



ESDU 06002

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Dwell-Rise-Fall-Dwell and Dwell-Fall-Rise-Dwell cam laws Sections 1 to 8: segments having equal rise and fall periods

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The Mechanisms Committee, which first met in 1975, is continuing the work that previously was the responsibility of the former Gearing and Mechanisms Committee, which first met in November 1964. Like the earlier Committee it has the cooperation of many engineers and scientists in industry, research establishments and universities from whom much assistance and information is being received.

DWELL-RISE-FALL-DWELL AND DWELL-FALL-RISE-DWELL CAM LAWS. SECTIONS 1 TO 8: SEGMENTS HAVING EQUAL RISE AND FALL PERIODS

Data Items in the series on Dwell-Rise-Fall-Dwell cam laws are:

Item 06002 ^[15] - Sections 1 to 8: segments having equal rise and fall periods,

Item 06003 ^[16] - Sections 9 to 13: segments having asymmetric rise and fall periods,

Item 06004 ^[17] - Sections 14 to 23: examples,

Item 06005 ^[18] - Computer program.

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DWELL-RISE-FALL-DWELL AND DWELL-FALL-RISE-DWELL CAM LAWS. SECTIONS 1 TO 8: SEGMENTS HAVING EQUAL RISE AND FALL PERIODS

1. NOTATION AND UNITS

The units used in this Item are predominantly SI but any coherent system of units may be used. All angles are measured positively in an anticlockwise sense and the cam is considered to rotate in an anticlockwise direction.

		Units
A, a, b, c	coefficients of SCCA cam laws (see Appendix C)	—
A_1, A_2, A_3	coefficients of the Triple Harmonic Motion cam law	—
D	distance from cam centre, O, to pivot point of swinging arm, measured in positive sense to right of cam centre in first quadrant	m
E	offset of follower line-of-action from cam centre, O, measured in positive sense to right of cam centre in first quadrant	m
H	total lift of translating follower corresponding to one motion segment	m
k	constant of value +1 or -1, taking the sign of D (see Equations (4.53c) and (4.54))	—
k_A	acceleration ratio (see Equations (4.2a) and (4.2b))	—
k_B	blending ratio (see Equations (4.1), (4.5) and (4.29))	—
L	distance between follower arm pivot and follower datum centre	m
N	cam speed	rev/min
N_1	component blending with dwell	—
N_2	component controlling reversal of velocity	—
O	cam centre of rotation and origin of co-ordinate axis system	—
R_a	prime circle radius	m
R_b	reference circle radius	m
R_r	roller radius	m
u	normalised cam angle (see Equations (3.2a) or (3.2b))	—
w	normalised follower displacement (see Equations (3.3a) or (3.3b))	—
w'	non-dimensional follower velocity	—

w''	non-dimensional follower acceleration	—
w'''	non-dimensional follower jerk	—
X_c, Y_c	axes fixed in, and rotating with, cam with origin at cam centre O	—
X_m, Y_m	axes fixed in machine, with origin at cam centre O	—
y	linear displacement of translating follower	m
y_b	height of translating follower datum above X_m -axis at innermost position of travel	m
y_m	height of translating follower datum above X_m -axis	m
y_0	height of translating follower datum above X_m -axis at start of motion segment	m
$\dot{y}$	linear velocity of translating follower	m/s
$\ddot{y}$	linear acceleration of translating follower	m/s ²
$\dddot{y}$	linear jerk of translating follower	m/s ³
Z_A	intermediate calculation parameter (see Equation (A1.4))	—
Z_1, Z_2	intermediate calculations parameters (see Equations (4.11) and (4.23), respectively)	—
Φ	total angle of cam rotation corresponding to one motion segment	degree
ϕ	intermediate angle of cam rotation from start of motion segment	degree
ϕ_0	angle of cam rotation at start of motion segment	degree
Ψ	total angle of follower swing associated with one motion segment	degree
ψ	angular displacement of swinging follower	degree
ψ_b	angle between X_m -axis and swinging follower datum at innermost position	degree
ψ_m	angle between X_m -axis and swinging follower datum	degree
ψ_0	angle between X_m -axis and swinging follower datum at start of motion segment	degree
$\dot{\psi}$	angular velocity of swinging follower	rad/s
$\ddot{\psi}$	angular acceleration of swinging follower	rad/s ²
$\dddot{\psi}$	angular jerk of swinging follower	rad/s ³
ω_c	angular velocity of cam	rad/s

Subscripts

<i>B</i>	refers to value at blending point
<i>CV</i>	refers to constant velocity component
<i>F</i>	refers to finish of constant velocity component
<i>fall</i>	refers to fall motion of follower
<i>i</i>	refers to intermediate value
<i>j</i>	refers to equal jerk in components at blending point (see Equation (4.29))
<i>max</i>	indicates maximum magnitude
<i>N1</i>	refers to notional dimension of component blending with dwell
<i>N2</i>	refers to notional dimension of component controlling reversal of velocity
<i>rise</i>	refers to rise motion of follower
<i>S</i>	refers to start of constant velocity component
<i>V</i>	refers to location of maximum follower velocity for single law motions
<i>V1</i>	refers to location of maximum follower velocity (Component <i>N1</i>)
<i>V2</i>	refers to location of maximum follower velocity (Component <i>N2</i>)

