1)$$

and

$$\text{engineering strain, } e = \frac{l - l_0}{l_0} = \frac{\Delta l}{l_0}. \quad (2.2)$$

In the early portion of the engineering stress-strain curve (see Sketch 2.1a), many materials obey Hooke's law, to a reasonable approximation, so that stress is proportional to strain with the constant of proportionality being the modulus of elasticity, E . Up to a point called the limit of proportionality, the material loads and unloads near-elastically. However, beyond this point, plastic deformation will begin and become appreciable at the yield stress, σ_y , or the proof stress. In a ductile material, the stress then needed to increase the strain continues to rise, a mechanism termed 'strain- or work-hardening'. The rate of strain-hardening diminishes up to a point called the tensile strength σ_{ts} . Past this point, the engineering stress-strain curve falls, giving the impression that the material is strain-softening; the material is, in fact, still strain-hardening, but the cross-section area of the test piece is decreasing and this more than offsets that effect.

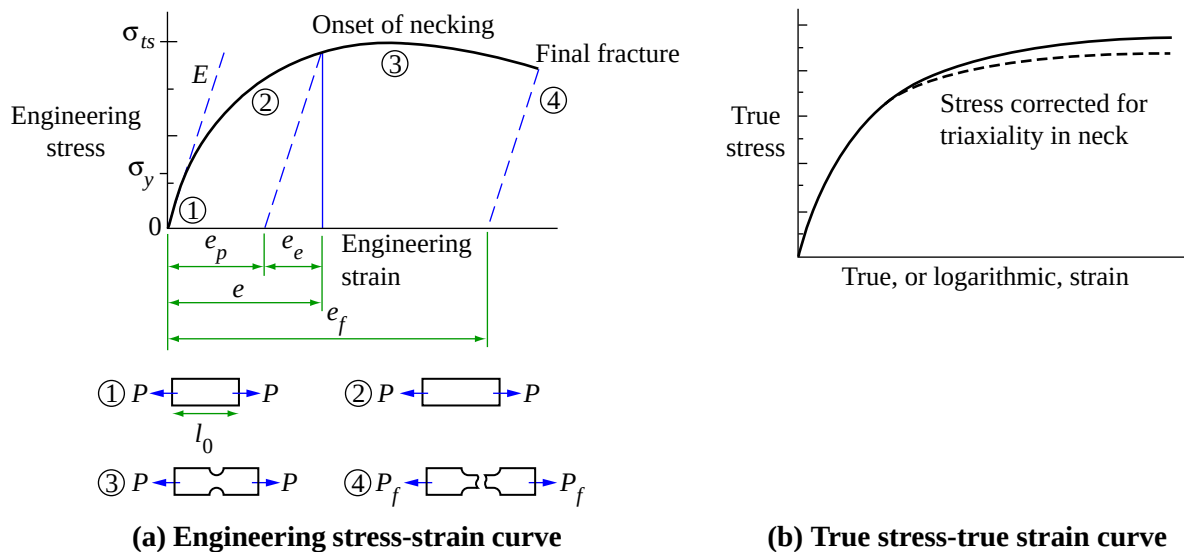
Beyond the elastic limit, the engineering stress-strain curve must be treated with caution since the specimen dimensions begin to experience substantial changes from their original proportions (Sketch 2.1a). True stress, calculated using instantaneous cross-sectional area, A , rather than engineering stress, gives a more direct measure of the response of the material in the plastic region:

$$\text{true stress, } \sigma = \frac{P}{A}. \quad (2.3)$$

A measure of strain that is used in conjunction with true stress is true, or logarithmic, strain, which takes the increment of strain to be the increment in displacement, dl , divided by the instantaneous length:

$$\text{true, or logarithmic, strain, } \varepsilon = \int_{l_0}^l \frac{dl}{l} = \ln \frac{l}{l_0}. \quad (2.4)$$

The true strain may be divided into its elastic, ε_e , and plastic, ε_p , components. A true stress-true strain curve is shown in Sketch 2.1b where the highly localised strains exceed the engineering strain and in the necked region triaxial stresses are introduced. In tension, the true stress is larger than the engineering stress, owing to decreases in the cross-sectional area resulting from deformation. Similarly, until necking occurs in the specimen, the true strain is smaller than the engineering strain because l is greater than l_0 . Note that true stress and strain are sometimes referred to as localized stress and strain. Note also, however, that if l_0 is suitably defined, one may also use the term 'local engineering strain'.



