



Extrapolating wind-tunnel data to full-scale Reynolds number. Part 1: Principles

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EXTRAPOLATING WIND-TUNNEL DATA TO FULL-SCALE REYNOLDS NUMBER. PART 1: PRINCIPLES

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EXTRAPOLATING WIND-TUNNEL DATA TO FULL-SCALE REYNOLDS NUMBER.

Part 1: Principles

1. INTRODUCTION

Prediction of the flow around a full-scale aircraft from wind-tunnel tests on a smaller scale model is subject to a number of difficulties. Some of these are caused by obvious differences between the two situations, such as the presence of constraining walls in the wind tunnel. Other difficulties are caused by a failure to achieve the equality in non-dimensional parameters, such as Mach number and Reynolds number, which is essential to ensure equivalence between the model and full-scale cases.

As Reynolds number is the only significant non-dimensional parameter to involve the scale of the model, the use of a model of reduced scale normally makes it difficult to achieve equality in Reynolds number while matching other non-dimensional parameters. In particular this means that there are likely to be discrepancies in the boundary-layer state between the full-scale and sub-scale test cases. The effects of this mismatch may be exacerbated by such factors as the geometric complexity of the configuration under investigation, the degree of supercritical flow present, the strength and location of any shock waves and the presence of separated flow. The problem is, therefore, likely to be particularly significant in the transonic flow regime.

In order to formulate the requirements for and interpret the ensuing results of a wind-tunnel test it is essential to have a proper understanding of the factors that influence boundary-layer behaviour, particularly with regard to transition and separation. Thus an awareness of possible discrepancies between the sub-scale and full-scale cases may be developed and, where possible, suitable corrections made to wind tunnel data.

A further difficulty in the assessment of Reynolds number effects is caused by the fact that they may be confused with other effects inherent in wind-tunnel testing. For example, the boundary-layer growth on the wind-tunnel walls causes an effective change in the working section area, which varies with the state of the wall boundary layer. This effect may then be misinterpreted as a genuine Reynolds-number effect. Similar problems may arise if, for example, the Reynolds number is controlled by varying the tunnel stagnation pressure. In this case the change in dynamic pressure may cause significant model deflections. A third example is the effect on the boundary-layer that may occur if the model is at a significantly different temperature from that of the freestream.

Interpretation of these “pseudo Reynolds number” effects requires a broad understanding of the wind tunnel testing process. For this reason some care is taken in this Data Item to examine the assumptions implicit in the establishment of the non-dimensional parameters, on which rests the concept of dynamic similarity. Next there is a broad classification of the discrepancies between wind tunnel and full scale, followed by a review of the mechanisms involved in boundary-layer transition and separation. Finally a detailed review of “pseudo Reynolds number” effects is given.

Although much of this Data Item is applicable to general conditions on aircraft configurations, the practical emphasis is on transonic flows around aircraft having lifting surfaces of moderate sweep ($\leq 35^\circ$) and aspect ratio ≥ 4 .

2. NOTATION

A	wave amplitude
a	speed of sound
A_N	wave amplitude at neutral stability point (see Section 5.1.1)
$B_{1...n}$	non-dimensional groups relating to body forces
b	span
C_p	pressure coefficient
C_L	lift coefficient
c	chord of aerofoil, or local wing chord
$\bar{c}$	wing mean chord
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
d_e	excrescence width, normal to local flow direction
e	excrescence height
$\vec{f}$	body force vector
H	specific enthalpy
k	thermal conductivity
l	characteristic length
M	Mach number
M_{shock}	Mach number immediately upstream of shock wave on aerofoil
N	Tollmien-Schlichting wave amplification exponent (Equation (5.2))
p	pressure
p_{out}	pressure at outlet of supersonic nozzle (Sketch 3.1)
$\vec{q}$	velocity vector
R	gas constant
Re	Reynolds number, ($= U_\infty \bar{c} / \nu$ for wing, or $= U_\infty c / \nu$ for aerofoil)

Re_e	excrescence Reynolds number, ($= u_e e / \nu$)
Re_l	Reynolds number, ($= U_\infty l / \nu$)
Re_x	Reynolds number based on distance, x , from leading edge of flat plate, ($= U_\infty x / \nu$)
$Re_{s_{stag}}$	Reynolds number based on distance from stagnation point, ($= U_\infty s_{stag} / \nu$)
Re_{x_e}	Reynolds number based on distance of excrescence from leading edge of flat plate and velocity at edge of boundary layer, ($= u_{\delta} x_e / \nu$)
Re_{δ_e}	Reynolds number based on height of excrescence and velocity at edge of boundary layer, ($= u_{\delta} e / \nu$)
Re_θ	Reynolds number based on momentum thickness, ($= U_\infty \theta / \nu$)
Re_{δ^*}	Reynolds number based on displacement thickness, ($= U_\infty \delta^* / \nu$)
Re_{δ_T}	flat plate transition Reynolds number, ($= u_{\delta} x_T / \nu$)
r_0	surface radius of curvature in streamwise direction
S_t	wind tunnel working section cross-sectional area
S_{ref}	reference area
s_{stag}	distance measured from stagnation point
T	temperature
T_u	freestream turbulence intensity, ($= \sqrt{u'^2} / U_\infty$)
T_δ	non-dimensional turbulence intensity, ($= (\overline{u'^2} + \overline{v'^2} + \overline{w'^2})^{1/2} / (3u_\delta)$)
t	time variable
$\hat{t}$	non-dimensional time variable, ($= t U_\infty / l$)
U_∞	freestream velocity
u, v, w	velocity components in x, y and z directions respectively
u_e	local velocity at excrescence height, within boundary layer
x, y, z	cartesian coordinates
x_e	distance from leading edge of flat plate to location of excrescence

