



ESDU 06010

Issued August 2006

With Amendment A

March 2012

Cycle counting methods for the estimation of fatigue life

**Associated software: ESDUpac A0610
ESDUpac B0610**



**ROYAL
AERONAUTICAL
SOCIETY**

Endorsed by
The Royal Aeronautical Society

AVA Documents
Document Sharing Hub

ESDU DATA ITEMS

Data Items provide validated information in engineering design and analysis for use by, or under the supervision of, professionally qualified engineers. The data are founded on an evaluation of all the relevant information, both published and unpublished, and are invariably supported by original work of ESDU staff engineers or consultants. The whole process is subject to independent review for which crucial support is provided by industrial companies, government research laboratories, universities and others from around the world through the participation of some of their leading experts on ESDU Technical Committees. This process ensures that the results of much valuable work (theoretical, experimental and operational), which may not be widely available or in a readily usable form, can be communicated concisely and accurately to the engineering community.

We are constantly striving to develop new work and review data already issued. Any comments arising out of your use of our data, or any suggestions for new topics or information that might lead to improvements, will help us to provide a better service.

THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Vibration and Acoustic Fatigue Committee. This Committee first met in 1966 and now has the following membership:

Chairman

Prof R.D. Adams — University of Bristol

Members

Dr N.S. Ferguson	— University of Southampton
Mr S. Georgiou	— SELEX Sensors and Airborne Systems Ltd, Luton
Mr A.R. Harris*	— Northrop Grumman, USA
Dr Hilmi Kurt-Elli	— Rolls Royce plc, Derby
Mr D.M.A. Millar*	— Independent
Dr M. Nash	— QinetiQ, Farnborough
Mr E. Smyth	— Bombardier Aerospace Short Brothers plc, Belfast
Prof R.G. White	— University of Southampton
Prof J.J. Wijker	— Dutch Space, Netherlands

* Corresponding Member

The construction and subsequent development of the Data Item was undertaken by

Dr J.S. Anderson	— Senior Engineer.
Dr M. Bratos-Anderson	— Consultant

The person with overall responsibility for the work in this subject area is Dr C.B. Chinoy, Head of the Aircraft Noise and Structural Dynamics Group.

CYCLE COUNTING METHODS FOR THE ESTIMATION OF FATIGUE LIFE

CONTENTS

	Page
1. NOTATION	1
2. INTRODUCTION	3
3. PEAK-VALLEY CYCLE COUNTING	5
4. RANGE COUNTING METHOD	7
5. RAINFLOW CYCLE COUNTING	8
5.1 Four-point Cycle Counting Method	10
5.2 Three-point Cycle Counting Method	11
5.3 Example of Rainflow Cycle Extraction	11
5.4 Probability Density Functions for Stress Cycle Ranges	13
6. COMPUTER PROGRAMS FOR ESTIMATION OF PDF AND FATIGUE LIFE BY RAINFLOW COUNTING	19
6.1 Input Data	20
6.1.1 Relationship between K_r and K_{rms}	20
6.2 Data Output	22
6.3 Examples	22
6.3.1 Example 1	22
6.3.2 Example 2	24
6.3.3 Example 3	27
6.3.4 Example 4	30
7. REFERENCES	33

ESDU 06010 A 2012

CYCLE COUNTING METHODS FOR THE ESTIMATION OF FATIGUE LIFE

1. NOTATION

		UNITS	
		SI	British
b	exponent associated with $S - N$ curve (material constant)		
D	cumulative fatigue damage		
D_{LCC}	mean fatigue damage estimated by level crossing counting		
D_{PC}	mean fatigue damage estimated by peak-valley counting		
D_{RC}	mean fatigue damage estimated by range counting		
D_{RFC}	mean fatigue damage estimated by rainflow cycle counting		
F_p	expected (mean) number of peaks per second	Hz	Hz
H	mean value of height of rise and fall of time history	m	in
K_a	constant in $S_a - N$ curve relationship (material constant)	(Pa) ^b	(1bf/in ²) ^b
K_r	constant in $S_r - N$ curve relationship (material constant)	(Pa) ^b	(1bf/in ²) ^b
K_{rms}	constant in $S_{rms} - N$ curve relationship (material constant)	(Pa) ^b	(1bf/in ²) ^b
N	total number of cycles in considered time history		
N_i	number of cycles to failure at S_{ri} stress range		
N_{max}	number of maxima		
N_{min}	number of minima		
N_{mc}	number of mean crossings		
n_i	number of cycles with defined stress range, S_{ri}		
P_i	probability		
$p_{pr}(S_r)$	probability density function of peaks	(Pa) ⁻¹	(1bf/in ²) ⁻¹
$p_r(S_r)$	probability density function for ranges of stress cycles (general)	(Pa) ⁻¹	(1bf/in ²) ⁻¹