Definition of terms and acronyms

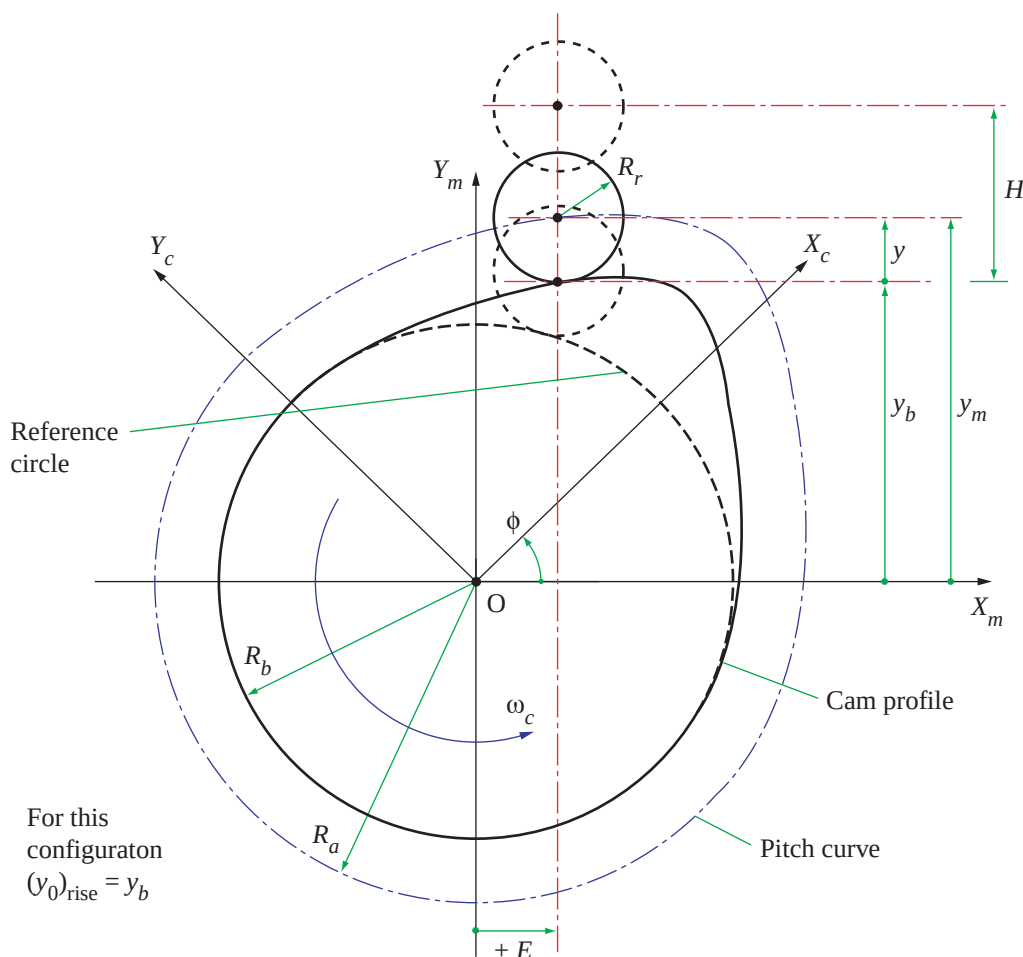
CCCA	Constant-Cosine-Constant Acceleration
DFD	Dwell-Fall-Dwell
DRD	Dwell-Rise-Dwell [†]
DFRD	Dwell-Fall-Rise-Dwell
DRFD	Dwell-Rise-Fall-Dwell
DFVRD	Dwell-Fall-Rise-Dwell with constant velocity components
DRVFD	Dwell-Rise-Fall-Dwell with constant velocity components
jerk	third derivative of the displacement equation with respect to time
SCCA	Sine-Constant-Cosine Acceleration
DHM	Double Harmonic Motion
SHM	Simple Harmonic Motion
THM	Triple Harmonic Motion

[†] DRD may also refer to Dwell-Return-Dwell in other ESDU Items in the cam series

2. INTRODUCTION

This Data Item provides guidance upon the analysis and selection of cam laws suitable for Dwell-Rise-Fall-Dwell (DRFD) and Dwell-Fall-Rise-Dwell (DFRD) follower motions having equal rise and fall periods, including ones having an intermediate constant-velocity (V) component in both the rise and fall actions, *i.e.* DRVFD and DFVRD, respectively. In this type of follower motion the follower is held stationary (dwell) whilst in contact with a constant radius segment of the cam profile concentric with the axis of rotation. It rises (or falls) during the subsequent motion segment through the maximum displacement (the lift) and then immediately falls (or rises) to return to the original dwell position. The content applies to all configurations of the cam mechanism[†], such as the disc cam driving a translating follower through a freely pivoted roller shown in Sketch 2.1, or a disc cam driving a swinging roller follower shown in Sketch 2.2. The terminology and dimensions used in this Data Item are defined in the Notation and superimposed on the drawings.

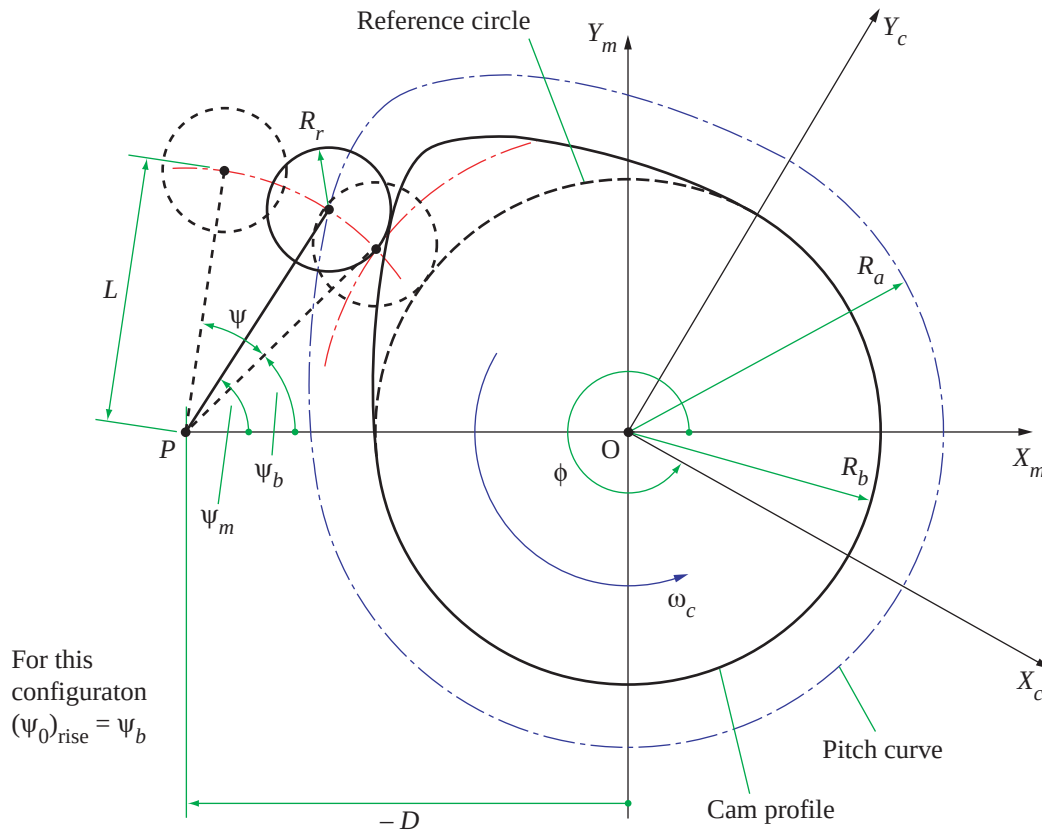
Besides the applications of cam laws to conventional cam mechanisms used throughout this series of Data Items, the laws given are equally applicable to the definition of the output motion of “electronic cams”. (See Items 82006^[6], 06003^[16], 06004^[17] and 06005^[18].)