x_n	distance from leading edge at which Tollmien-Schlichting instability occurs
x_{pg}	distance from beginning of pressure gradient to separation point
x_{shock}	distance from aerofoil leading edge to origin of sidewall shock wave
x_T	distance from leading edge of flat plate to full transition (fully turbulent flow)
$\hat{x}$	non-dimensional coordinate, ($= x/l$)
$\hat{y}$	non-dimensional coordinate, ($= y/l$)
$\hat{z}$	non-dimensional coordinate, ($= z/l$)
ΔM	Mach number error (Equation (8.1))
Y	strain related dissipation term in energy equation (Equation (3.5))
α	incidence, or non-dimensional wave number, ($\alpha = 2\pi\delta^*/\lambda$)
β	angle between shock wave and upstream velocity vector
γ	ratio of specific heats, ($= \frac{c_p}{c_v}$)
δ	boundary layer thickness
δ^*	boundary layer displacement thickness
θ	boundary layer momentum thickness
θ_s	change in flow direction through shock wave
Λ	wing leading edge sweep angle (see Sketch 5.12)
λ	wavelength of Tollmien-Schlichting wave
μ	shear coefficient of viscosity
μ_b	bulk coefficient of viscosity
ν	kinematic viscosity, ($= \frac{\mu}{\rho}$)
ρ	density
σ	relative humidity
τ_w	shear stress at wall

Other Subscripts

<i>attachment line</i>	denotes value relating to attachment line at wing leading edge
<i>N</i>	denotes Reynolds number for neutral stability
<i>T</i>	denotes value at which transition to fully turbulent flow occurs
<i>crit</i>	denotes critical value
<i>crit1</i>	denotes value at which a vortex street is first generated by an excrescence
<i>crit2</i>	denotes value at which a fully turbulent wake is generated by an excrescence
<i>eq</i>	denotes wall equilibrium value
<i>plen</i>	denotes value in plenum chamber
<i>stag</i>	denotes stagnation value
<i>s_{stag}</i>	denotes distance from stagnation point
<i>w</i>	denotes value at wall
δ	denotes value at edge of boundary layer
∞	denotes freestream value
1	denotes value upstream of shock wave
2	denotes value downstream of shock wave
<i>mean N to T</i>	denotes average value between neutral stability and onset of transition

Superscripts

'	denotes non-steady velocity or component thereof (e.g. $u' = (u - \bar{u})$)
-	denotes time averaged quantity (e.g. $\bar{u} = \frac{1}{2\Delta t} \int_{t-\Delta t}^{t+\Delta t} u \cdot dt$, where Δt is large compared with the period of the lowest frequency under consideration)

3. DYNAMIC SIMILARITY IN INCOMPRESSIBLE AND COMPRESSIBLE FLOWS

3.1 Dimensional Analysis

Standard texts, *e.g.*^{1,2}, introduce the concepts of dimensional analysis and dynamic similarity by appealing to the fact that any expression relating physical quantities must be dimensionally consistent. For example, expressions describing the behaviour of variables in a system governed by Newtonian physics must be homogeneous in three basic dimensions, such as (mass, length and time) or (force, length and time) *etc.* The parameters relevant to the problem are usually chosen by an intuitive appreciation of the physical basis of the problem.

To illustrate this, consider the classic problem of steady flow past a body immersed in an incompressible freestream. The variable under examination is the pressure $p(x, y, z)$. The relevant parameters may be chosen as the freestream speed, U_∞ , density ρ , viscosity μ and a characteristic dimension, l . Dimensional analysis, using the methods of^{1,2}, gives:

$$\frac{p(\hat{x}, \hat{y}, \hat{z})}{\rho U_\infty^2} = f\left(\frac{\rho U_\infty l}{\mu}\right) = f(Re_l). \quad (3.1)$$

This is usually written alternatively in terms of pressure coefficient:

$$C_p(\hat{x}, \hat{y}, \hat{z}) = \frac{p - p_\infty}{\frac{1}{2}\rho U_\infty^2} = f(Re_l). \quad (3.2)$$

In this expression the pressure coefficient and the Reynolds number are both non-dimensional and the pressure coefficient variation is specified in terms of a coordinate system $(\hat{x}, \hat{y}, \hat{z})$ scaled with respect to the characteristic length, l . There is a direct equivalence between two flows around geometrically similar bodies (subject to the stated restrictions) which have the same freestream direction and Reynolds number, even though the values of U_∞ , l , ρ and μ may differ between these flows.

