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Summary of the model for performance of an aircraft tyre rolling or braking on dry or precipitate contaminated runways



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SUMMARY OF THE MODEL FOR PERFORMANCE OF AN AIRCRAFT TYRE ROLLING OR BRAKING ON DRY OR PRECIPITATE CONTAMINATED RUNWAYS

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SUMMARY OF THE MODEL FOR PERFORMANCE OF AN AIRCRAFT TYRE ROLLING OR BRAKING ON DRY OR PRECIPITATE CONTAMINATED RUNWAYS

1. NOTATION AND UNITS

		Dimensions
a	half-length of static footprint of tyre	L
a_{F_1}	parameter in correlation of micro-texture function (see Reference 3)	—
a_0, a_1	constants in stress equation	—
a_2	constant associated with shear strength of snow	—
B	Beta distribution (see Reference 6)	—
C_D	decelerating force coefficient due to fluid displacement by single tyre	—
c	surface roughness factor	—
c_0, c_1, c_{01} c_{10}, c_{11}, c_{12}	constants in drag equation	—
D	inflated diameter of tyre	L
d	depth	L
d_E	effective size of solids in surface	L
$(d_{TEX})_{MIN}$	minimum depth of macro-texture (see Reference 3)	L
$(d_{TEX})_0$	macro-texture before grooving (see Reference 3)	L
d_{TREAD}	tyre tread depth	L
F_1	micro-texture sharpness parameter (see Reference 3)	—
$[F_1]_U$	upper bound of function F_1 (see Reference 3)	—
f_λ	parameter relating to phase angle between stress and strain (see Section 3.4)	—
f_{SNOW}	parameter used to determine shear strength of snow	—
G	decelerating force on single tyre	M L T ⁻²
G	effective shear modulus or modulus of rigidity of tread or contaminant	M L ⁻¹ T ⁻²
i	row number in multiple-wheel undercarriage	—
j	column number in multiple-wheel undercarriage	—
m	number of wheels in the line-astern arrangement of multiple-wheel undercarriage (see Section 3.2)	—
m	parameter for calculating $(\mu_{REF})_{SNOW}$	—
N	speed number $\left(= \left(\rho_C V^2 / (\pi p_{ABS}) \right)^{1/2} \right)$	—

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n	number of wheels in line-abreast arrangement of multiple-wheel undercarriage (see Section 3.2)	—
n	polytropic index for process of compressing slush or snow (see Table 5.2)	—
p	tyre inflation pressure	$M L^{-1} T^{-2}$
$Pr[\]$	probability of event in $[\]$	—
q	kinetic pressure	$M L^{-1} T^{-2}$
q_v	pressure in Zone 2 (tyre footprint on wet surface)	$M L^{-1} T^{-2}$
R	dimensionless empirical factor	—
s	slip ratio	—
T	temperature	K
T_{gt}	glass-transition temperature (see Reference 4)	K
T_r	reference temperature associated with f_{λ} term (see Reference 4)	K
V	velocity	L^{-1}
w	tyre width (inflated)	L
Z	normal (to runway) load on tyre	$M L T^{-2}$
z	value from the standard Normal distribution (see Reference 7)	—
$\alpha_i, i = 0, 1, 2, 3$	constants in correlations	—
δ	static tyre (vertical) deflection	L
$E[\]$	expected value of quantity in $[\]$	—
ϵ	strain	—
η	empirical motion efficiency (see Table 5.2.3)	—
η	loss coefficient (see Tables 5.3.3, 5.4.3, 5.5.3 and 5.6.3)	—
$\eta_i, i = 0, 1, 2$	constants for a tyre in slip used to calculate Φ_{01} factor	—
Θ	temperature parameter	—
θ_1	constant in temperature parameter equation	—
κ	tyre construction parameter	—
Λ	tyre loading parameter	—
λ	phase angle between stress and strain (see Section 3.4)	rad
λ_{273}	constant in empirical motion efficiency equation	—
μ	coefficient of friction or retarding force coefficient (ratio of rolling resistance force to normal load)	—
$(\mu_{REF})_0$	coefficient for braking on compressed snow or ice	—
ν	Poisson's ratio	—
ρ	density	$M L^{-3}$
σ	compressive strength	$M L^{-1} T^{-2}$