Sketch 2.1 Disc cam for a DRFD motion driving a translating roller follower

[†] Flat-faced sliding followers are unsuitable for DFRD cams.

The choice of a suitable cam law for the application, along with the selection of a suitable cam and follower type (see Item 82024^[7]) forms the basis of the mechanism design process. The cam law is the equation defining the relationship between the output displacement of the follower and the angle of cam rotation. One cam law can define the complete motion segment or, in some applications, an improved motion may be derived by blending components of suitable cam laws.



Sketch 2.2 Disc cam for a DRFD motion driving a swinging roller follower

The definition of follower motion determines many of the input values for continuing the design of the mechanism (see Item 01002^[13], which also provides guidance upon the criteria for selecting a cam law). The procedure for designing a blended cam profile to include an intermediate precision point is given in Item 93002^{[10]†}. A particular mechanism is readily assessed using the computer-aided cam design program, OSCAM (Item 00014^[12]), to determine the critical values of pressure angle, profile curvature and driving torque.

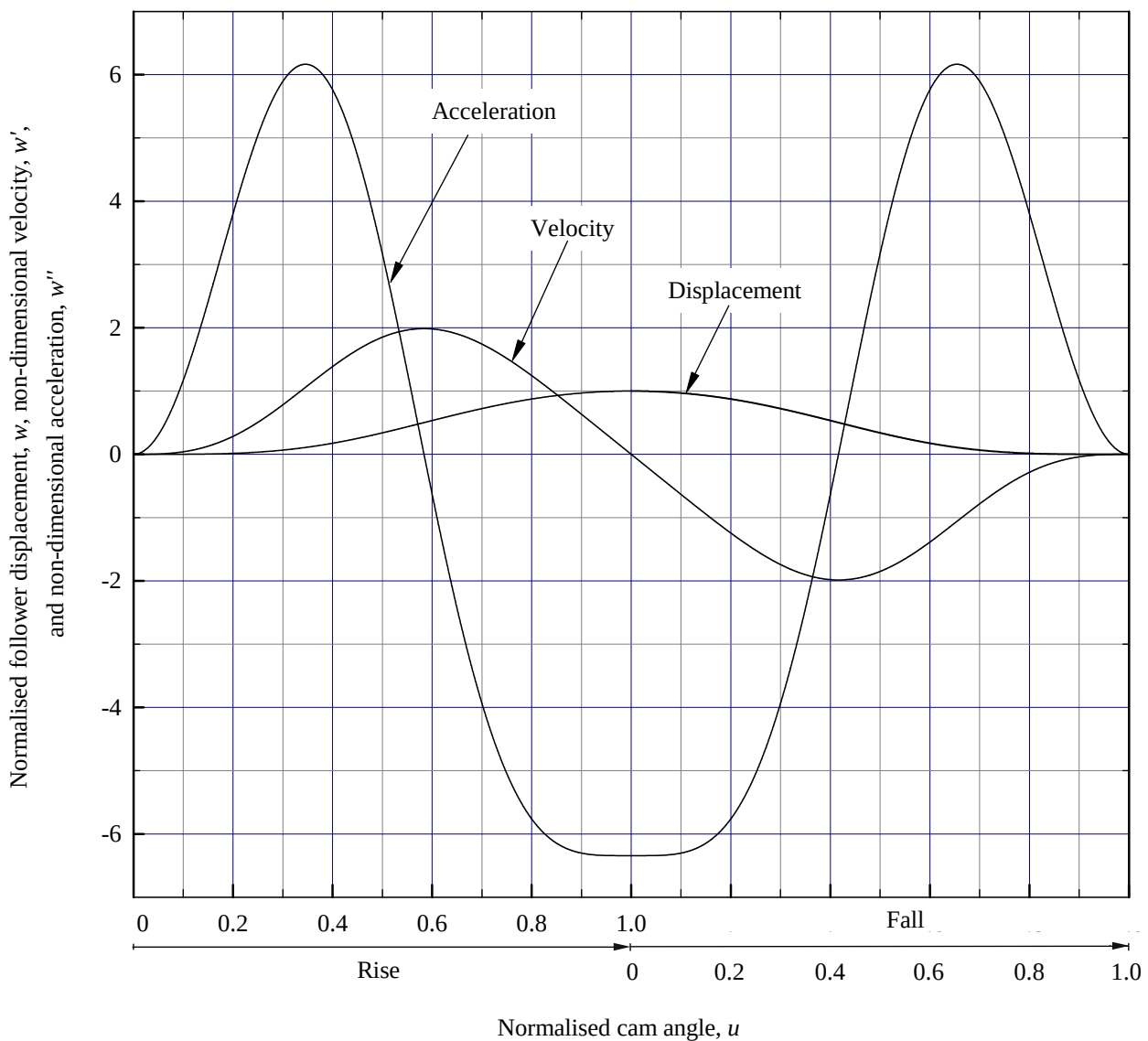
Asymmetric cam motions have unequal rise and fall periods. By implication, the mechanical functions of the rise and fall actions differ. For example, the rise of a DRFD motion could control a feed operation requiring a constant velocity drive to the follower for part of that movement to be succeeded immediately by a fall motion restoring the follower to the inner dwell position (see Example 2 of Item 03010^[14] and Example 7 of Item 06004^[17]).