A preferable (to intuition) method of selecting the relevant parameters for inclusion in the dimensional analysis is by appeal to the governing equations. In fluid dynamics these are well known, although an analytic solution to a particular problem may not exist. A proper examination of these equations, and the assumptions implicit in their formulation, promotes a clearer understanding of the appropriate parameters to be included in the dimensional analysis and of the significance of the resulting dimensionless groups. This approach is adopted here. The complete governing equations are extremely complex. Care is taken to introduce them in a general form but details of derivation and the structure of particular terms are not presented, as these details are not essential for the purpose of this Data Item.

3.2 Examination of the Governing Equations

If air flows past an object which is large compared with the mean free path of the molecules of which the air is comprised, then the dynamics of the individual molecules may be ignored and the air treated as a continuous distribution of matter in space, or *continuum*. Since the mean free path in the Standard Atmosphere³ is only 6.68×10^{-7} m even at an altitude of 18000 m, this assumption may reasonably be made for conventional aircraft. The motion of such a continuum is governed by partial differential equations expressing the conservation of mass and momentum. In the case of a Newtonian fluid (defined as having strain-independent coefficients of viscosity, μ , μ_b) these equations are⁴:

the Navier-Stokes equation:

$$\frac{d\vec{q}}{dt} = -\frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2\vec{q} + \frac{(\mu_b + \frac{1}{3}\mu)}{\rho}\nabla(\nabla\cdot\vec{q}) + \vec{f} \quad (3.3)$$

(where the total derivative) $\frac{d\vec{q}}{dt} = \frac{\partial\vec{q}}{\partial t} + (\vec{q}\cdot\nabla)\vec{q}$

and the continuity equation:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{q}) = 0 \quad (3.4)$$

The Navier-Stokes Equation (3.3) (sometimes referred to as the momentum equation) expresses the momentum changes due to body and boundary forces. The second and third terms on the right hand side of Equation (3.3) are the “viscosity” terms, caused by molecular diffusion, and are discussed further in Section 3.3.2.2. The continuity Equation (3.4) simply expresses the conservation of mass.

Expanding the vector form of the Navier-Stokes Equation (3.3) gives three momentum equations, corresponding to the three coordinate directions. Together with the continuity Equation (3.4) and appropriate boundary and initial conditions, these equations may be solved (although not, in general, analytically) for the three components of the velocity, $\vec{q}$, and the pressure, p , *provided that the flow is incompressible* (i.e. ρ is constant).

In the case of a compressible flow, other information is required, since ρ is no longer constant. The energy equation⁴ is therefore introduced:

$$\frac{dH}{dt} = \frac{1}{\rho}\frac{dp}{dt} + \frac{1}{\rho}Y + \frac{1}{\rho}\nabla\cdot(k\nabla T) \quad (3.5)$$

This equation expresses the change of internal energy in the fluid caused by heat transfer and boundary stresses. The term, Y , is the viscous energy dissipation governed by the coefficients of viscosity μ and μ_b . The last term deals with the energy exchange due to thermal conductivity. The significance of μ_b is discussed further in Section 3.3.2.2.

For a perfect gas the specific enthalpy may be written:

$$H = c_p T \quad (3.6)$$

The introduction of the temperature, T , both here and in the heat conduction term, requires the addition of the equation of state:

$$\frac{p}{\rho} = RT \quad (3.7)$$

For a perfect gas, R is constant. If c_p is also constant, the gas is calorically perfect. These conditions are closely approximated by dry air at temperatures and pressures typical of the operating conditions for conventional aircraft.

Because of the introduction of relationships relating to thermodynamic processes in addition to Newton's laws, it appears that the list of basic dimensions should be extended to become mass, length, time and temperature. The introduction of these two equations also implies that the additional gas properties, R and c_p , should be included in the list of relevant parameters in the dimensional analysis.

However, since $R = (c_p - c_v) = c_p \left(\frac{\gamma - 1}{\gamma} \right)$ and the speed of sound $a^2 = \gamma RT$:

$$H = \frac{a^2}{\gamma - 1}. \quad (3.8)$$

Also, if the thermal conductivity, k , is a constant, the last term in the energy equation (3.5) can be written as $\frac{1}{\rho} \nabla \cdot (\nabla(kT))$ where

$$kT = \frac{ka^2}{\gamma R}. \quad (3.9)$$

Since these are the only two terms containing temperature in the governing equations, the speed of sound, a , can be used as a variable instead of T . This reduces the list of fundamental dimensions back to the original mass, length, time, as a can be described solely in terms of length and time. The relevant fluid property relating to heat transfer then becomes, $\frac{k}{\gamma R}$ and that relating to H is γ .