Sketch 2.1 Stress-strain relationships

The total strain, ε , in a tensile test can be separated into an elastic component, ε_e , which is recoverable on unloading, and an inelastic, or plastic, component, ε_p , which cannot be reclaimed on unloading. The total strain can thus be written as:

$$\varepsilon = \varepsilon_e + \varepsilon_p, \quad (2.5)$$

where the elastic strain is determined from the true stress and the modulus of elasticity, E :

$$\varepsilon_e = \frac{\sigma}{E}. \quad (2.6)$$

Note that for small strains, engineering strain and true strain are approximately equal. For most metals of practical interest, a log-log plot of true stress versus true plastic strain can be drawn as a straight line, and is expressed using a power law expression:

$$\sigma = K_{sc} \varepsilon_p^n, \quad (2.7)$$

where K_{sc} is the strength coefficient
and n is the strain-hardening exponent.

Rearranging Equation (2.7) gives the plastic strain:

$$\varepsilon_p = \left(\frac{\sigma}{K_{sc}} \right)^{1/n}. \quad (2.8)$$

From the expression for elastic and plastic strains, Equations (2.6) and (2.8), the total strain may be written as:

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K_{sc}} \right)^{1/n}. \quad (2.9)$$

During testing, deformation is uniform along the test piece (this is one of the assumptions beneath Equations (2.1) to (2.9)) until a neck forms and then all subsequent deformation takes place in the neck. The neck becomes smaller, local true stress increasing all the time, until the test piece fails. The true fracture strength, σ_f , is the true stress at final fracture:

$$\sigma_f = \frac{P_f}{A_f}. \quad (2.10)$$

The engineering strain at fracture, e_f , includes the deformation in both the necked and un-necked regions. Since the strain in the neck is larger than the strain in the un-necked material, the value of e_f depends on the fraction of the length of the test piece that has necked. Therefore, e_f is a function of the specimen geometry as well as the material. The true fracture ductility, e_f , which is greater than e_f , can be defined in terms of the initial cross-sectional area and the area at fracture:

$$\varepsilon_f = \ln \frac{A_0}{A_f} = \ln \frac{1}{1-RA}, \quad (2.11)$$

where $RA = \text{reduction in area} = \frac{A_0 - A_f}{A_0}$ (note that this ratio is often expressed as a percentage).

The strength coefficient, K_{sc} , can also be expressed in terms of the true stress at fracture and the true strain at fracture by substituting σ_f and ε_f into Equation (2.7):

$$\sigma_f = K_{sc} \varepsilon_f^n; \quad (2.12)$$

on re-arranging:

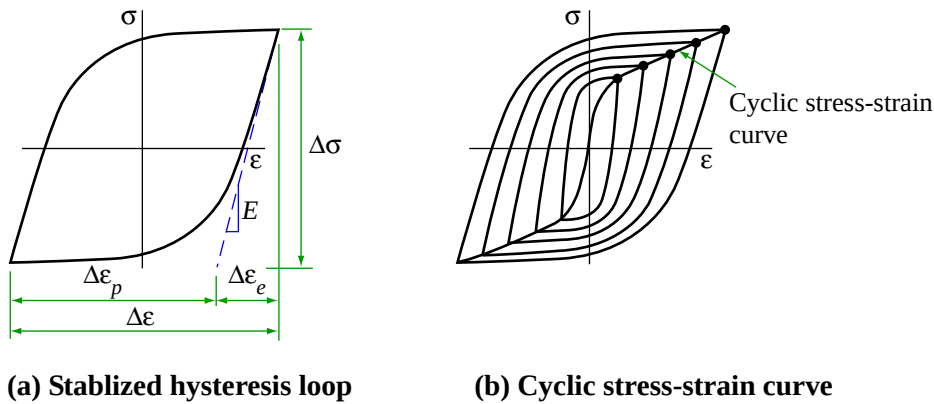
$$K_{sc} = \frac{\sigma_f}{\varepsilon_f^n} \quad (2.13)$$

2.2 Cyclic Stress-Strain Behaviour

To understand the behaviour of structures subject to repeated loading into the inelastic range, cyclic stress-strain curves are used. The cyclic stress-strain properties are determined by testing smooth, polished test pieces under axial cyclic strain-controlled conditions using completely reversed constant-amplitude strain-controlled tests.

The response of the material to inelastic loading is in the form of a hysteresis loop. The area within the hysteresis loop is the energy per unit volume dissipated during a cycle; it is a measure of the work of plastic deformation done on the material. After some number of cycles (depending on the test conditions, material *etc.*) the same stresses are induced for given applied strains and the hysteresis loop is said to have stabilized, see Sketch 2.2a. If it is not possible to establish when the hysteresis loop has stabilized, then the cyclic stress and strain amplitudes are assumed to be stable at approximately half life-to-failure.

