



Normal force and pitching moment of non-circular bodies: elliptical cross-sections at supersonic speeds

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NORMAL FORCE AND PITCHING MOMENT OF NON-CIRCULAR BODIES: ELLIPTICAL CROSS-SECTIONS AT SUPERSONIC SPEEDS

CONTENTS

	Page
1. NOTATION AND UNITS	1
2. INTRODUCTION	3
3. METHOD	3
3.1 Linear and Non-linear Components $[C_N]_L$, $[C_N]_{NL}$, $[(C_m)_0]_L$ and $[(C_m)_0]_{NL}$	4
3.2 Factor K_L	6
3.3 Factor K_{NL}	6
3.4 Centre of Pressure Position	7
3.5 Pitching Moment Coefficient about Axis x_m	7
4. ACCURACY AND APPLICABILITY	7
5. DERIVATION AND REFERENCES	9
5.1 Derivation	9
5.2 Reference	10
6. EXAMPLE	11
FIGURES 1a to 3	14-16

NORMAL FORCE AND PITCHING MOMENT OF NON-CIRCULAR BODIES: ELLIPTICAL CROSS-SECTIONS AT SUPERSONIC SPEEDS

1. NOTATION AND UNITS

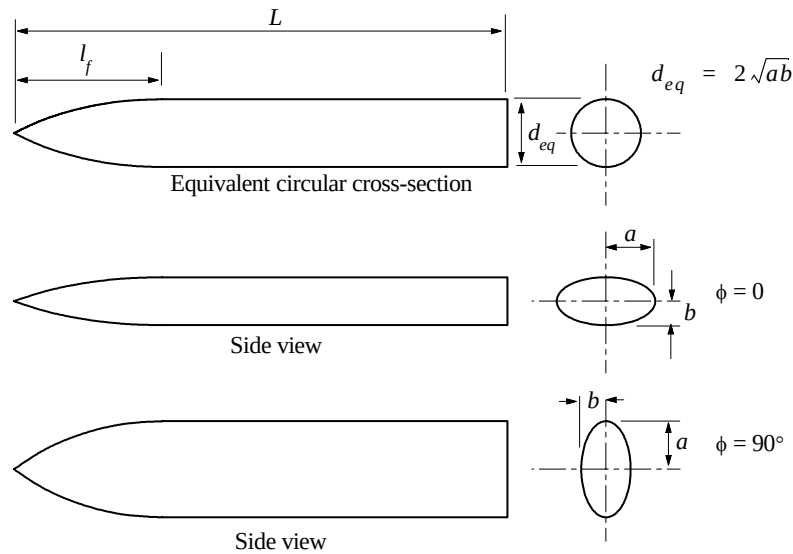
		<i>SI</i>	<i>British</i>
a	semi-major axis of ellipse, see Sketch 1.1	m	ft
a_f	local semi-major axis of ellipse on forebody	m	ft
b	semi-minor axis of ellipse, see Sketch 1.1	m	ft
b_f	local semi-minor axis of ellipse on forebody	m	ft
C_{Cf}	centroid of forebody planform area coefficient, $2x_{Pf}/l_f$		
C_{CL}	centroid of planform area coefficient, $2x_P/L$, see Equation (3.9)		
$(C_m)_0$	pitching moment coefficient, based on S_m and d_{eq} , about reference point $x_m = 0$, positive nose-up		
$(C_m)_{0\alpha}$	pitching-moment-curve slope, $d(C_m)_0/d\alpha$	rad ⁻¹	rad ⁻¹
C_m	pitching moment coefficient, based on S_m and d_{eq} , about reference point x_m , positive nose-up		
C_N	normal force coefficient, based on S_m		
C_{Nc}	cross-flow contribution to normal force coefficient of circular body, based on S_P		
$C_{N\alpha}$	normal-force-curve slope, $dC_N/d\alpha$	rad ⁻¹	rad ⁻¹
C_{Pf}	forebody planform area coefficient, $S_{Pf}/l_f d_{eq}$		
C_{PL}	planform area coefficient, S_P/Ld_{eq} , see Equation (3.8)		
d_{eq}	equivalent circular body diameter, $2\sqrt{ab}$, see Sketch 1.1	m	ft
F	tangent-ogive parameter, see Table 3.1		
l_f	forebody length, see Sketch 1.1	m	ft
L	overall length, see Sketch 1.1	m	ft
K_L	ratio of linear contribution to normal force of non-circular body to that of circular body with same cross-sectional area distribution, see Section 3.2		