For the flow of a uniform air stream around a given shape, Equations (3.3), (3.4), (3.5), (3.7), (3.8) and (3.9) determine the distribution of the values of the variables $\vec{q}$, p , ρ and a over the flow field. To define the problem fully, further information is required. First it is necessary to specify boundary conditions at any solid surface. These are the zero velocity ("no slip") condition, and conditions sufficient to determine heat transfer rates at the surface. Unless there is zero heat transfer at the surface (adiabatic surface conditions) this requirement makes it necessary to introduce the surface temperature, T_w . Next, the freestream conditions U_∞ , ρ_∞ , a_∞ are required. T_∞ must also be specified if the solid boundaries are not adiabatic. In this case the temperature must now be introduced as a fourth dimension, extending the list of fundamental dimensions to *mass, length, time and temperature*. However, this is only necessary if heat transfer at the walls is of significance and gives rise to the addition of a single additional group, a temperature ratio, in the dimensional analysis (Equation (3.10)). Finally the fluid properties μ , μ_b , $\frac{k}{\gamma R}$, γ and the characteristic length, l must be specified. In the case of an unsteady flow, appropriate initial conditions may also be required. Using the parameters in the dimensional analysis gives, for the pressure coefficient in compressible flow*:

$$C_p(\hat{x}, \hat{y}, \hat{z}, \hat{t}) = f \left(Re_l, M_\infty, \gamma, \frac{\mu_b}{\mu}, \frac{(k/\gamma R)}{\mu}, \frac{T_w}{T_\infty}, B_1, B_2, \dots \right)$$

* As this expression is generalised for unsteady flows, C_p depends on the non-dimensional time parameter $\hat{t} = (tU_\infty/l)$ in addition to the three space-coordinates $\hat{x}, \hat{y}, \hat{z}$. Thus $\hat{t}$ is included on the left hand side of this equation and in Equation (3.10). The groups B_n relate to the body forces, $\vec{f}$, and will depend on the nature of the body force, e.g. gravitational, electromagnetic etc. (See Section 3.3.1)

Since γ already appears on the right hand side of this equation and $R = c_p \left(\frac{\gamma-1}{\gamma} \right)$, the non-dimensional group $\frac{(k/\gamma R)}{\mu}$ may be replaced with the more familiar Prandtl number $\frac{\mu c_p}{k}$, so that:

$$C_p(\hat{x}, \hat{y}, \hat{z}, \hat{t}) = f \left(Re_l, M_\infty, \gamma, \frac{\mu_b}{\mu}, \frac{\mu c_p}{k}, \frac{T_w}{T_\infty}, B_1, B_2, \dots \right). \quad (3.10)$$

3.3 Simplifying the Governing Equations

Certain simplifying assumptions have already been made in the governing equations; for example the assumption that the fluid is a continuum and that the thermal conductivity, k , is constant, as are γ and R . It is also assumed that the coefficients of viscosity, μ and μ_b are constant, although it is known that μ typically depends on temperature (Section 3.3.2.2). In this section, terms in Equation (3.10) are examined with regard to their relevance to the flow fields encountered in typical aeronautical applications. This allows further simplifying assumptions to be made while identifying those respects in which the finally adopted model differs from the exact solution.

3.3.1 Body forces

Normally the only body forces of interest in the flow of air around a conventional aircraft are those due to gravity. Examination of orders of magnitude for the terms in Equation (3.5) indicates that the body force contribution is typically less than 0.05% of the dp/dt term. The contribution of the body forces is similarly insignificant in the momentum equation (Equation (3.3)). In most practical cases the B_n terms in Equation (3.10) may be therefore ignored.

3.3.2 Molecular-diffusion effects

In air, the primary physical reason for the existence of thermal conductivity and viscosity is molecular diffusion between adjacent “layers” of fluid. Thermal conductivity is caused by the diffusion of molecules across a temperature gradient and viscous stresses by the diffusion of molecules within a shear layer or boundary layer.

3.3.2.1 Thermal conductivity

The thermal conductivity of air is 0.02534 W/mK at Standard sea level conditions⁵. The heat transfer term $\nabla \cdot (k \nabla T)$ in Equation (3.5) depends on the second derivative of temperature, T , or, from Equation (3.9) the second derivative of a^2 . Again, examination of the orders of magnitude for Equation (3.5) shows that, for transonic flow, the heat conduction terms are typically of the order of 0.001% of the dp/dt term. This is because of the low values of both k and the second derivatives of temperature. The contribution of the thermal conductivity terms is therefore negligible except in cases where the temperature difference between a boundary, such as a solid wall, and the surrounding flow is significant, or where shock waves are present. In the latter case the second derivative of the temperature across the shock wave is very high. However, because the thickness of the shock wave is comparable to the mean free path of the molecules, the assumption of continuum flow is violated and the set of differential Equations, (3.3), (3.4) and (3.5) must be replaced by the oblique shock wave equations (Section 3.3.3) which relate flow parameters across a shock wave. Thus the thermal conductivity may be neglected in regions in which the differential equations are valid. An exception to this is where solid surfaces have a temperature significantly different from that of the local flow. This is referred to again in Section 4.2.1