If the tips of stabilized hysteresis loops from tests at several strain ranges are connected, a cyclic stress-strain curve can be obtained, see Sketch 2.2b.



Sketch 2.2

The total width of a hysteresis loop is the total strain range, $\Delta\varepsilon$, and the total height of the hysteresis loop is $\Delta\sigma$, or the total stress range. From these the strain amplitude and the stress amplitude are:

$$\text{strain amplitude} = \frac{\Delta\varepsilon}{2}, \quad (2.14)$$

and

$$\text{stress amplitude} = \frac{\Delta\sigma}{2}. \quad (2.15)$$

Similarly to Equation (2.5), the total strain range is the sum of the elastic and plastic strain ranges:

$$\Delta \varepsilon = \Delta \varepsilon_e + \Delta \varepsilon_p, \quad (2.16)$$

or, in terms of amplitudes,

$$\frac{\Delta \varepsilon}{2} = \frac{\Delta \varepsilon_e}{2} + \frac{\Delta \varepsilon_p}{2}. \quad (2.17)$$

Using Hooke's law, the elastic term in Equation (2.17) can be replaced by $\Delta \sigma / E$ to give:

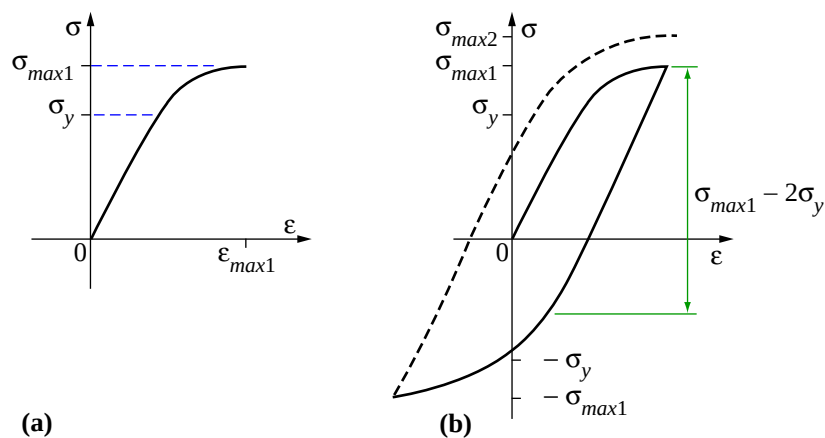
$$\frac{\Delta \varepsilon}{2} = \frac{\Delta \sigma}{2E} + \frac{\Delta \varepsilon_p}{2}. \quad (2.18)$$

2.2.1 The Bauschinger effect

The response of a metal test piece first loaded in tension past the yield strength, σ_y , to a maximum stress, σ_{max1} , corresponding to a strain ε_{max1} , is shown in Sketch 2.3a. When the material is subsequently unloaded and then compressively loaded to $-\sigma_{max1}$, inelastic strains develop before $-\sigma_y$ is reached, thus modifying the shape of the hysteresis curve, as shown in Sketch 2.3b. On loading again in tension to a strain of ε_{max1} , a new maximum stress of σ_{max2} is attained. Similar behaviour will occur until the hysteresis loop becomes stable. In the same way, if the experiment was repeated with initial loading in compression to $-\sigma_{max}$ yield would take place at $-\sigma_y$ but would again occur before σ_y was reached in the tensile loading direction.

This phenomenon is known as the Bauschinger effect (Reference 12). The point to note is that inelastic strains develop before the nominal yield stress is reached. A simplified hypothesis from Massing states that reversed yield first takes place at a stress level $2\sigma_y$ from σ_{max1} , as shown in Sketch 2.3b.

The Bauschinger effect gave the first indication that a single reversal of inelastic strain could change the stress-strain behaviour of metals and was a forerunner to the understanding of cyclic-hardening/cyclic-softening behaviour in metals (Reference 15).