In all asymmetric cam motions, the portion of the cam profile driving the more complex action is designated the "primary" motion segment and that controlling the reverse follower movement is designated the "secondary" motion segment. The procedures contained in this Data Item and the associated ESDU computer program (see Item 06005^[18]) also apply to primary motion segments of cams having unequal rise and fall periods. The procedures for completing the definition of the secondary motion segment are given in Item 06003^[16]. These procedures are included in the same ESDU computer program.

† The cam laws for DRFD and DFRD motions involving a precision point must meet the criteria specified in this Data Item.

2.1 Cam Law Selection

This Data Item deals specifically with the selection of a cam law for the rise and fall motion segments of a cam profile which control those parts of a DRFD or a DFRD motion. The content is confined to output motions having equal rise and fall periods, using the same cam law or blended cam laws for both segments by making appropriate changes to the kinematic equations. Because the follower dwells at only one limit of its travel, the boundary conditions defining the motion differ from those for DRD cams given in Item 82006^[6]. In contrast to a DRD motion, the follower acceleration should be an absolute maximum at the cam angle where the follower displacement equals the lift, H , or follower swing, Ψ , co-incident with the reversal of velocity. Hence it is important to appreciate the importance of avoiding the apparently attractive means of deriving a DRFD motion from a DRD rise followed immediately by a DFD fall as this would result in an unnecessarily high acceleration and drive torque and be liable to induce vibration^[19,20]. A typical non-dimensional follower motion, displacement, velocity and acceleration, corresponding to a symmetrical single-law motion segment is shown in Sketch 2.3.



**Sketch 2.3 DRFD follower motion
for Triple Harmonic Motion cam law**

This Data Item deals specifically with the selection of a cam law for the rise and fall motion segments which form part of a Dwell-Rise-Fall-Dwell or a Dwell-Fall-Rise-Dwell motion. The cam laws to be found in the Information Sheets (Section 7[†]) which generate the equations of motion for the rise or fall segment of a DRFD motion as continuous functions are:

Double Harmonic Motion

Triple Harmonic Motion

2-4-5 Polynomial

2-4-5-6 Polynomial

3-5-6-7-8 Polynomial

and, additionally

Constant-Cosine-Constant Acceleration motion

which is suitable only for the component of blended motions controlling the reversal of velocity at the maximum follower displacement.

The potential for blending components of different DRD and DRFD cam laws to derive complete rise and fall motion segments provides greater flexibility to suit the proposed motion to the application. The approach adopted in this Data Item involves blending components of two separate motions at their transition points to form the resultant motion segment. This method ensures continuity of displacement, velocity and acceleration as the follower attains maximum velocity at the transition point where the acceleration is zero. It is not necessary for the two motions to have equal notional segment angles or lift, or to be defined by the same cam law, making it possible to combine components of motions which only satisfy the boundary conditions for the relevant portion of the motion.

2.2 Reasons for Blending

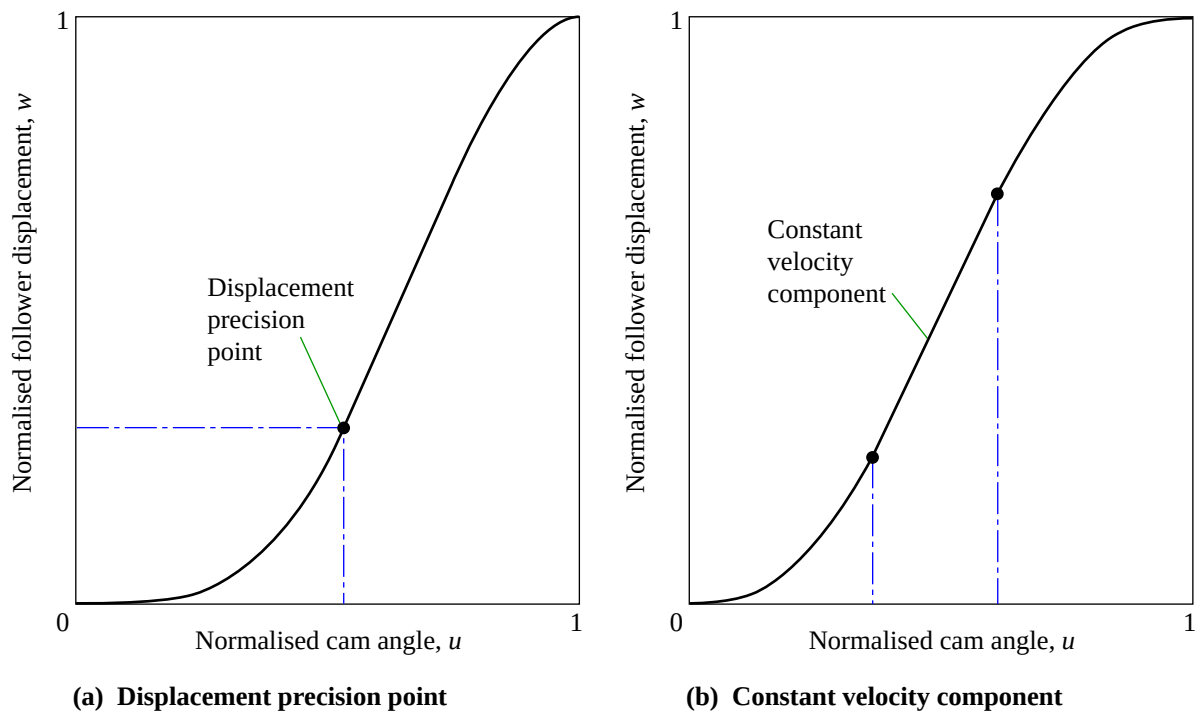
One cam law may not generate the required follower motion for the complete rise or fall segment of a DRFD action. Examples include a displacement curve required to incorporate a specified precision point, Sketch 2.4(a), or a constant velocity component, Sketch 2.4(b). In such cases it can be advantageous to investigate the possibility of forming the segment from blended components of the same or different cam laws. Further reasons for profile blending and potential benefits are summarized as follows.