3.3.2.2 Coefficients of viscosity, μ and μ_b

The coefficient of viscosity, μ , is associated with the shear stresses on a fluid element caused by molecular diffusion across the element boundary. These stresses arise when there is a velocity gradient normal to the direction of flow. For dry air, under Standard sea level conditions⁵, the value of μ is 17.894×10^{-6} Ns/m².

The bulk coefficient of viscosity, μ_b , is associated with the normal stresses, additional to those due to the equilibrium pressure, which are produced by molecular diffusion across the fluid element boundary. These stresses arise because of the rate of dilation, and the mean of their principal components is equal⁴ to $\frac{\mu_b}{\rho} \frac{d\rho}{dt}$. Because these stresses are only significant when very high rates of dilation are present, μ_b is very difficult to measure experimentally⁴, but arguments based on the kinetic theory of gases indicate⁴ that it is of similar magnitude to μ .

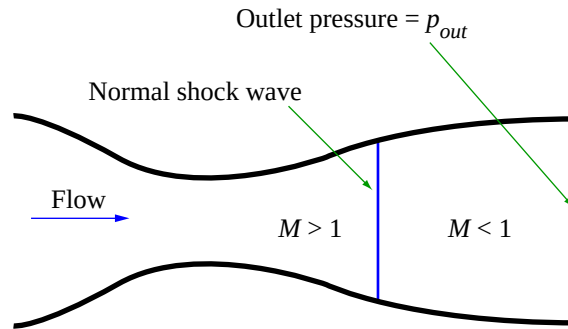
The coefficient of viscosity, μ , is temperature dependent and varies according to Sutherland's law⁵. Strictly interpreted, this would modify Equations (3.3) and (3.5) to include the derivative terms of μ and, similarly other properties, such as μ_b . However these derivative terms are of little significance and are usually ignored.

Because the coefficients of viscosity are very small, the viscous terms are only of significance in Equation (3.3) when the associated velocity derivatives are very high. Generally this condition only occurs for the second term on the right hand side of Equation (3.3) in boundary layers or shear layers. The third term is primarily concerned with volume dilation rates. This term is generally insignificant but becomes large across a shock wave. However, this region must be determined using the oblique shock wave equations (Section 3.3.3). Similar arguments to those given in Section 3.3.2.1 allow terms involving μ_b to be discounted in the energy Equation (3.5).

3.3.3 Oblique shock wave equations

Across a shock wave the governing differential equations must be replaced by the oblique shock wave equations. These equations express mass and energy continuity and momentum balance across the shock considered as a discontinuity and are a generalisation of the Rankine-Hugoniot equations⁶. Examination of the full oblique shock wave equations⁶ shows that the ratios p_2/p_1 , ρ_2/ρ_1 , M_2/M_1 and a_2/a_1 together with the local inclination, β , of the shock wave to the upstream velocity vector, may all be written in the form $f(M_1, \gamma, \theta_s)$. The oblique shock wave equations thus provide a link between the values of p , ρ , M and a in the two regions of continuum on either side of the shock wave. The change in flow direction, θ_s , across the shock wave is defined by the change in the velocity vector from $\vec{q}_1$ to $\vec{q}_2$, whilst the variation of $\vec{q}$ within the continuous regions is defined by Equations (3.3) to (3.5).

To obtain a solution to the governing equations, the introduction of shock waves may be required in order to allow the boundary conditions to be satisfied. A simple example of this is the one-dimensional supersonic flow in a nozzle, where the position of the normal shock wave in the duct is determined by the requirement for the pressure at the duct exit to equal the value, p_{out} , imposed as a boundary condition on the exit plane (Sketch 3.1).



Sketch 3.1 Formation of Normal Shock Wave in a Supersonic Duct

Since the oblique shock wave equations do not introduce any new parameters, Equation (3.10) applies even when shock waves are present.

Although the flow mechanism within the shock wave results in a significant increase in entropy, neither viscosity nor heat transfer coefficients appear in the oblique shock wave equations. Thus the simplifications introduced in Sections 3.3.1 and 3.3.2 still apply.

