



Dwell-Rise-Fall-Dwell and Dwell-Fall-Rise-Dwell cam laws Sections 14 to 23: examples

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Mechanisms Committee and has the following membership:

Chairman	
Dr C.W. Stammers	— University of Bath
Vice-Chairman	
Dr G. Druce	— Independent, Hampshire
Members	
Dr C. Burvill	— The University of Melbourne, Australia
Mr P.J. Carden	— Ricardo Consulting Engineers Ltd, Shoreham-by-Sea
Mr A.A. Fogarasy	— Independent, Tyne & Wear
Prof F. Freudenstein*	— Columbia University, New York, USA
Dr M.J. Gilmartin	— Liverpool John Moores University
Dr M.R. Smith†	— University of Newcastle-upon-Tyne (Member Emeritus)
Prof Zhonghe Ye*	— Fuzhou University, P.R. China
* Corresponding Member	
† Member Emeritus	

The technical work involved in the assessment of the available information and the construction and subsequent development of the Data Item was carried out under contract to ESDU by a Member of the Mechanisms Committee

Dr G. Druce.

The person with overall responsibility for the work in this subject area is Mr C.J. Loughton, Head of the Mechanical Motion and System Dynamics Group.

The Mechanisms Committee, which first met in 1975, is continuing the work that previously was the responsibility of the former Gearing and Mechanisms Committee, which first met in November 1964. Like the earlier Committee it has the cooperation of many engineers and scientists in industry, research establishments and universities from whom much assistance and information is being received.

DWELL-RISE-FALL-DWELL AND DWELL-FALL-RISE-DWELL CAM LAWS. SECTIONS 14 TO 23: EXAMPLES

Data Items in the series on Dwell-Rise-Fall-Dwell cam laws are:

- Item 06002^[15] - Sections 1 to 8: segments having equal rise and fall periods,
- Item 06003^[16] - Sections 9 to 13: segments having asymmetric rise and fall periods,
- Item 06004^[17] - Sections 14 to 23: examples,
- Item 06005^[18] - Computer program.

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DWELL-RISE-FALL-DWELL AND DWELL-FALL-RISE-DWELL CAM LAWS. SECTIONS 14 TO 23: EXAMPLES

14. NOTATION AND UNITS

The units used in this Item are predominantly SI but any coherent system of units may be used. All angles are measured positively in an anticlockwise sense and the cam is considered to rotate in an anticlockwise direction.

Equations, sketches, tables, appendices, *etc.* identified by section numbers not found in this Item are to be found in: Sections 1 to 8 and Appendices A to D in Item 06002^[15] and Sections 9 to 13 in Item 06003^[16].

		Units
A	coefficient of SCCA cam law ($= 64\pi^2/(72 + 7\pi^2)$)	—
E	follower offset	m
g	acceleration due to gravity	m/s ²
H	total lift of translating follower corresponding to one motion segment	m
J	coefficient for Modified Sinusoidal Acceleration motion equations ($J = \pi/(\pi + 4)$)	—
k_A	acceleration ratio (see Equations (4.1) and (11.25))	—
k_B	blending ratio (see Equation (4.1), (4.5) and (4.31))	—
$N1$	component blending with dwell	—
$N2$	component between peak velocity and reversal of follower velocity	—
$P1$	parametric component blending with dwell	—
$P2$	parametric component controlling peak velocity and reversal of follower velocity	—
u	normalised cam angle (see Equation (3.2a))	—
w	normalised follower displacement (see Equations (3.3a) or (3.3b))	—
w'	non-dimensional follower velocity	—
w''	non-dimensional follower acceleration	—
w'''	non-dimensional follower jerk	—
y	linear displacement of translating follower	m
$\dot{y}$	linear velocity of translating follower	m/s

$\ddot{y}$	linear acceleration of translating follower	m/s ²
$\dddot{y}$	linear jerk of translating follower	m/s ³
Z_1	intermediate calculation parameter (see Equations (4.11) and (18.5))	—
Z_2	intermediate calculation parameter (see Equations (4.23) and (19.1))	—
Z_3	intermediate calculation parameter (see Equation (11.10) and (22.31))	—
Z_4	intermediate calculation parameter (see Equation (11.32))	—
Φ	total angle of cam rotation corresponding to one motion segment	degree
ϕ	intermediate angle of cam rotation corresponding to one motion segment	degree
ϕ_0	angle of cam rotation at start of motion segment	degree
Ψ	total angle of follower swing associated with one motion segment	degree
ψ	angular displacement of swinging follower	degree
$\dot{\psi}$	angular velocity of swinging follower	rad/s
$\ddot{\psi}$	angular acceleration of swinging follower	rad/s ²
$\dddot{\psi}$	angular jerk of swinging follower	rad/s ³
ω_c	angular velocity of cam	rad/s

Subscripts

B	refers to value at blending point
cal	calculated value
CV	refers to constant velocity component
$dwell$	refer to dwell motion of follower
F	refers to finish of constant velocity component
$fall$	refers to fall motion of follower
j	refers to equal jerk in components at blending point (see Equation (4.29))
max	indicates maximum magnitude
$N1$	refers to notional dimension of component blending with dwell
$N2$	refers to notional dimension of component controlling reversal of velocity
$P1$	refers to parametric dimension of component blending with dwell

<i>P2</i>	refers to parametric dimension of component controlling reversal of velocity
<i>p</i>	refers to parametric dimension
<i>rev</i>	refers to dimensional value at reversal of follower velocity
<i>rise</i>	refers to rise motion of follower
<i>S</i>	refers to start of constant velocity component
<i>sec</i>	refers to Secondary Motion Segment
<i>V</i>	refers to normalised location of maximum follower velocity for single law motions
<i>V1</i>	refers to normalised location of maximum follower velocity (Component <i>N1</i>)
<i>V2</i>	refers to normalised location of maximum follower velocity (Component <i>N2</i>)