- (a) It becomes possible to vary the relative magnitudes of the peak follower acceleration and retardation corresponding to a given segment angle, either to obtain a superior balance between the two components or to reduce the peak value acting towards the cam and so reduce the spring force required to maintain contact (see Examples 3, 5 and 6 in Item 06004^[17]). The consequences of such changes are explained in Items 92014^[9] and 93002^[10].
- (b) It becomes possible to vary the follower displacement corresponding to a specific cam angle, for example, to lengthen the time during which a valve is held wide open, or to ensure that a specific displacement occurs at the required cam angle (see Example 5 in Item 06004^[17] and Item 92014^[9]).

[†] The equations for other cam laws generating a continuous DRFD motion have been published in the general literature. These are omitted here because, for example, either a small accumulative error occurs, or the equations of motion do not suit the convention adopted to make this Item compatible with that for DRD laws given in previous Items, e.g. Item 82006^[6], and software essential to blend segments of DRD and DRFD laws.

- (c) It becomes possible to incorporate an intermediate constant velocity component between the components controlling the follower acceleration and retardation, either to suit a design requirement or to reduce the peak velocity (but increase the acceleration), (see Item 93002^[10] and also Examples 2 and 6 in Item 06004^[17]).
- (d) The segment angle required for the follower to reach a given lift (for a cam with specified values of reference circle radius and maximum allowable pressure angle) can be minimised by blending (see Item 92014^[9]).
- (e) The minimum allowable reference circle radius required for a cam with specified segment angle, follower lift and maximum allowable pressure angle can be reduced by blending (see Item 92014^[9] and Example 2 in that Item).
- (f) The maximum allowable follower lift that can be obtained with a specified segment angle, reference circle radius and maximum allowable pressure angle can be increased by blending (see Item 92014^[9]).
- (g) Blending provides a means of obtaining a more rapid follower displacement from dwell than possible with a single-law motion (see Example 3 in Item 06004^[17]).

In some cases the decision to use a blended profile is determined by the suitability of a particular cam law for that component of the motion. In the absence of further constraints, to minimise vibration the (unique) “notional” dimensions of segment angle and lift for each component should be found by equating the follower velocity and jerk at the blending points. As an alternative, interposing a constant velocity component between components controlling the acceleration and retardation can be beneficial.



Sketch 2.4 Special motion requirements

Constant velocity sections are sometimes employed in cam profiles for use with translating flat-faced followers because in this arrangement the maximum travel of the contact point on the follower is directly proportional to the peak velocity. Thus the available follower face diameter places a restriction on the peak velocity and so, if it is important to maximise the area under the lift curve, a constant velocity section is useful.

2.3 Use of this Data Item

The theoretical background to alternative forms of the mechanism and follower motion, including definitions and terminology, is explained in Section 3. The design procedures for alternative types of blended motion are analysed in Sections 4 and 5 and their use illustrated by the examples in Item 06004^[17]. It is strongly recommended that the calculations be performed using the ESDU computer program (see Item 06005^[18]), exporting the files defining the motion to other programs as appropriate to complete the design of the mechanism. Item 06005^[18] provides information regarding the use of the computer program including the composition of program input and interpretation of program output.

2.4 Cam Laws for DRFD and DFRD Motions

The cam laws and blended motions included in this Data Item are all designed to ensure that the jerk is finite throughout the DRFD motion. Thus the cam laws of SHM (see Item 82006^[6]) and CCCA (see Section 7) are only suitable for the central component of a blended motion controlling the reversal of follower velocity, because the jerk is infinite at the start and finish of these single-law motions. Conversely, finite-jerk DRD cam laws, such as Cycloidal and Modified Sinusoidal Acceleration motions (see Item 82006^[6]) are only suitable for those components of a blended DRFD motion which join a dwell.

The boundary conditions defining a complete finite-jerk DRFD motion and a complete finite-jerk DFRD motion are given in Tables 3.1 and 3.2, respectively.

When reading the data from Tables 3.1 and 3.2 it is vital to realise that the peak follower velocity of a single-law motion does not necessarily occur at mid-displacement, *i.e.* when $u_V = 0.5$ and/or $w_V = 0.5$. Also, there are cases, *e.g.* the 3-4-5-6 Polynomial cam law, where values quoted in this Data Item, such as the normalised cam angle and follower displacement at the transition point, have been rounded to six decimal places. Moreover, it is feasible to blend components of cam laws having different values of u_V and w_V at their transition points, see Example 2 in Item 06004^[17].

The choice of cam law is influenced by the interaction between the peak values of follower velocity and acceleration with the segment angle and lift. Hand calculations are convenient for this purpose. A concise form of the equations is obtained by expressing parameters where possible as products of π as given in Appendix D.

3. THEORETICAL BACKGROUND

3.1 Definitions and Terminology

3.1.1 Motion segments and single-law motion

For the purpose of this Data Item a motion segment is defined as that part of a cam motion cycle during which the follower moves from one position of zero follower velocity (such as a dwell or a position where the velocity reverses direction) to another position of follower zero velocity. A motion segment may be described as Dwell-Rise-Dwell (DRD), Dwell-Fall-Dwell (DFD), one part of a Dwell-Rise-Fall-Dwell (DRFD) action or one part of a Dwell-Fall-Rise-Dwell (DFRD) action. The most common way to specify follower motion, and hence determine the cam profile is to use one of a recommended series of equations, known as cam laws, which relate the cam angle of rotation to the follower motion. To define a single-law motion segment, that is one governed by a single cam law, it is only necessary to specify the cam law, the follower lift and the segment angle.

3.1.2 Dimensionless form of displacement, velocity and acceleration

Sketches 2.1 and 2.2 show two pairs of co-ordinate axes having origins co-incident with the centre of cam rotation. The X_m , Y_m axes are fixed to the machine frame whilst the X_c , Y_c axes are fixed to the cam and rotate with it.

A cam law is the mathematical function defining the normalised follower displacement, w , during a single motion segment in terms of the normalised cam rotation, u . It is conveniently expressed as

$$w = f\{u\} . \quad (3.1)$$

This function is evaluated readily by substituting the equation for the relevant cam law. In this Data Item the cam laws are identified by their generic names, for example Double Harmonic Motion or 3-4-5-6 Polynomial. The equations of motion for commonly-used DRFD cam laws are given as formulae and graphically in Section 7 and, for DRD cam laws, in Item 82006^[6].