3.4 Simplified Dimensional Analysis and Dynamic Similarity

The oblique shock wave equations are used to replace the governing differential equation across any shock wave. This means that there are no regions in which thermal conductivity, k , and the coefficient of viscosity, μ_b , are of importance. Further, it is established in Section 3.3.1 that the body forces, B_n , are insignificant. These parameters may now be omitted and Equation (3.10) reduces to:

$$C_p(\hat{x}, \hat{y}, \hat{z}, t) = f(Re_l, M_\infty, \gamma). \quad (3.11)$$

Thus, to obtain an equivalence between the flows around two geometrically similar objects, not only must the previously stated assumptions regarding the governing equations be valid but the Reynolds number, Re_l , the Mach number, M_∞ , and ratio of specific heats, γ , must be identical in both cases. In addition the boundary and initial conditions must also be equivalent. If all these conditions are met, the two flows are said to be *dynamically similar*.

4. DISCREPANCIES BETWEEN WIND-TUNNEL AND FULL-SCALE RESULTS

Discrepancies between wind-tunnel and full-scale results may be caused by a failure to ensure dynamic similarity or by other factors such as instrumentation errors. In this Data Item, only problems caused by failure to achieve dynamic similarity are addressed.

From the analysis in Section 3 it is seen that failure to achieve dynamic similarity can have three basic causes:

- violation of assumptions in governing equations,
- failure to match boundary (or initial) conditions,
- failure to match the non-dimensional parameters, Re_l , M_∞ or γ .

These are discussed further in Sections 4.1, 4.2 and 4.3.

4.1 Assumptions in Governing Equations

The assumptions in the governing equations are not limiting in tests up to supersonic speeds. An exception to this is the effect of humidity. Not only does humidity modify the value of the ratio of specific heats, γ , but latent heat may provide a significant additional term in the energy equation (Equation (3.5)) due to possible condensation and also subsequent evaporation. Other factors, such as energy release due to molecular dissociation, are not significant until hypersonic speeds (M_∞ greater than approximately 6) are reached. Humidity effects are discussed further in Section 8.3.3.

4.2 Matching Boundary and Initial Conditions

4.2.1 Boundary conditions

For a subsonic freestream it is not possible to match the boundary conditions between the “free-flight” case and a wind-tunnel test. In the “free-flight” case, the boundary conditions are applied at an infinite distance from the vehicle while, in the case of a wind-tunnel test, the best that can be done is to ensure uniform conditions at the entry to the working section with the tunnel empty. In all tunnel tests there is also an unsteady component in U_∞ due to turbulence in the tunnel circuit. Steps may be taken to minimise this but it cannot be eliminated. A further complication is that flow in the working section of a wind tunnel is bounded by solid walls, a constant pressure boundary (in an ‘open-jet’ wind tunnel) or, in the case of a transonic tunnel, perforated or slotted walls.

The above features are inherent in wind-tunnel testing. The other important part of the boundary representation is the model itself. Because of the small scales used in most wind tunnel tests, it is not possible to reproduce fine detail that may have a significant effect on the flow, particularly in the boundary layer. Generally, too, the model may deflect under aerodynamic load in a manner that is unrepresentative of the full-scale aircraft.

In Section 3.3.2.1 the significance of heat transfer from a solid surface is mentioned. This can be important, particularly in intermittent tunnels where there may be substantial differences between the temperatures of the air and the model surface.

4.2.2 Initial conditions

In the majority of cases, wind-tunnel tests are conducted with constant freestream conditions and, although a naturally unsteady flow (such as a wake oscillation) may result, specific matching of initial conditions is not required. In some cases, such as store release, time dependency is an essential element of a test. In this particular example, the initial conditions are self evident, although additional scaling parameters have to be introduced into Equation (3.10) in order to account for the dynamics of the store.

4.3 Matching Non-dimensional Parameters

4.3.1 Matching γ

According to kinetic theory⁶, γ is dependent on the number of degrees of freedom of each molecule. According to these assumptions, for a gas comprising monatomic molecules, $\gamma = 1.67$, for diatomic molecules $\gamma = 1.40$ and for triatomic molecules $\gamma = 1.33$. Air is comprised mainly of diatomic molecules (O_2 and N_2) and consequently $\gamma \approx 1.40$. Close matching in γ is therefore obtained in tunnels using dry air or any other diatomic gas, such as nitrogen, which is frequently used as the working fluid in cryogenic wind tunnels.

4.3.2 Matching Re_l and M_∞

In any flow in which compressibility effects are important and shock waves are likely to be present, it is essential to match Mach number between tests at sub-scale and full scale. The difficulty in obtaining simultaneous Reynolds and Mach number matching in most conventional wind tunnels is well known. Where there is little or no control over the density or the coefficient of viscosity, the only way in which equality of Reynolds and Mach number may be maintained between the wind-tunnel test and full scale is to increase the freestream speed in inverse proportion to the model scale. This leads to an unacceptable mismatch in Mach number. Simultaneous Reynolds and Mach number matching may be achieved by the use of pressurised tunnels, which enable the density to be controlled, or cryogenic tunnels in which the air is cooled to allow a simultaneous reduction in viscosity and increase in density. The use of wind tunnels of this type confers an obvious advantage in the prediction of full-scale behaviour. However economic considerations often dictate that a significant proportion of a test program must be conducted in less expensive facilities, where a mismatch in Reynolds number between test and full scale must be accepted.