Definition of terms and acronyms

CCCA	Constant-Cosine-Constant Acceleration
DFD	Dwell-Fall-Dwell
DHM	Double Harmonic Motion
DRD	Dwell-Rise-Dwell
DFRD	Dwell-Fall-Rise-Dwell
DRFD	Dwell-Rise-Fall-Dwell
DFVRD	Dwell-Fall-Rise-Dwell with constant velocity components
DRVFD	Dwell-Rise-Fall-Dwell with constant velocity components
dwell	refers to dwell of follower
jerk	third derivative of the displacement equation with respect to time
SHM	Simple Harmonic Motion
THM	Triple Harmonic Motion

15. INTRODUCTION

This Data Item[†] contains examples illustrating the application of Dwell-Rise-Fall-Dwell (DRFD) and Dwell-Fall-Rise-Dwell (DFRD) cam motions, including blended motions and those having a constant velocity component interposed between components controlling the acceleration and retardation of the follower (DRVFD and DFVRD as appropriate). The analysis and the equations of single cam laws which satisfy the boundary conditions for these types of motion are given in Item 06002^[15]. The equations of Dwell-Rise-Dwell (DRD) cam laws, some of which are suitable for Component N1 of blended profiles adjoining the dwell, are given in Item 82006^[6].

Besides the applications of cam laws to conventional mechanisms used throughout this series of Data Items, the laws given are equally applicable to the definition of the output motion of “electronic cams”. (See Items 82006^[6], 06002^[15], 06003^[16] and 06005^[18].)

The examples are confined to the establishment of the dimensions needed to define the motion, together with the provision of curves illustrating the follower displacement, velocity, acceleration and jerk relative to the cam angle for that motion segment. It is strongly recommended that the ESDU computer program (see Item 06005^[18]), be used for these calculations, including sensitivity analyses to optimise the motion. All analyses can be continued by exporting the motion file to other ESDU computer programs, *e.g.* OSCAM (see Item 00014^[12]). See also “Guide to the use of ESDU Data Items on cam design”, Item 01002^[13].

Examples 1 to 6 apply to follower motions having equal rise and fall periods, whereas Example 7 illustrates procedures for analysing an asymmetric follower motion having unequal rise and fall periods.

Example 1 applies the analysis of Section 3 to solve single-law applications of both DRFD and DFRD motions. The calculations are performed by hand: different polynomial cam laws chosen from Section 7 are used to illustrate the differences between them. To avoid cumulative error, the intermediate stages of hand calculations were taken to several decimal places further than the accuracy required for the final results.

Hand calculations are also used in Example 2 to illustrate the analysis of a blended motion incorporating an intermediate constant velocity component (see Section 5). In this case the maximum follower velocity and acceleration and the maximum permitted lift are specified. The analysis is used to find the segment angle and lift, together with the notional values of these dimensions for the components of the profile. Example 2 also illustrates the combination of a DRD cam law for Component N1 adjoining the dwell (Cycloidal motion, see Item 82006^[6]), a constant velocity component and a DRFD cam law for Component N2 controlling the reversal of velocity (Double Harmonic Motion, see Section 7). These cam laws were selected to demonstrate the application of cam laws having different values of u_V and w_V at the blending point (see Appendix D).

The direct application of a single-law motion for a given segment angle and lift immediately determines the magnitude and location of the peak follower velocity and acceleration. Example 3 illustrates the procedure for introducing some flexibility into the output motion by applying the principles of blended profiles whilst using the same cam law for both components. The Acceleration Ratio, k_A , is varied to illustrate a means of mitigating the large difference between the peak values of acceleration and retardation of the Double Harmonic Motion cam law.

The equations defining the motion of a cam follower driven by a cam with a blended profile are functions of the notional segment angle and notional lift of the appropriate component. As these dimensions differ from the actual segment angle and lift of the complete blended segment, the follower motion cannot be

[†] Equations, sketches, tables, appendices, *etc.* identified by section numbers not found in this Item are to be found in: Sections 1 to 8 and Appendices A to D in Item 06002^[15] and Sections 9 to 13 in Item 06003^[16].

expressed in terms of the non-dimensional parameters, w , w' , etc. applied to single-law segments. Instead, the directly comparable (but dimensionalised) parameters, y_P , $\dot{y}_P$, etc. are derived in Example 3 by putting the cam speed, ω_c , segment angle, Φ , and lift, H , or follower swing, Ψ , all equal to unity to obtain the corresponding values.

Profile blending also provides a means of increasing the variety of cam motions available for DRFD motions specified solely by segment angle and lift. In Example 4 the DRD Cycloidal motion is used for Component N1 from the blend with the dwell to the transition point at maximum follower velocity. Although a single-law DRD application of SHM imposes infinite jerk at the start and finish of the motion, this motion is eminently suitable for Component N2 from the transition point of a blended DRFD action to the reversal of follower velocity as the peak retardation occurs at maximum displacement. In the case of motions having equal rise and fall periods, the displacement curve for the fall is a mirror-image of that of the rise. Hence, there is no discontinuity of jerk through the blending point. (See Item 82006^[6] for both of these cam laws.) In Example 4, the notional dimensions of each component, in the absence of other constraints, are determined by equating both the velocity and jerk of the follower of each motion where they blend at the transition point. To assist comparison with other motions and application of the results, the specification of this profile is derived in a parametric form directly analogous with the normalised form of cam angle and lift and the non-dimensional parameters for follower velocity, acceleration and jerk associated with single-law motions.