$\sigma[\]$	standard deviation of quantity in []	—
$(\sigma_{SNOW})_{273}$	compressive strength of dry loose snow with equivalent specific gravity to moist/wet/very wet loose snow at temperature of 273 K	$M L^{-1} T^{-2}$
$\hat{\sigma}_{SNOW}$	specific gravity of snow	—
τ	shear strength	$M L^{-1} T^{-2}$
$1/\tau$	surface drainage parameter ($= d_C/(d_C + d_{TEX})$)	—
Φ	parameter used to calculate $(\mu_{REF})_{ICE}$	—
Φ_0, Φ_{01}	parameters associated with $(\mu_{SKID})_{WET}$	—
Φ_1, Φ_{10}	parameters associated with q_v	$M L^{-1} T^{-2}$
ϕ_E	measure of surface roughness ($= d_{TEX}/(d_{TEX} + d_E)$)	—
ϕ_{WET}	fractional water content of snow	—

Subscripts

<i>ABS</i>	absolute
<i>AMB</i>	ambient
<i>BRAKE</i>	brake
<i>C</i>	contaminant
<i>COM</i>	compression or compressed
<i>DISP</i>	displacement of fluid
<i>DRY</i>	dry (uncontaminated) runway
<i>FLUID</i>	fluid
<i>G</i>	ground
<i>ICE</i>	ice
<i>REF</i>	reference
<i>ROLL</i>	rolling wheel
<i>RUT</i>	rut in snow after passage of wheel
<i>SKID</i>	skidding, no wheel rotation
<i>SLIP</i>	slipping, some wheel rotation and some wheel slide
<i>SNOW</i>	snow
<i>STATIC</i>	on point of moving
<i>TEX</i>	effective surface macro-texture
<i>TOT</i>	total for single wheel
<i>TREAD</i>	tyre tread material
<i>WET</i>	wet, or water, or slush, covered runway

2. INTRODUCTION

An empirical model is defined that describes the decelerating forces acting on single aircraft type tyres when rolling or braking on hard surfaces typical of runways and taxiways. The model is valid for runways contaminated with water, slush, snow and ice but can also be applied to runways that are free from any contaminant.

Analysis of the decelerating forces acting on the tyres that belong to a multiple-wheel undercarriage *for rolling and braking on dry surfaces* is covered in Section 3.2. The analysis of the braking forces from a multiple-wheel undercarriage operating on a *precipitate-contaminated runway surface* is described in Reference 9.

Refer to Reference 13 for information about modelling a tyre in yaw on a dry or wet runway surface.

Further information regarding retarding forces acting on aircraft tyres and definitions of runway contaminants can be found, respectively, in References 5 and 14.

In order to calculate decelerating forces acting on a tyre operating on a contaminated runway surface, in addition to an approximation to the operating slip ratio for the braking system for cases where the tyre is braked, it is necessary to know values of nine variables, all of which are used in customary calculations of aircraft performance. These are

- depth of macro-texture of runway surface, d_{TEX} ,
- depth of contaminant, d_C ,
- density of contaminant, ρ_C ,
- ground speed, V_G ,
- tyre inflation pressure, p ,
- normal (to runway) load on tyre, Z ,
- nominal tyre width, w ,
- nominal tyre diameter, D
- runway surface temperature, T_G .

For the case where the tyre is operating on a dry runway surface, all of the aforementioned variables, with the exception of the depth and density of the contaminant, are required.

The model is founded on experiments with aircraft type tyres analysed in the context of observations based in rubber technology. It has been substantiated over a range of experimental data so that aircraft and ground-test machine performance can be predicted for a wide range of operational envelopes. Development and substantiation is described in Derivations[†] 1 and 2, both of which contain an extensive bibliography.