The Notation is given in Section 1: the sign convention and equivalent equations for the various configurations of translating and swinging followers are given in Section 3.2 of Item 95001^[11].

Translating followers

For a disc cam driving a radial translating follower as shown in Sketch 2.1, the normalised cam angle and normalised follower displacement for each DRFD rise motion segment are given by

$$u_{rise} = \frac{\phi - \phi_0}{\Phi} \quad (3.2a)$$

$$\text{and} \quad w_{rise} = \frac{y_m - y_0}{H} . \quad (3.3a)$$

Making the usual assumption that the cam rotates at constant angular velocity, the dimensionless velocity parameter for a translating follower is defined as

$$w' = \frac{\dot{y}}{\omega_c H} \left(\frac{\pi \Phi}{180} \right) . \quad (3.4a)$$

Similarly, the dimensionless acceleration parameter for a translating follower is defined as

$$w'' = \frac{\ddot{y}}{\omega_c^2 H} \left(\frac{\pi \Phi}{180} \right)^2 \quad (3.5a)$$

and the dimensionless jerk parameter for a translating follower is defined as

$$w''' = \frac{\dddot{y}}{\omega_c^3 H} \left(\frac{\pi \Phi}{180} \right)^3. \quad (3.6a)$$

Swinging followers

For a disc cam driving a swinging follower as shown in Sketch 2.2, the normalised cam angle and normalised follower displacement for each motion segment are given by

$$u_{rise} = \frac{\phi - \phi_0}{\Phi} \quad (3.2b)$$

and
$$w_{rise} = \frac{\psi_m - \psi_0}{\Psi}. \quad (3.3b)$$

Making the usual assumption that the cam rotates at constant angular velocity, the dimensionless velocity parameter for a swinging follower is defined as

$$w' = \frac{\dot{\psi}}{\omega_c \Psi} \Phi. \quad (3.4b)$$

The dimensionless acceleration parameter for a swinging follower is defined as

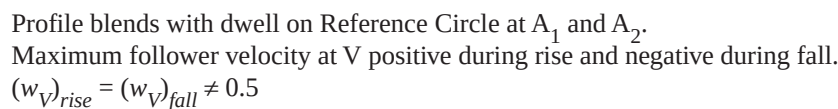
$$w'' = \frac{\ddot{\psi}}{\omega_c^2 \Psi} \frac{\pi}{180} \Phi^2. \quad (3.5b)$$

and the dimensionless jerk parameter for a swinging follower is defined as

$$w''' = \frac{\dddot{\psi}}{\omega_c^3 \Psi} \left(\frac{\pi}{180} \right)^2 \Phi^3. \quad (3.6b)$$

3.1.3 Equations for motion segments of a DRFD profile

The follower displacement curve A_1BA_2 for a complete DRFD follower cycle is given in Sketch 3.1. For compatibility with the presentation for DRD and DFD motions (see Item 82006^[6]), the rise and fall are distinct motion segments.



Sketch 3.1 shows the follower displacement through the complete DRFD movement. The cam angle, ϕ , is measured from the start of the rise, Point A₁, where the normalised cam angle of the rise motion segment, u_{rise} , the normalised follower displacement, w_{rise} , the non-dimensional follower velocity, w'_{rise} , and the non-dimensional follower acceleration, w''_{rise} , are all zero. The follower has positive velocity during the rise to attain the lift, H , at point B after the cam has rotated through the angle of lift, Φ . At this point, the values of u_{rise} and w_{rise} are both unity, w'_{rise} is zero and w''_{rise} has its maximum negative value.

A separate set of equations apply to the fall action represented by the curve BA₂ on Sketch 3.1, when the follower falls to the inner dwell position. Since this Data Item considers only motions having equal rise and fall periods the segment angle for the fall equals that for the rise, Φ . Cam rotation through this angle causes the follower to return through the lift to the original dwell position. Point B is the datum for measuring the cam angle of the fall motion segment. The follower displacement continues to be measured from the X_m -axis and so at Point B, $u_{fall} = 0$ and $w_{fall} = 1$, and the follower velocity, $w' = 0$. Moreover, the magnitude and sign of the follower acceleration, $\ddot{y}$, must be the same as at the end of the rise. The follower has negative velocity during the fall as it returns through the lift, H , to reach point A₂ at the start of the dwell. At this point, the normalised cam angle has the value $u_{fall} = 1$, whilst the values of the normalised follower displacement, w_{fall} , the non-dimensional follower velocity, w'_{fall} , and acceleration, w''_{fall} , are all zero. Hence the normalised cam angle, u_{fall} , at an intermediate position during the fall is related to the cam angle, ϕ_{fall} , by

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The corresponding normalised follower displacement is given by

$$w_{fall} = f\{1 - u_{fall}\}, \quad (3.7)$$

the non-dimensional velocity parameter is given by

$$w'_{fall} = -f'\{1 - u_{fall}\} \quad (3.8)$$

the non-dimensional acceleration parameter by

$$w''_{fall} = f''\{1 - u_{fall}\} \quad (3.9)$$

and the non-dimensional jerk parameter by

$$w'''_{fall} = -f'''\{1 - u_{fall}\}. \quad (3.10)$$

The critical boundary conditions for a DRFD cam motion are summarised in Table 3.1.