The relationship between kinematic performance and the proportions of the components forming a blended motion are investigated in Examples 5 and 6. A general solution showing the variation of the peak values of follower velocity and acceleration for acceleration ratios over the range, $0.4 \leq k_A \leq 2.5$, is given in Example 5 for a combination of Modified Sinusoidal Acceleration motion (see Item 82006^[6]) and Constant-Cosine-Constant Acceleration motion (see Section 7). The significance of the acceleration ratio upon the cam angle corresponding to follower displacements of 80% and 90% of the lift is investigated. This study has been extended beyond the practical range to show the trend clearly. For comparison, the corresponding values for $k_A = 1$ of the single-law Triple Harmonic Motion are superimposed upon Sketch 20.4.

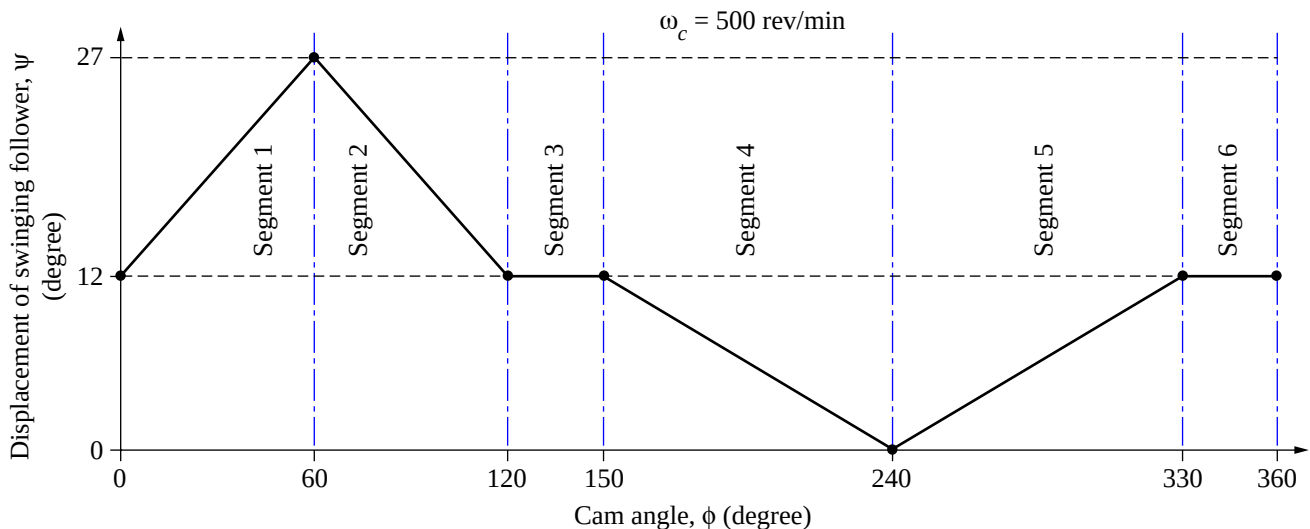
The significance upon the output motion of interposing constant velocity components of varying duration into a blended rise action is examined in Example 6. The combination of Cycloidal motion and SHM is used in the same manner as for Example 4.

Example 7 illustrates the application of the procedures for determining asymmetric DRFD follower motions (see Item 06003^[16]). In Example 7 the working output motion of the end effector, incorporating an intermediate constant-velocity component, is driven by the rise action from the dwell to the maximum displacement. The fall begins immediately. The solution of the rise using the principles given in Item 06002^[15] provides some starting data for the design of the idle fall motion. Additional design constraints are imposed on the fall by a restriction upon the peak retardation and the required minimum duration of the dwell between operating cycles. Comparison of alternative profiles demonstrates the need for a blended profile. It is concluded by a sensitivity analysis to illustrate an approach to finding a compromise between conflicting requirements.

Throughout these examples it is assumed that the cam rotates at a nominally 'constant' speed.

16. EXAMPLE 1 – CONTROL OF SEPARATE DRFD AND DFRD MOTIONS

This manually worked example[†] illustrates the application of the equations defining DRFD cam laws to the active motion segments of a cam rotating at 500 rev/min to drive a swinging rolling follower with separate DRFD and DFRD actions. The follower dwells at the same intermediate position between the two actions. The timing diagram (see Item 03010^[14]) for the follower motion during one complete rotation of the cam is given in Sketch 16.1.



Sketch 16.1 Timing diagram for cam driving a swinging follower with separate DRFD and DFRD actions

Sketch 16.1 shows that the profile consists of six motion segments. The motion starts with a DRFD action from a dwell, which is an intermediate follower position above the prime circle. It begins with a rise action during Segment 1 and is followed immediately by a fall in Segment 2 to the initial start position. The angles of rise and fall are both 60° and the following swing is 15°. After a dwell in Segment 3, the follower begins a DFRD action. The fall occurs during Segment 4 and the rise in Segment 5 followed by a dwell in Segment 6. At the end of Segment 5 the follower is restored to the starting position. The angles of fall and rise are both 90° and the follower swing is 12°. Consequently, the cam profile touches the Reference Circle only at the junction of Segments 4 and 5 where the follower velocity reverses, midway through the DFRD action. This inner limit of follower travel is designated as the reference datum from which the follower displacement is measured.

In the absence of constraints requiring any intermediate precision points or constant velocity components in either action, the 2-4-5 Polynomial Cam law was chosen for Segments 1 and 2 and the 3-4-5-6 Polynomial Cam law for Segments 4 and 5. These laws comply with the boundary conditions defined in Tables 3.1 and 3.2. With these cam laws the jerk is finite throughout the motion and the peak values of the non-dimensional velocity and acceleration parameters, $|w'_{max}|$ and $|w''_{max}|$, respectively, have similar magnitude to those of established DRD cam laws (see Tables in Appendix D).