Where values are known for all the variables listed above, the model provides estimates of the decelerating forces due to rolling or braking. However, in cases where, say, runway conditions are unknown, the model can be used to deduce those conditions from ground-test machine measurements. Subsequently, those conditions can be re-applied to the model to predict aircraft performance.

In the particular case of winter precipitation, the model includes the statistical variation of the mechanical

[†] The model presented here and in Derivation 2 includes developments of some of the work presented in Derivation 1. These developments simplify the model and remove unnecessary limitations.

properties of snows and the frictional characteristics for runways that are completely covered with ice or snow. These statistical variations are identified from experiments on aircraft and ground-test machines.

Section 3 states the basis of the methods of estimating the various contributions that tyres make to decelerating forces. The model is defined in the Tables of **Section 5**. Examples of, and information relating to, the use of the methods defined in this Data Item are given in References 10 to 12 and 16. A computer program which incorporates the method has also been developed (see Reference 17).

3. BASIS OF MODEL

3.1 Longitudinal Forces Acting on Single Tyre

The resultant deceleration force, G_{TOT} , acting on a single, unyawed tyre is assumed to be represented by

$$G_{TOT} = G_{ROLL} + G_{FLUID} \text{ (or } G_{SNOW}) + G_{BRAKE} \quad (3.1)$$

These contributions to G_{TOT} are assumed to be independent of one another, as follows.

Tyre rolling resistance, $G_{ROLL} (= Z \mu_{ROLL})$, arises from the flexing of the tyre and is assumed to be a function of the peripheral speed of the tyre $(= (1-s)V_G)$. This resistance arises in all runway surface conditions, dry, wet, or contaminated. See Table 5.2, Section 5.2.1[†].

Contaminant drag forces, G_{FLUID} or G_{SNOW} , are generated by the displacement and/or compression of contaminant lying in the tyre's path and are assumed to be functions of the ground speed[‡], V_G . See Table 5.2, Section 5.2.2 (water and slush) or Section 5.2.3 (snow).

Braking force, $G_{BRAKE} (= Z \mu_{SKID} \text{ or } Z \mu_{SLIP})$ due to braking action alone. This is assumed to be a function of the translation speed of the tyre footprint, $V (= sV_G)$. See Table 5.3 (dry surfaces), Table 5.4 (water- or slush-covered surfaces) and Table 5.5 (ice- or snow-covered surfaces).

Values of constants, used in the relationships that define the model, are given in Table 5.1. In the case of the reference coefficient of friction, μ_{REF} , a more appropriate value for a particular aircraft or ground-test machine may be deduced from any *friction limited* test (see Section 4.2 of Reference 8) using the relevant equations(s) from Table 5.3 or Table 5.4. Such values would then be used in preference to those quoted in Table 5.1.

3.2 Longitudinal Forces Acting on Multiple-Wheel Undercarriage Configurations

For a multiple-wheel undercarriage configuration account must be taken of any effects of that configuration on the decelerating force experienced by the tyres that belong to the undercarriage.

A typical multiple-wheel undercarriage will feature m wheels in a line-astern arrangement and n wheels in a line-abreast arrangement. These terms are defined in Sketch 3.1, where the x -axis refers to the line-astern direction and the y -axis refers to the line-abreast direction. The undercarriage row number, i , is defined in the line-astern direction and the undercarriage column number, j , is defined in the line-abreast direction. The tyre in the upper-left corner of the undercarriage tyre array in the direction of travel is defined

[†] In many cases "established performance" for a particular aircraft type incorporates a value(s) of "aircraft μ_{ROLL} " derived to be consistent with the overall thrust/drag balance in ground-run tests. Such values can be used in place of those derived from the equation given in Section 5.2.1.

[‡] The model does not explicitly discriminate between compression and displacement effects for snow-covered surfaces but does make this distinction in the case of slush, *i.e.* a mixture of water, ice and air.