4.3.3 Dependence of the boundary layer on Reynolds number

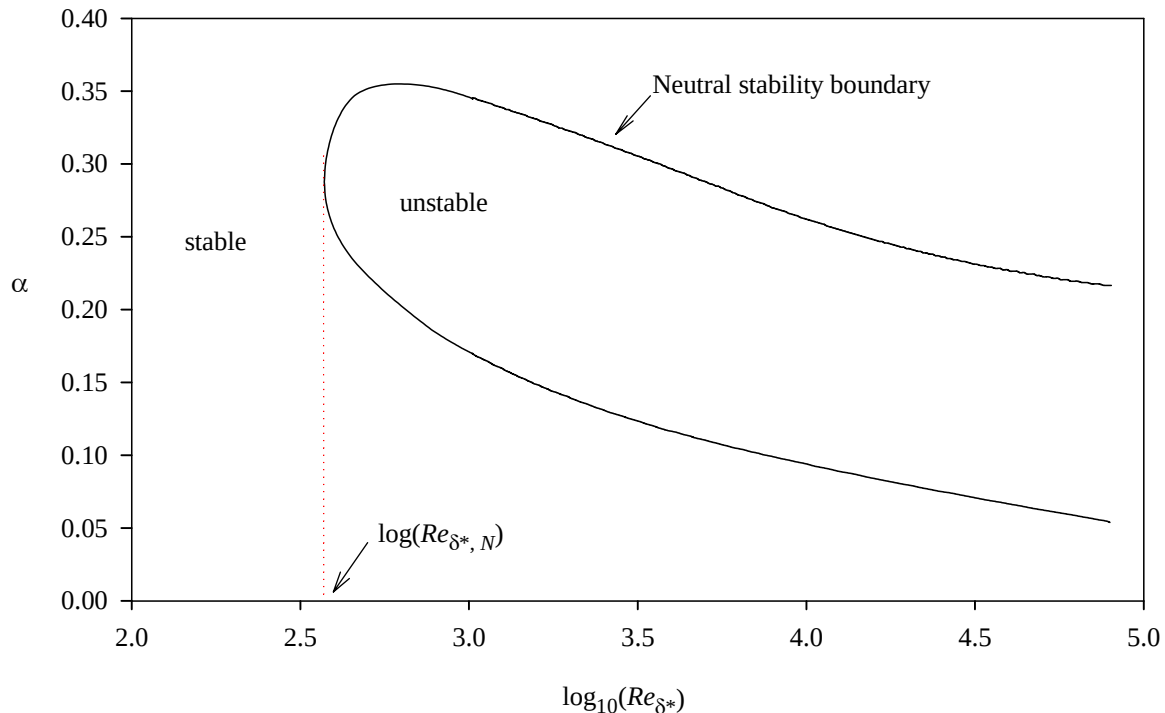
In Section 3.3.2.2 it is seen that viscous stresses are significant only in boundary layers and shear layers where large second derivatives in velocity, normal to the local stream direction, are present. These conditions are met in the boundary layer close to a solid surface. Across this layer, there is a rapid increase in velocity from zero at the surface to the edge of the layer, where viscous stresses become negligible. It is known that the flow in the boundary layer may be laminar or turbulent and that it may also separate from the surface, forming a separation bubble or, if reattachment does not take place, a wake. The state of the flow within the boundary layer can thus have a very important bearing both on local flow near the surface and in the wider flow field. Both Reynolds number and external flow characteristics, such as freestream turbulence, influence transition from laminar to turbulent flow in the boundary layer and the separation location. The streamwise pressure gradients, external to the boundary layer, are also of critical importance and are particularly important at transonic speeds when shock waves occur. A number of different mechanisms may be involved in transition and separation, so that accounting for discrepancies between model tests and full scale may be difficult. A brief review of the transition and separation processes, partly based on the account of Young⁷, is given in Sections 5 and 6 to help in the subsequent analysis of these discrepancies.

5. BOUNDARY-LAYER TRANSITION

Generally wind-tunnel tests are made at a Reynolds number considerably below the full-scale value and in an air stream with significant turbulence and acoustic noise. This is likely to lead to substantial differences in the boundary layer transition location between the sub-scale and full-scale cases, unless transition is fixed by artificial means. In a wind tunnel test at low Reynolds number, transition locations on a wing are, typically, significantly further downstream, in terms of non-dimensional distance, than is the case at full scale. It is also likely that the transition mechanisms differ between the two cases. Therefore, a correct interpretation of wind-tunnel results necessitates an understanding of the nature of these mechanisms and the factors that influence them.

The process of transition from laminar to turbulent flow within the boundary layer is caused by the growth of small disturbances, which extract energy from the surrounding fluid stream. The disturbances may originate from a number of sources and various factors may influence their damping or rate of amplification. In Sections 5.1 to 5.1.3 the transition mechanisms for two-dimensional boundary layers are discussed while Sections 5.3 to 5.5 relate to transition in three-dimensional boundary layers, with particular emphasis on transition in the flow over a swept wing.