[†] Equations, sketches, tables, appendices, etc. identified by section numbers not found in this Item are to be found in: Sections 1 to 8 and Appendices A to D in Item 06002^[15] and Sections 9 to 13 in Item 06003^[16].

Expressed in rad/s, the cam speed of 500 rev/min is

$$\omega_c = 2\pi \frac{500}{60} = 52.3599 \text{ rad/s} . \quad (16.1)$$

To avoid cumulative error, values needed for subsequent calculations are given to greater accuracy than the magnitudes of dimensions, velocities, *etc.* that are rounded to 2 decimal places.

Segment 1 – Rise of DRFD action ($0 \leq \phi \leq 60^\circ$)

The cam angle, ϕ , is measured from the start of this motion segment. The follower motion during the rise of a DRFD action is defined by Equations (3.2b) to (3.6b). The follower motion during the subsequent fall motion is defined by the parametric Equations (3.7) to (3.10). The boundary conditions defining the follower motion during a DRFD action are given in Table 3.1. Reading from the timing diagram, Sketch 16.1, the angle of cam rotation for this motion segment is

$$\Phi_1 = 60 \frac{\pi}{180} = 1.0472 \text{ rad} \quad (16.2)$$

and the follower swing is

$$\Psi_1 = (27 - 12) \frac{\pi}{180} = 15 \frac{\pi}{180} = 0.2618 \text{ rad}. \quad (16.3)$$

From Equation (7.9), the follower displacement from the reference point for the 2-4-5 Polynomial cam law is given by

$$\begin{aligned} \psi_1 &= \psi_{\text{dwell}} + \Psi_1 \left(1 - \frac{10(1-u_1)^2}{3} + 5(1-u_1)^4 - \frac{8(1-u_1)^5}{3} \right) \\ &= 12 + 15 \left(1 - \frac{10(1-u_1)^2}{3} + 5(1-u_1)^4 - \frac{8(1-u_1)^5}{3} \right), \end{aligned} \quad (16.4)$$

in which the normalised cam angle parameter, u_1 , is

$$u_1 = \frac{\phi}{\Phi_1}. \quad (16.5)$$

From Equation (3.4b), the follower velocity is given by

$$\begin{aligned} \dot{\psi} &= \frac{w'_1 \omega_c \Psi_1}{\Phi_1} \\ &= \frac{w'_1 \times 52.3599 \times 0.2618}{1.0472} \text{ rad/s}, \end{aligned} \quad (16.6)$$

where w'_1 is found from Equation (7.10) as follows

$$w'_1 = \frac{20}{3} (1 - u_1) - 20(1 - u_1)^3 + \frac{40}{3} (1 - u_1)^4.$$

From Equation (3.5b) the follower acceleration is given by

$$\begin{aligned} \ddot{\psi}_1 &= \frac{w''_1 \omega_c^2 \Psi_1}{\Phi_1^2} \\ &= \frac{w''_1 \times 52.3599^2 \times 0.2618}{1.0472^2} \text{ rad/s}^2, \end{aligned} \quad (16.7)$$

where w''_1 is found from Equation (7.11) as

$$w''_1 = -\frac{20}{3} + 60(1 - u_1)^2 - \frac{160}{3} (1 - u_1)^3.$$

The magnitude of the normalised and non-dimensional parameters defining the boundary conditions of this cam law, given in Table D1.3, were found by numerical analysis. By substituting $|w'_{max}| = 1.7332$ in Equation (16.6), the maximum follower velocity is 22.69 rad/s which occurs at a cam angle of

$$\phi = 60^\circ \times 0.578465 = 34.71^\circ. \quad (16.8a)$$

Similarly, by substituting $|w''_{max}| = 4.583$ in Equation (16.7), the maximum intermediate follower acceleration is positive and has a magnitude of $\ddot{\psi} = 2999.78 \text{ rad/s}^2$. It occurs at a cam angle of

$$\phi = 60^\circ \times 0.25 = 15^\circ. \quad (16.8b)$$

The peak follower retardation occurs at the extreme displacement, *i.e.* when $\phi = 60^\circ$, given by substituting $w''_{max} = -20/3$ in Equation (16.7) to obtain $\ddot{\psi} = 4363.32 \text{ rad/s}^2$.

Segment 2 – Fall of the DRFD action ($60^\circ \leq \phi \leq 120^\circ$)

The cam angle and follower swing are the same as those for Segment 1, so the absolute magnitudes of the follower velocity and acceleration are the same, except that the velocity is towards the reference datum and so is negative. To ensure there is no discontinuity in the jerk, the acceleration at the start of Segment 2 must equal that at the end of Segment 1. In addition, the acceleration must be zero at the end of Segment 2 to blend smoothly with the subsequent dwell in Segment 3. The normalised follower displacement in Segment 2 is defined by Equation (3.7), *i.e.*

$$w_{fall} = f\{1 - u_{fall}\}, \quad (16.9)$$

where
$$u_{fall} = u_2 = \frac{\phi - 60}{\Phi_2}.$$

Hence the $(1 - u)$ term of Equation (7.9) for the 2-4.5 Polynomial cam law becomes

$$f(1 - (1 - u_2)) = f(u_2). \quad (16.10)$$

Substituting, the follower displacement during the fall is given by

$$\begin{aligned}\Psi_2 &= \Psi_{\text{dwell}} + \Psi_2 \left(1 - \frac{10}{3} u_2^2 + 5 u_2^4 - \frac{8}{3} u_2^5 \right) \\ &= 12 + 18 \left(1 - \frac{10}{3} u_2^2 + 5 u_2^4 - \frac{8}{3} u_2^5 \right).\end{aligned}\quad (16.11)$$