TABLE 3.1 Boundary conditions for DRFD cam motions having equal rise and fall periods

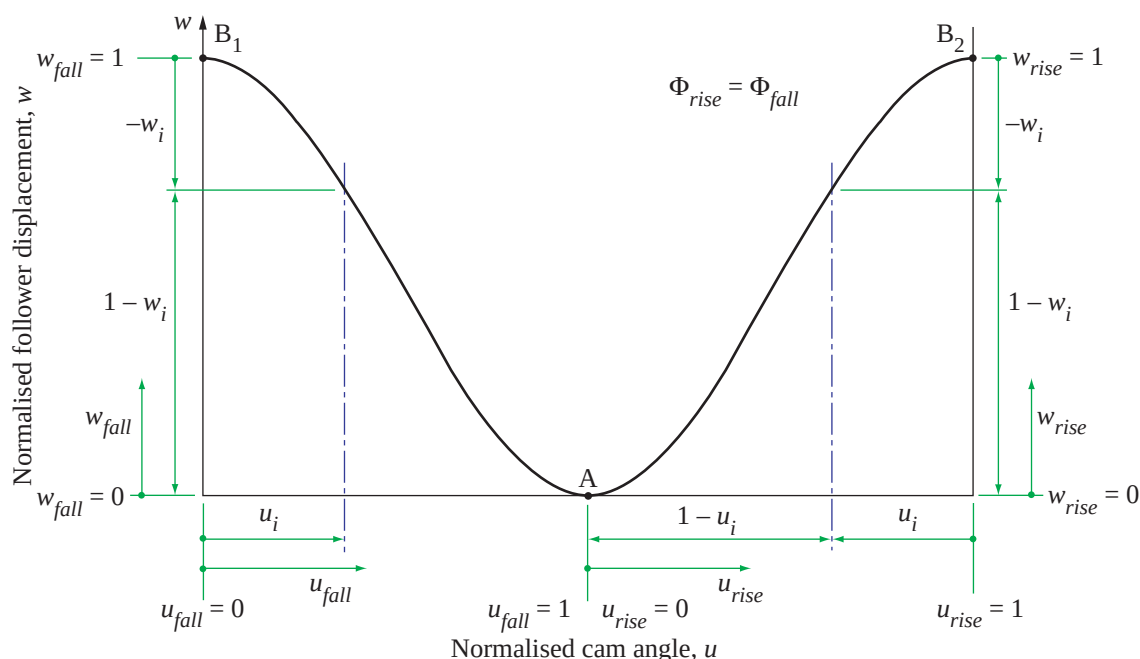
	u	w	w'	w''	Notes
Rise	0	0	0	0	Profile blends with the inner dwell
	u_V	w_V	Maximum +ve	0	
	1	1	0	Maximum -ve [†]	Follower velocity reverses
Fall	0	1	0	Maximum -ve [†]	
	$1 - u_V$	w_V	Maximum -ve	0	
	1	0	0	0	Profile blends with the inner dwell

[†] For zero jerk the follower accelerations must be continuous.

3.1.4 Equations for motion segments of a DFRD profile

The follower displacement curve B_1AB_2 for a complete DFRD follower cycle is given in Sketch 3.2. For compatibility with the presentation for DRD and DFD motions (see Item 82006^[6]) the rise and fall are treated as distinct motion segments.

Referring to Sketch 3.2, which shows the follower displacement through the complete DFRD movement, the cam angle, ϕ , is measured from the start of the fall, Point B_1 , where the follower leaves the dwell at maximum cam radius with negative velocity.



Profile touches the Reference Circle at A and blends with the dwell at B₁ and B₂

Sketch 3.2 Displacement during complete DFRD motion

At point B₁, the normalised cam angle, u_{fall} , the non-dimensional follower velocity, w'_{fall} , and acceleration, w''_{fall} , are all zero but, as the follower is displaced by the lift, H , above the Reference Circle, the normalised follower displacement is $w_{fall} = 1$. The cam profile touches the Reference Circle at the end of the fall motion segment, Point A. At this point, the normalised cam angle is $u_{fall} = 1$, the normalised lift, w_{fall} , and the non-dimensional follower velocity, w'_{fall} , are both zero and the non-dimensional follower acceleration, w''_{fall} , has the maximum positive value. Therefore the normalised displacement equation must be changed to the following form

$$w_{fall} = 1 - f\{u_{fall}\} . \quad (3.11)$$

To correspond, the non-dimensional velocity parameter is given by

$$w'_{fall} = -f'\{u_{fall}\} , \quad (3.12)$$

the non-dimensional acceleration parameter by

$$w''_{fall} = -f''\{u_{fall}\} \quad (3.13)$$

and the non-dimensional jerk parameter by

$$w'''_{fall} = -f'''\{u_{fall}\} . \quad (3.14)$$

A separate set of equations applies to the rise action represented by the Curve AB₂ on Sketch 3.2 for the movement of the follower to the dwell position at maximum cam radius. Since this Data Item considers only motions having equal rise and fall periods, the segment angle, Φ , for the rise equals that for the fall.

At the start of the rise motion segment, Point A, the normalised cam angle, u_{rise} , the normalised follower displacement, w_{rise} , and the non-dimensional follower velocity, w'_{rise} , are all zero, whilst the magnitude and direction of the follower acceleration, $\ddot{y}_{rise}$, must be the same as that at the end of the fall motion segment. Cam rotation through the angle of lift, M , causes the follower to rise with positive velocity through the lift, H , to the original dwell position, Point B₂, where the values of both u_{rise} and w_{rise} are unity and of w'_{rise} and w''_{rise} are zero. Consequently, for the rise motion, the normalised cam angle is given by

$$u_{rise} = \frac{\phi - \phi_0}{\Phi} \quad (3.15)$$

and the normalised follower displacement is given by

$$w_{rise} = 1 - f\{1 - u_{rise}\} . \quad (3.16)$$

The non-dimensional velocity parameter is given by

$$w'_{rise} = f'\{1 - u_{rise}\} . \quad (3.17)$$

To correspond, the non-dimensional acceleration parameter is

$$w''_{rise} = -f''\{1 - u_{rise}\} \quad (3.18)$$

and the non-dimensional jerk parameter by

$$w'''_{rise} = f'''\{1 - u_{rise}\} . \quad (3.19)$$

The critical boundary conditions for a DFRD cam motion are summarised in Table 3.2.

The manual calculations for both DRFD and DFRD motions are explained in Example 1 of Item 06004^[17].

**TABLE 3.2 Boundary conditions for DFRD cam motions
having equal rise and fall periods**

	u	w	w'	w''	Notes
Fall	0	1	0	0	Profile blends with the outer dwell
	u_V	$1 - w_V$	Maximum -ve	0	
	1	0	0	Maximum +ve [†]	Follower velocity reverses
Rise	0	0	0	Maximum +ve [†]	
	$1 - u_V$	$1 - w_V$	Maximum +ve	0	
	1	1	0	0	Profile blends with the outer dwell

[†] For zero jerk these accelerations must be continuous.