Issued May 2005 – 16 pages

K_{NL}	ratio of non-linear contribution to normal force of non-circular body to that of circular body with same cross-sectional area distribution, see Section 3.3		
M_{∞}	freestream Mach number		
$R_{d_{eq}}$	Reynolds number based on freestream conditions and equivalent body diameter		
R_L	Reynolds number based on freestream conditions and body length		
S_m	maximum cross-sectional area of body, πab	m ²	ft ²
S_p	planform area of equivalent circular body	m ²	ft ²
S_{pf}	planform area of equivalent circular forebody	m ²	ft ²
x	distance aft of forebody apex, see Sketch 1.1	m	ft
x_{cp}	distance of centre of pressure position aft of forebody apex	m	ft
x_m	distance of moment reference point aft of forebody apex	m	ft
x_p	distance of centroid of planform area aft of forebody apex	m	ft
x_{pf}	distance of centroid of forebody planform area aft of forebody apex	m	ft
α	angle of attack	deg, rad	deg, rad
ϕ	body roll angle ($\phi = 0$ corresponds to major axis horizontal), see Sketch 1.1	deg	deg

Subscripts

$[]_L$	denotes linear contribution
$[]_{NL}$	denotes non-linear contribution



Sketch 1.1 Body geometry

2. INTRODUCTION

This Item presents a semi-empirical prediction method for normal force and pitching moment of forebody-cylinder combinations of elliptic cross-section at supersonic speeds. As the data available for bodies with elliptical sections are relatively limited both in number and scope, the method is based wholly on an adaptation of the prediction method for circular bodies given in Item Nos 89008 (Derivation 5), 89014 (Derivation 6) and their associated computer program in ESDUpac A9034, which itself is described in Item No. 90034 (Derivation 7). Those Items provide a prediction method applicable at angles of attack up to 90 degrees and Mach numbers up to 5.

Although the method is derived from test data for bodies with either tangent-ogival or conical profile forebodies, the method can, with care, be applied to other forebody profiles, see Section 4.

The method is presented in Section 3 and its accuracy and applicability are discussed in Section 4. The use of the method is illustrated in Section 6 by means of a worked example.

A prediction method for bodies of square and triangular cross-sections is given in Item No. 04007 (Reference 8).

3. METHOD

Derivation 3 presents a method for extending empirical prediction techniques for normal force and pitching moment of axisymmetric bodies to those of other cross-sectional shapes. Such methods generally assume that the forces and moments may be evaluated as the sum of two components: the first is a potential flow contribution, which dominates at low angle of attack, and the second a viscous cross-flow contribution which dominates at high angle of attack. These methods may readily be extended to deal with non-axisymmetric cross-sections by applying factors to both the linear and non-linear components, *i.e.*

$$C_N = K_L[C_N]_L + K_{NL}[C_N]_{NL} \quad (3.1)$$

and

$$(C_m)_0 = K_L[(C_m)_0]_L + K_{NL}[(C_m)_0]_{NL} \quad (3.2)$$

3.1 Linear and Non-linear Components $[C_N]_L$, $[C_N]_{NL}$, $[(C_m)_0]_L$ and $[(C_m)_0]_{NL}$

From Item Nos 89008 and 89014, the linear and non-linear contributions to normal force for a body of circular cross-section are given by

$$[C_N]_L = C_{N\alpha} \sin \alpha \cos \alpha \quad (3.3)$$

and

$$[C_N]_{NL} = \frac{4}{\pi} \frac{L}{d_{eq}} C_{PL} C_{Nc} \quad (3.4)$$

respectively.

The corresponding linear and non-linear contributions to pitching moment for a body of circular cross-section are given by

$$[(C_m)_0]_L = (C_m)_{0\alpha} \sin \alpha \cos \alpha \quad (3.5)$$

and

$$[(C_m)_0]_{NL} = -\frac{2}{\pi} \left(\frac{L}{d_{eq}} \right)^2 C_{PL} C_{CL} C_{Nc} \quad (3.6)$$

respectively.

In Equations (3.3) and (3.5) the normal-force-curve slope, $C_{N\alpha}$, and the pitching-moment-curve slope, $(C_m)_{0\alpha}$, are calculated from Item No. 89008 (or ESDUpac A9034) for the equivalent circular body, which is defined as the axisymmetric body with the same distribution of cross-sectional area. The diameter of the equivalent body, d_{eq} , is given by

$$d_{eq} = 2\sqrt{ab} \quad (3.7)$$

where a and b are the semi-major and semi-minor axes, respectively.

In Equations (3.4) and (3.6) C_{Nc} is the cross-flow contribution to normal force coefficient for a body of circular cross-section, based on S_P , and is presented in Item No. 89014. The coefficients C_{PL} and C_{CL} are the planform area coefficient (S_P/Ld_{eq}) and the centroid of planform area coefficient ($2x_P/L$), respectively, and may be calculated using Equations (3.8) and (3.9) with the forebody data for C_{Pf} and C_{Cf} given in Table 3.1, which covers a number of common forebody profiles. Data for other forebody profiles are given in Item No. 77028 (Derivation 4). The equations for C_{PL} and C_{CL} are

$$C_{PL} = (C_{Pf} - 1)(l_f/L) + 1 \quad (3.8)$$

and

$$C_{CL} = \frac{(C_{Pf} C_{Cf} - 1)(l_f/L)^2 + 1}{(C_{Pf} - 1)(l_f/L) + 1} \quad (3.9)$$