The follower velocity for the fall is derived from Equations (3.4b) and (3.8) to give

$$\begin{aligned}\dot{\Psi}_2 &= - \frac{w'_2 \omega_c \Psi_2}{\Phi_2} \\ &= - \frac{w'_2 \times 52.3599 \times 0.2618}{1.0472} \text{ rad/s},\end{aligned}\quad (16.12)$$

where w'_2 is found from Equations (7.10) and (3.9) as

$$w'_2 = - \frac{20}{3} u_2 + 20 u_2^3 - \frac{40}{3} u_2^4. \quad (16.13)$$

The follower acceleration for the fall is derived from Equations (3.5b) and (3.9) to give

$$\begin{aligned}\ddot{\Psi}_2 &= \frac{w''_2 \omega_c^2 \Psi_2}{\Phi_2^2} \\ &= \frac{w''_2 \times 52.3599^2 \times 0.2618}{1.0472^2} \text{ rad/s}^2,\end{aligned}\quad (16.14)$$

where w''_2 is found from Equations (7.11) and (3.9) as

$$w''_2 = - \frac{20}{3} + 60 u_2^2 - \frac{160}{3} u_2^3. \quad (16.15)$$

The magnitudes of the peak follower velocity and acceleration equal those for Segment 1, except that the velocity is negative. From Table D1.3 the maximum follower velocity, $\dot{\Psi} = -22.69 \text{ rad/s}$, occurs at a cam angle of

$$\phi = 60^\circ + (120^\circ - 60^\circ)(1 - 0.578465) = 85.29^\circ. \quad (16.16a)$$

The maximum follower retardation occurs at the extreme displacement, *i.e.* at $\phi = 60^\circ$ and $u_2 = 0$. So, from the analysis of Segment 1, $\ddot{\Psi} = -4363.32 \text{ rad/s}^2$, at this position. As in Segment 1 the peak follower acceleration, $\ddot{\Psi} = 2999.78 \text{ rad/s}^2$, which occurs at a cam angle of

$$\phi = 60^\circ + (120^\circ - 60^\circ) \times (1 - 0.25) = 105^\circ. \quad (16.16b)$$

Segment 3 – Dwell ($120^\circ \leq \phi \leq 150^\circ$)

As shown in the timing diagram, Sketch 16.1, the follower is held at the intermediate dwell position throughout Segment 3 so the follower displacement is given by $\psi_3 = \psi_{dwell} = 12^\circ$.

Segment 4 – Fall of DFRD action ($150^\circ \leq \phi \leq 240^\circ$)

The cam angle, ϕ , is measured from the start of Segment 1. The follower motion during the fall of a DFRD action is shown in Sketch 3.2 and defined by the parametric Equations (3.11) to (3.14) and that for the subsequent rise is defined by the parametric Equations (3.15) to (3.19). During this segment the follower moves towards the reference datum, so it has negative velocity. Reading from the timing diagram, Sketch 16.1, the angle of cam rotation for this motion segment is

$$\Phi_4 = (240^\circ - 150^\circ) \frac{\pi}{180^\circ} = 1.5708 \text{ rad} \quad (16.17)$$

and the follower swing is

$$\Psi_4 = 12^\circ \frac{\pi}{180^\circ} \equiv 0.2094 \text{ rad}. \quad (16.18)$$

From Equations (3.11) and (7.13), the follower displacement from the reference point for the 3-4-5-6 Polynomial cam law is given by

$$\begin{aligned} \psi_4 &= \psi_{dwell} - \Psi_4 \left(8 u_4^3 - 12 u_4^4 + 6 u_4^5 - u_4^6 \right) \\ &= 12 - 12 \left(8 u_4^3 - 12 u_4^4 + 6 u_4^5 - u_4^6 \right), \end{aligned} \quad (16.19)$$

where the normalised cam angle parameter, u_4 , is

$$u_4 = \frac{\phi - 150}{\Phi_4}. \quad (16.20)$$

The follower velocity for the fall is derived from Equations (3.4b) and (3.12) to give

$$\dot{\psi}_4 = - \frac{w'_4 \times 52.3599 \times 0.2094}{1.5708} \text{ rad/s}, \quad (16.21)$$

where w'_4 is found from Equations (7.14) and (16.20) as

$$w'_4 = 24 u_4^2 - 48 u_4^3 + 30 u_4^4 - 6 u_4^5. \quad (16.22)$$

The follower acceleration for the fall is derived from Equations (3.5b) and (3.13) to give

$$\ddot{\psi}_4 = - \frac{w''_4 \times 52.3599^2 \times 0.2094}{1.5708^2} \text{ rad/s}^2, \quad (16.23)$$

where w_4'' is found from Equations (7.11) and (16.20) as

$$w_4'' = 48 u_4 - 144 u_4^2 + 120 u_4^3 - 30 u_4^4 . \quad (16.24)$$

The magnitude of the normalised and non-dimensional parameters defining the boundary conditions of this cam law, given in Table D1.3 were found by numerical analysis. By substituting $|w_{max}'| = 1.7173$ in Equation (16.21), the maximum follower velocity is -11.99 rad/s which occurs at a cam angle of

$$\phi = 150^\circ + (240^\circ - 150^\circ) \times 0.5528 = 199.75^\circ . \quad (16.25a)$$

Similarly, by substituting $|w_{max}''| = 4.8$ in Equation (16.23), the peak intermediate follower acceleration is $\ddot{\psi} = -1117.01$ rad/s² which occurs at a cam angle of

$$\phi = 150^\circ + (240^\circ - 150^\circ) \times 0.2254 = 170.29^\circ . \quad (16.25b)$$

The maximum follower retardation, at $u_4 = 1$, is found by substituting $|w_{max}''| = 6$ in Equation (16.23) to give $+1396.26$ rad/s².