The diagram illustrates the blending of two displacement components, Component N1 and Component N2, to form a smooth cam profile. The vertical axis represents Follower displacement (y) and the horizontal axis represents Cam angle (ϕ).

Key Points and Parameters:

- Point C:** Start of Component N1 at $(0, H)$.
- Point E:** Start of Component N2 at (ϕ_{fall}, H) .
- Point B:** Blending point where the two components meet.
- Point D:** End of Component N2 at $(\phi_{fall} + \phi_{N2}, H_{N2})$.
- Point A:** End of Component N1 at $(\phi_{N1}, 0)$.

Displacement and Angle Parameters:

- H : Total follower displacement.
- H_{N1} : Displacement of Component N1.
- H_{N2} : Displacement of Component N2.
- $w_{V1}H_{N1}$: Vertical distance from the base to the start of Component N1.
- $(1 - u_{V2})H_{N2}$: Vertical distance from the blending point B to the end of Component N2.
- $(1 - u_{V2})\phi_{N2}$: Horizontal distance from the start of Component N2 to the blending point B.
- ϕ_{N2} : Cam angle of Component N2.
- $u_{V1}\phi_{N1}$: Horizontal distance from the blending point B to the end of Component N1.
- $k_B\phi_{fall}$: Horizontal distance from the start of Component N1 to the blending point B.
- ϕ_{fall} : Cam angle of the fall phase.
- ϕ_{N1} : Cam angle of Component N1.

Boundary Conditions:

- At $\phi_{fall} = 0$: $u_{fall} = 0$, $u_{N2} = 1$.
- At $\phi_{fall} + \phi_{N2} = \phi_{N1}$: $u_{fall} = 1$, $u_{N1} = 0$.

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The procedures adopted in this Data Item require that the blending points always coincide with the position of maximum follower velocity to ensure continuity of follower velocity and acceleration.

In the case of motions formed of two components, *i.e.* without an intermediate section at constant follower velocity, the possibilities include three methods illustrated by examples in Item 06004^[17] as follows;

- (a) equating the jerk at the blending point (see Section 4.3 and Example 4 in Item 06004^[17]),
- (b) specifying the cam angle from the end of the dwell to the blending point as a ratio of the actual segment angle defined by the blending ratio, k_B , given by (see Example 3 in Item 06004^[17])

$$k_B = \frac{\phi_B}{\Phi}, \quad (4.1)$$

- or (c) or specifying the ratio of the absolute values of peak acceleration and retardation defined by the acceleration ratio, k_A , given by (see Example 5 in Item 06004^[17])

$$k_A = \frac{\left| \ddot{y}_{N1} \right|_{max}}{\left| \ddot{y}_{N2} \right|_{max}} \text{ for translating followers,} \quad (4.2a)$$

$$k_A = \frac{\left| \ddot{\psi}_{N1} \right|_{max}}{\left| \ddot{\psi}_{N2} \right|_{max}} \text{ for swinging followers.} \quad (4.2b)$$

in which it is noted that the acceleration ratio, k_A , is not directly related to either the peak acceleration or the peak retardation of the corresponding single-law motion.

When blending different cam laws it is not necessary for the normalised cam angle and normalised follower displacement to be the same for each law, *i.e.* it is possible for $u_{V1} \neq u_{V2}$ and/or for $w_{V1} \neq w_{V2}$, see Example 2 in Item 06004^[17].

The procedure for determining the notional prime circle radius of each component is covered in Section 4.5.

4.2 Proportions of Blended Profiles Defined by Acceleration Ratio or Blending Ratio

When using the non-dimensional equations of the cam laws to compute the dimensions and output motion of a specific cam having a blended profile it is essential to substitute the notional parameters into the equations. Hence, the normalised cam angle of the actual cam, u , must be expressed as the notional value, u_{N1} or u_{N2} , as appropriate.

The following analysis applies to a translating follower, the corresponding equations for a swinging follower are obtained by substituting Ψ for H , ψ for y , *etc.*

The relationship expressing the acceleration ratio, k_A , as a function of the blending ratio, k_B , is given in Appendix A, Equation (A1.9), as

$$k_B = \frac{u_{V1}}{u_{V1} + (1 - u_{V2})k_A Z_A},$$

where Z_A is given by Equation (A1.4)

$$Z_A = \frac{(w'_{N1})_{\max} |(w''_{N2})_{\max}|}{(w'_{N2})_{\max} |(w''_{N1})_{\max}|}.$$

Critical values of the normalised follower displacement, and of the non-dimensional velocity, acceleration and jerk for commonly-used cam laws are given in Appendix D.

By inspection of Sketch 4.1 or Sketch 4.2 it is seen that

$$\Phi = u_{V1}\Phi_{N1} + (1 - u_{V2})\Phi_{N2} \quad (4.3)$$

$$\text{and} \quad H = w_{V1}H_{N1} + (1 - w_{V2})H_{N2}. \quad (4.4)$$

Also, since at the blending point

$$\phi_B = k_B\Phi \equiv u_{V1}\Phi_{N1}, \quad (4.5)$$

the notional segment angle for Component N1 is given, in terms of the blending ratio, by

$$\Phi_{N1} = \frac{\phi_B}{u_{V1}} = k_B \frac{\Phi}{u_{V1}}. \quad (4.6)$$

Similarly, the notional segment angle for Component N2 is given by

$$\Phi_{N2} = \frac{(1 - k_B)\Phi}{1 - u_{V2}}. \quad (4.7)$$

Two components of follower motion are blended together at the point of maximum follower velocity. Equating the expressions (obtained by re-arranging Equation (3.5a)) for follower velocity at the blending point and substituting the notional dimensions gives

$$\dot{y}_{\max} = \frac{(w'_{N1})_{\max} \omega_c H_{N1}}{\Phi_{N1} \pi / 180} = \frac{(w'_{N2})_{\max} \omega_c H_{N2}}{\Phi_{N2} \pi / 180}, \quad (4.8)$$

which leads to

$$\frac{\Phi_{N1}}{\Phi_{N2}} = \frac{(w'_{N1})_{\max} H_{N1}}{(w'_{N2})_{\max} H_{N2}}. \quad (4.9)$$