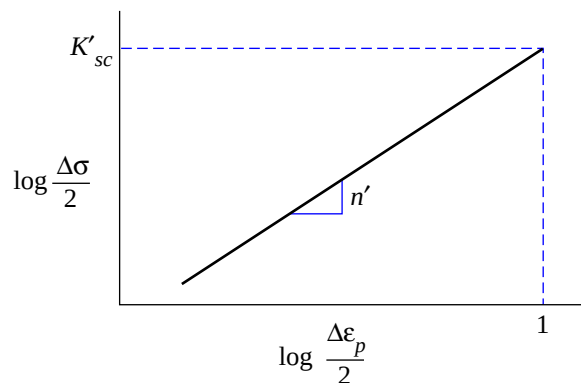


Sketch 2.3 The Bauschinger effect

In metals, the Bauschinger effect takes place by either short- or long-range mechanisms. Firstly, short-range effects can exist in which the resistance to dislocation movement is less in the reverse direction than in the forward direction, for example, recovery of dislocation debris by the backward motion of dislocations, or the re-establishment of local order when individual dislocations retrace their path through sheared precipitates. Secondly, long-range effects can exist, whereby internal sources of stress such as dislocation pile-ups at grain boundaries and Orowan loops around strain precipitates assist the reverse movement of dislocations (Reference 7).

2.3 Cyclic Stress-Strain Relationship

As for the monotonic stress-strain curve (Equation (2.7)), a log-log plot of the completely-reversed stabilized cyclic true stress-true plastic strain curve (Sketch 2.2b) can often be approximated to a straight line, see Sketch 2.4.



Sketch 2.4 Log-log plot of true cyclic stress versus true cyclic plastic strain

A power law expression for the cyclic stress-strain relationship is:

$$\frac{\Delta\sigma}{2} = K'_{sc} \left(\frac{\Delta\epsilon_p}{2} \right)^{n'} \quad (2.19)$$

where $\Delta\sigma/2$ is the stabilized cyclic stress amplitude,

$\Delta\epsilon_p/2$ is the stabilized cyclic plastic strain amplitude,

K'_{sc} is the cyclic strength coefficient

and n' is the cyclic strain-hardening exponent.

Rearranging Equation (2.19) gives:

$$\frac{\Delta\epsilon_p}{2} = \left(\frac{\Delta\sigma/2}{K'_{sc}} \right)^{1/n'} \quad (2.20)$$

Substituting this expression for the plastic strain amplitude into Equation (2.18) gives:

$$\frac{\Delta\epsilon}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma/2}{K'_{sc}} \right)^{1/n'} \quad (2.21)$$

2.4 Strain-Life Relationship

Coffin and Manson (References 1 and 2) determined that plastic strain-life data could be linearized using a log-log plot of plastic strain against fatigue life. A power law relationship between plastic strain and life can be defined as:

$$\frac{\Delta \varepsilon_p}{2} = \varepsilon'_f N_f^c, \quad (2.22)$$

where $\Delta \varepsilon_p/2$ is the plastic strain amplitude,
 N_f is the number of cycles to failure (1 cycle = 2 reversals),
 ε'_f is the fatigue ductility coefficient
 and c is the fatigue ductility exponent.

The constants ε'_f and c are fatigue properties of the material. Equation (2.22) is a modification of the original equation proposed individually by Coffin and Manson (References 1 and 2) which related the number of cycles to failure to the strain **range**. In Equation (2.22), ε'_f is the intercept of the straight line relationship between the logarithm of number of cycles to failure and logarithm of plastic strain amplitude. ε'_f may be calculated approximately from the fracture ductility of the material, ε_f , obtained from a tensile test (see Section 5). If the plastic strain range was used in Equation (2.22), then ε'_f would be twice the size.

For cycling in the elastic region, as is the case for high-cycle fatigue, the stress-life data when plotted on a log-log scale become linear. Using the true stress amplitude, the plot is linearized and expressed by the Basquin equation:

$$\frac{\Delta \sigma}{2} = \sigma'_f N_f^b, \quad (2.23)$$

where $\Delta \sigma/2$ is the true stress amplitude,
 σ'_f is the fatigue strength coefficient
 and b is the fatigue strength exponent.

The constants σ'_f and b are also fatigue properties of the material.

Dividing the Basquin equation by the modulus of elasticity gives an equation for the elastic strain-life curve:

$$\frac{\Delta \varepsilon_e}{2} = \frac{\sigma'_f}{E} N_f^b. \quad (2.24)$$

The total strain amplitude can now be expressed using Equations (2.22) and (2.24) as:

$$\frac{\Delta \varepsilon}{2} = \frac{\sigma'_f}{E} N_f^b + \varepsilon'_f N_f^c. \quad (2.25)$$

Equation (2.25) is called the strain-life equation and provides the basis of the strain-life method; it is also sometimes referred to as the Coffin-Manson equation. The exponents in Equation (2.25) may be derived from log-transformed LCF test data, as shown in Sketch 2.5. Values obtained in this way will only be valid over the range of the experimental data, thus emphasizing the need for a broad range of experimental data