Segment 5 – Rise of the DFRD action ($240^\circ \leq \phi \leq 330^\circ$)

The cam angle and the follower swing are the same as those for Segment 4, so the absolute magnitudes of the peak follower velocity and accelerations are also the same. During this segment the follower moves away from the reference datum, so it has positive velocity. To comply with the boundary conditions in Table 3.2 and so ensure no discontinuity in the jerk, the acceleration at the start of Segment 5 must equal that at the end of Segment 4. In addition, the acceleration must be zero at the end of Segment 5 to blend smoothly with the subsequent dwell. The normalised follower displacement is defined by Equation (3.16), i.e. as

$$w_5 = 1 - f(1 - u_5) . \quad (16.26)$$

Substituting $1 - u_5$ for u in Equation (7.13), the follower displacement from the intermediate dwell position is given by

$$\begin{aligned} \psi_5 &= \psi_{dwell} - \Psi_5 \left(8(1-u_5)^3 - 12(1-u_5)^4 + 6(1-u_5)^5 - (1-u_5)^6 \right) \\ &= 12 - 12 \left(8(1-u_5)^3 - 12(1-u_5)^4 + 6(1-u_5)^5 - (1-u_5)^6 \right) , \end{aligned} \quad (16.27)$$

where the normalised cam angle parameter, u_5 , is

$$u_5 = \frac{\phi - 240}{\Phi_5} . \quad (16.28)$$

The segment angle, Φ_5 , is given by Equation (16.17) and the follower swing, Ψ_5 , by Equation (16.18).

The follower velocity for the rise is derived from Equation (3.4b) to give

$$\dot{\psi}_5 = \frac{w'_5 \times 52.3599 \times 0.2094}{1.5708} \text{ rad/s}, \quad (16.29)$$

where w'_5 is found from Equations (3.17), (7.14) and (16.28) as

$$w'_5 = 24(1 - u_5)^2 - 48(1 - u_5)^3 + 30(1 - u_5)^4 - 6(1 - u_5)^5. \quad (16.30)$$

The follower acceleration for the rise is derived from Equation (3.5b) to give

$$\ddot{\psi}_5 = -\frac{w''_5 \times 52.3599^2 \times 0.2094}{1.5708^2} \text{ rad/s}^2, \quad (16.31)$$

where w''_5 is found from Equations (3.18), (7.15) and (16.28) as

$$w''_5 = 48(1 - u_5) - 144(1 - u_5)^2 + 120(1 - u_5)^3 - 30(1 - u_5)^4. \quad (16.32)$$

The magnitude of the normalised and non-dimensional parameters defining the boundary conditions of this cam law, given in Table D1.3 were found by numerical analysis. By substituting $|w'_{max}| = 1.7173$ in Equation (16.29), the maximum follower velocity is +11.99 rad/s which, from Table D1.3, occurs at a cam angle of

$$\phi = 240^\circ + (330^\circ - 240^\circ)(1 - 0.5528) = 280.25^\circ. \quad (16.33a)$$

Similarly, by substituting $|w''_{max}| = 4.8$ in Equation (16.31), the peak intermediate follower acceleration is $\ddot{\psi} = 1117.01 \text{ rad/s}^2$ which, from Table D1.3, occurs at a cam angle of

$$\phi = 240^\circ + (330^\circ - 240^\circ)(1 - 0.2254) = 309.71^\circ. \quad (16.33b)$$

The maximum follower retardation, at $\phi = 240^\circ$, i.e. $(1 - u_5) = 0$, is found by substituting $|w''_{max}| = 6$ in Equation (16.31) to give +1396.26 rad/s².

The curves showing the follower displacement, velocity and acceleration during a complete rotation of the cam are given in Sketch 16.2.

Segment 6 – Dwell ($330^\circ \leq \phi \leq 360^\circ$)

The follower is held at the intermediate dwell position whilst the cam rotates through 30° . As shown in Sketch 16.1, the design requirement for Segments 4 and 5 specifies the dwell to be 12° above the innermost position of the follower and so the follower displacement is given by

$$\psi_{dwell} = \psi_6 = 12^\circ. \quad (16.34)$$

It is noted that at either end of the 2-4-5 Polynomial cam law joining a dwell the jerk has a finite discontinuity (see Sketch 7.3).

Commentary

The 2-4-5 and 3-4-5-6 Polynomial cam laws were chosen for the DRFD and DFRD actions, respectively, because both comply with the boundary conditions specified in Tables 3.1 and 3.2. However, inspection of Sketches 7.3 and 7.4 shows finite discontinuities in the jerk curves of both motions at the start of the rise. Therefore the fourth derivatives of their cam law displacement functions are infinite at this position. The jerk is found by re-arranging Equation (3.6b) to give

$$\ddot{\psi} = w''' \omega_c^3 \Psi\left(\frac{180}{\pi\Phi}\right)^3. \quad (16.35)$$

Considering Segment 1, from Table D1.3 the magnitude of the jerk of 2-4-5 Polynomial Motion at $u = 0$ is $w''' = 40$. Substituting the values from the analysis above into Equation (16.35), the magnitude of the step change of jerk at the start of Segment 1 is