5.1 Boundary-Layer Instability – Tollmien-Schlichting Waves

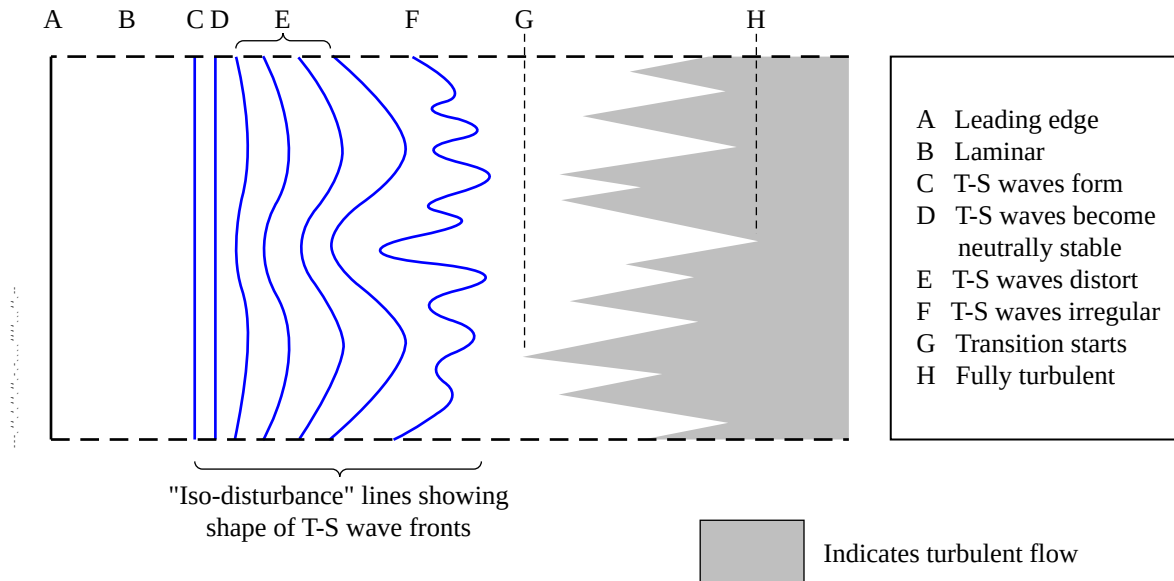


Sketch 5.1 Stability of boundary layer on a flat plate for disturbances having a range of non-dimensional wave number, α (derived from Reference 7)

For a two-dimensional boundary layer on a smooth flat plate, theoretical analysis^{7,8} shows that small sinusoidal disturbances (known as Tollmien-Schlichting waves) are amplified, at some wave numbers α , if the Reynolds number is above a critical value, ($Re_{\delta^*} \approx 500$) (Sketch 5.1). The results shown in Sketch 5.1 are derived using the Orr-Sommerfeld equation⁷, which defines the stability criterion for small disturbances. The growth of these disturbances with time, at a fixed location, is examined (temporal analysis) as this simplifies the analysis. A similar result is obtained if the development of disturbances in the downstream direction is analysed for a flow with steady mean characteristics (spatial analysis).

5.1.1 Development and prediction of transition

When stability is neutral, the boundary layer does not become turbulent immediately. Further downstream the Tollmien-Schlichting waves distort progressively and turbulent spots appear, which coalesce in the transition region to form fully turbulent flow (G to H in Sketch 5.2).



Note: The geometric form of the T-S wave distortion (D to F) and the turbulent breakdown (G to H) vary with time.

Sketch 5.2 Development of transition from Tollmien-Schlichting (T-S) instability (derived from Reference 7)

If the extent of the transition region is small, the transition position, at which the boundary layer becomes fully turbulent, may be estimated by the use of simple empirical correlations. One example is the empirical method due to Michel⁹, which was obtained using a correlation between the Reynolds number based on the boundary-layer momentum-thickness at transition, $Re_{\theta,T}$, and that based on distance from the stagnation point $Re_{s_{stag}}$:

$$Re_{\theta,T} = 1.174 \left(1 + \frac{2400}{Re_{s_{stag}}} \right) Re_{s_{stag}}^{0.46} \quad (5.1)$$

This correlation has been shown to work well for a number of aerofoils but its performance in compressible flows is uncertain.

An alternative method that is commonly used to predict the onset of transition due to Tollmien-Schlichting waves is linear stability analysis combined with the e^N method^{10,11}, in which it is assumed that the amplitude of the waves increases with $(x - x_N)$ as:

$$\frac{A}{A_N} = e^N \quad (5.2)$$

The variation of N with Reynolds number, for different wave numbers, can be derived using the Orr-Sommerfeld equation and the envelope to these curves is plotted on Sketch 5.3.