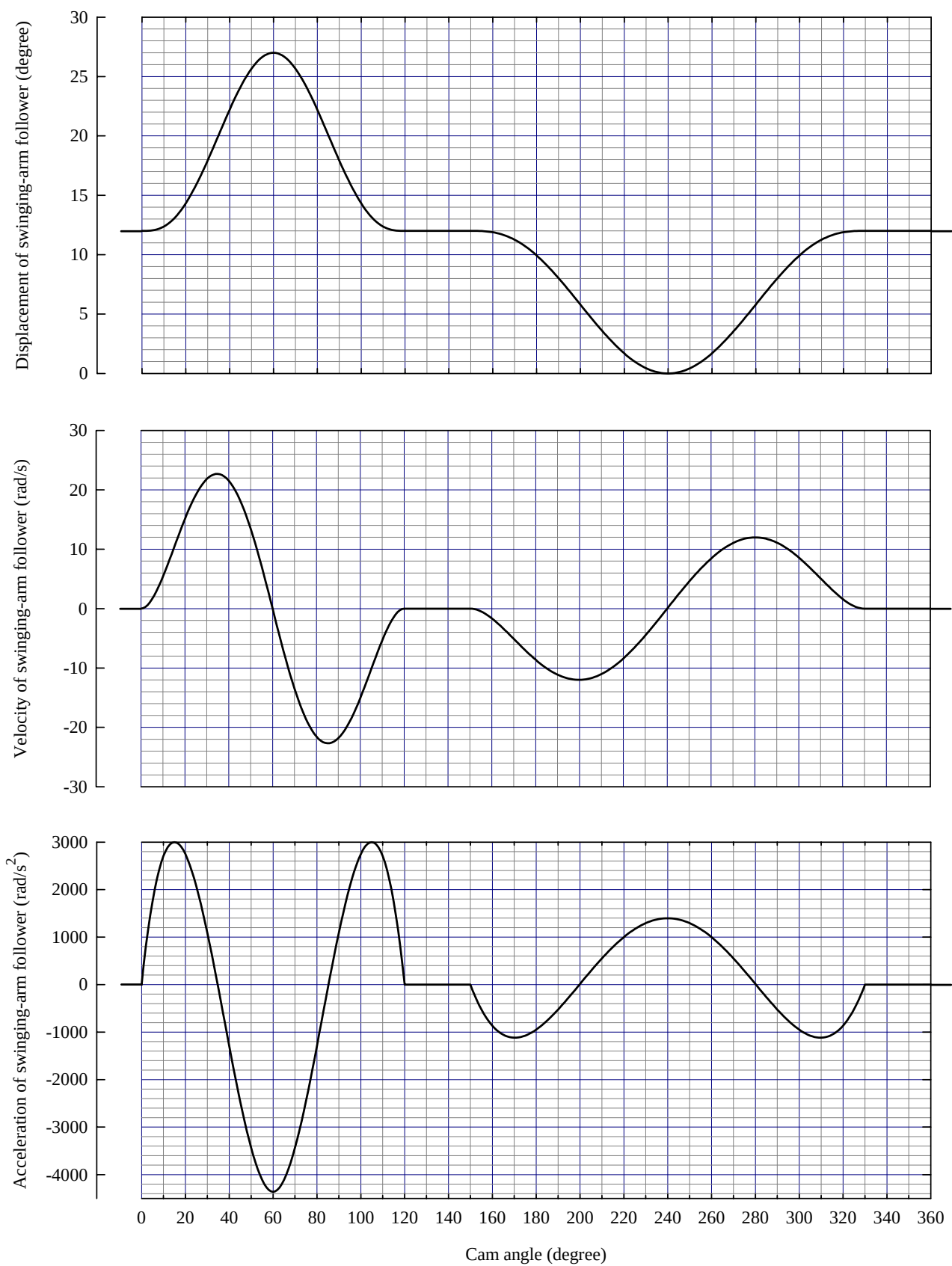
$$\ddot{\psi}_{\phi=0} = \frac{40 \times 52.3599^3 \times 0.2618}{1.0472^3} = 1308892 \text{ rad/s}^3.$$

Similarly, the magnitude of the step change of jerk at the start of the fall in Segment 4 is 372265 rad/s³.

Section 6 contains the warning “transient oscillations will occur as a result of a discontinuity in the cam law displacement function or any derivative of that function. The lower the order of the derivative in which the first discontinuity occurs, the more serious the resulting vibrations”. Replacing the polynomial cam laws used in Example 1 by one having zero jerk at all limits of follower travel *e.g.* Triple Harmonic Motion (see Section 7.2), advances the first discontinuity to the fourth derivative. This benefit is liable to be obtained at the expense of increased accelerations. These are greatest in Segments 1 and 2, so the critical values of the two cam laws are compared in Table 16.1.

TABLE 16.1 Comparison of critical values of follower motion in Segment 1

Cam Law	Cam Angle (degree)	Follower Velocity (rad/s)	Follower Acceleration (rad/s ²)	Follower Jerk (rad/s ³)
2-4-5 Polynomial Motion	0	0	0	1308992
	15		2999.78	0
	34.71	22.69	0	-724970
	60	0	-4363.32	0
Triple Harmonic Motion	0	0	0	0
	20.73		4036	0
	30.09	26	0	-1270197
	60	0	-4153	0



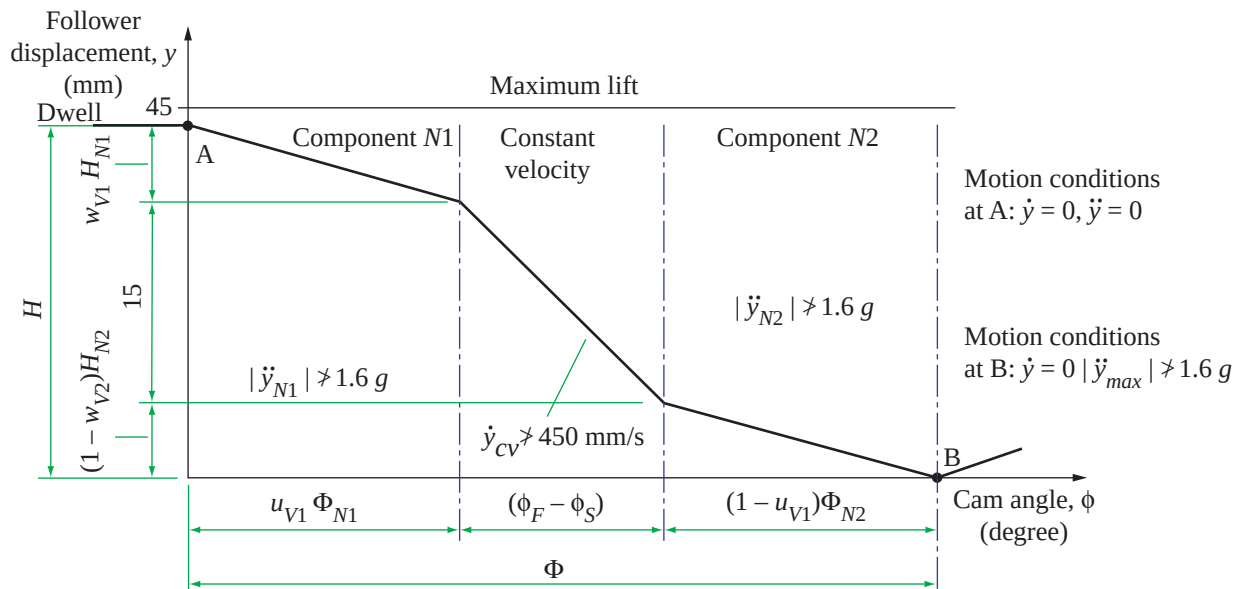
Sketch 16.2 Displacement, velocity and acceleration of follower for Example 1

17. EXAMPLE 2 – PRINCIPAL DIMENSIONS OF FALL MOTION SEGMENT FOR DFVRD MOTION TO SUIT SPECIFIED MAXIMA

This example[†] is concerned with the fall motion segment of a cam having a blended DFVRD profile to drive a translating follower. The rise action is the mirror-image of the fall. For a cam speed of 180 rev/min, the maximum follower velocity must not exceed an absolute value of 450 mm/s, nor may the peak follower acceleration exceed an absolute value of 1.6 g . The follower displacement during the constant velocity components of the fall and rise motions is 15 mm. The lift must not exceed 45 mm. The remaining dimensions are to be determined. Cycloidal motion (see Item 82006^[6]) is specified for the components blending with the dwell and Double Harmonic motion (see Section 7.1) for the components controlling the reversal of follower velocity.

These data are superimposed on the partial timing diagram (see Item 03010^[14]) given in Sketch 17.1 for the DFVRD fall action only. The symbols identify unknown dimensions which need to be found.

This combination of specified parameters is analysed in Case 1 of Appendix B2.



Sketch 17.1 Partial timing diagram for fall of DFVRD action with translating follower

Basic Dimensions

The cam speed, ω_c , is given by

$$\omega_c = 180 \text{ rev/min} \equiv 2\pi \frac{180}{60} = 6\pi = 18.849556 \text{ rad/s.} \quad (17.1)$$

The follower velocity during the constant velocity component is taken to be the maximum permitted value of

$$\dot{y}_{CV} = 450 \text{ mm/s.} \quad (17.2)$$

[†] Equations, sketches, tables, appendices, etc. identified by section numbers not found in this Item are to be found in: Sections 1 to 8 and Appendices A to D in Item 06002^[15] and Sections 9 to 13 in Item 06003^[16].

Similarly, the peak follower acceleration is taken to be the maximum permitted value of

$$|\ddot{y}|_{\max} = 1.6 \, g \equiv 15\,700 \, \text{mm/s}^2 \quad (17.3)$$

Component N1 – Cycloidal Motion

Dwell to blend at transition point.

From Table D1.1, the significant parameters are given by the following.

$$\text{Normalised cam angle start to blending point, } u_{V1} = 0.5. \quad (17.4a)$$

$$\text{Normalised follower displacement at blending point, } w_{V1} = 0.5. \quad (17.4b)$$

$$\text{Maximum non-dimensional follower velocity, } |w'_{\max}| = 2.0. \quad (17.4c)$$

$$\text{Maximum non-dimensional follower acceleration, } |w''_{\max}| = 2\pi. \quad (17.4d)$$

The notional segment angle, Φ_{N1} , from Equation (B2.3) is

$$\Phi_{N1} = \frac{|\dot{y}_{CV}| |w''_{N1}|_{\max} \omega_c}{|\ddot{y}_{N1}|_{\max} |w'_{N1}|_{\max}} \left(\frac{180}{\pi} \right)$$

and so substituting known values into this equation gives

$$\Phi_{N1} = \frac{450 \times 2\pi \times 6\pi}{15\,700 \times 2} \frac{180}{\pi} = 97.2493^\circ. \quad (17.5)$$

The notional lift, H_{N1} , from Equation (B2.4) is

$$H_{N1} = \frac{|\dot{y}_{CV}| \Phi_{N1}}{|w'_{N1}|_{\max} \omega_c} \left(\frac{\pi}{180} \right)$$

and so substituting known values into this equation gives

$$H_{N1} = \frac{450 \times 97.2493}{2 \times 6\pi} \frac{\pi}{180} = 20.2603 \, \text{mm}. \quad (17.6)$$

Constant Velocity Component

From Equation (B2.7), the cam angle corresponding to the constant velocity component is given by

$$|\phi_F - \phi_S| = \frac{|y_F - y_S| \omega_c}{|\dot{y}_{CV}|} \frac{180}{\pi} = \frac{15 \times 6\pi}{450} \frac{180}{\pi} = 36^\circ. \quad (17.7)$$