Issued August 2006 – 36 pages
With Amendment A March 2012 – 35 pages

UNITS

		SI	British
$p_{rfr}(S_r)$	probability density function for stress ranges of rainflow cycles	$(\text{Pa})^{-1}$	$(\text{lbf/in}^2)^{-1}$
$S(t)$	sample record of stress process	Pa	lbf/in^2
S_a	stress amplitude	Pa	lbf/in^2
S_m	mean stress of stress cycle	Pa	lbf/in^2
S_r	stress range	Pa	lbf/in^2
S_{ri}	stress range (i^{th} class of cycle)	Pa	lbf/in^2
S_{rms}	rms stress	Pa	lbf/in^2
T_{fl}	fatigue life	s	s
γ	irregularity factor		
ε	spectral width parameter		
σ_s	standard deviation of sample record, $S(t)$	Pa	lbf/in^2

Subscripts used:

a	refers to amplitude of stress (strain) cycle
i	numerates class of stress cycles
m	refers to mean values
pr	refers to range of stress cycle in peaks approach
r	refers to range of stress cycle
rf	refers to rainflow cycle
rfr	refers to rainflow cycle range

Abbreviations used in the Item:

LCC	level crossing counting
PC	peak-valley cycle counting
PDF	probability density function
PSD	power spectral density function
RC	range counting
RFC	rainflow cycle counting

2. INTRODUCTION

In fatigue analysis counting the number of cycles of stress at a given amplitude is of fundamental importance in predicting the life of a structure. As real structures may experience wide band random loading, it is useful to analyse the stress response of the structure in the time domain as a sum of constant amplitude events. In other words the time history of the stress can be regarded as damage equivalent sets of stress cycles with a constant amplitude (range) within each set.

The random nature of the stress response necessitates a probabilistic approach to fatigue life estimation. Fatigue life is directly related to the number of cycles in the stress history. In this Item the different methods of counting cycles of stress are considered. A stress range is associated with each cycle. The main purpose of this Item is to find the distribution of the ranges of stress cycles in the time history and hence to determine the probability density function. The PDF is an essential element in the estimation of the fatigue life of the structure.

There are many methods established in cycle counting procedures (References 2, 3, 4, 6, 8, 11, 15, 17, 20 and 25). Level-crossing counting, peak counting, simple range counting, as well as rainflow counting and other procedures related to rainflow counting, are considered in the American Standard (Reference 20). The basic ideas of rainflow methods are presented in an ESDU Item (Reference 17). It should be mentioned that some procedures for counting of stress cycles should be treated with caution, since all counting methods, except rainflow counting and its variants, are not free from errors (References 3, 4, 9, 10 and 18).

A graphic way of visualising the rainflow counting method is to consider rain falling on the roof of a pagoda. The rain drops off one tier of the pagoda onto the larger tier below, and so on. If the pagoda is then turned on its side the outline of the tiers represents the stress and the axis of the pagoda is time. The path of a raindrop defines a stress cycle. Rainflow counting is widely used in fatigue analysis and has become accepted as the most reliable method of dealing with random stress histories from structures with many modes of vibration.

An important application of rainflow counting is in the prediction of the fatigue life of different structures, such as offshore structures and aircraft. Rainflow counting has been used in relation to acoustic fatigue of military aircraft (References 22 and 24). A stress time series is not always available, but a power spectral density function of the stress or strain responses may have been measured. From the PSD of a random, Gaussian process a time series can be simulated and rainflow counting applied to the simulated time series. The resulting probability density function for ranges of rainflow cycles can be compared with that obtained by using spectral methods, discussed in ESDU 06011 (Reference 30).

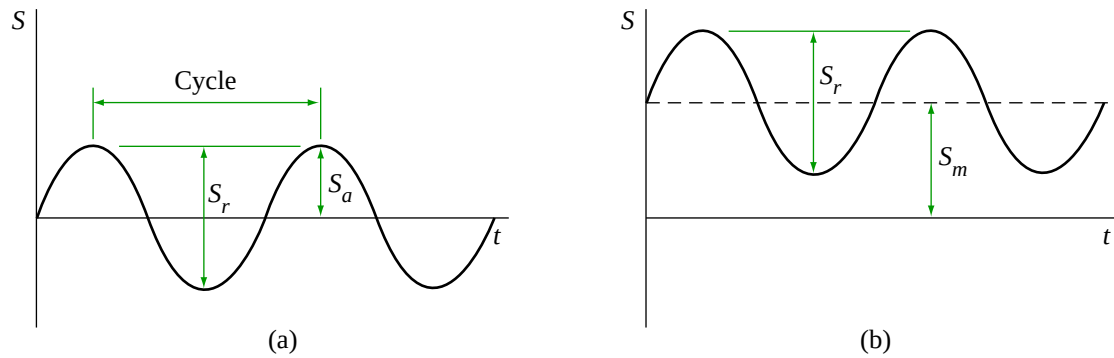
The procedure of rainflow cycle counting refers to the analysis of the variable amplitude response of a structure in the time domain. In agreement with the Miner hypothesis, discussed in ESDU 06009 (Reference 29), the fatigue damage is accumulated linearly after each cycle of stress. The fatigue damage is expressed as

$$D = \sum_i \frac{n_i}{N_i} = N \sum_i \frac{P_i}{N_i}, \quad (2.1)$$

where $N = \sum_i n_i$, and where P_i is the probability that the stress cycle range has magnitude S_{ri} during the process, n_i is the number of cycles in the process (response) with a defined stress range, S_{ri} , and N_i denotes the number of cycles with the same stress range, S_{ri} , to failure.

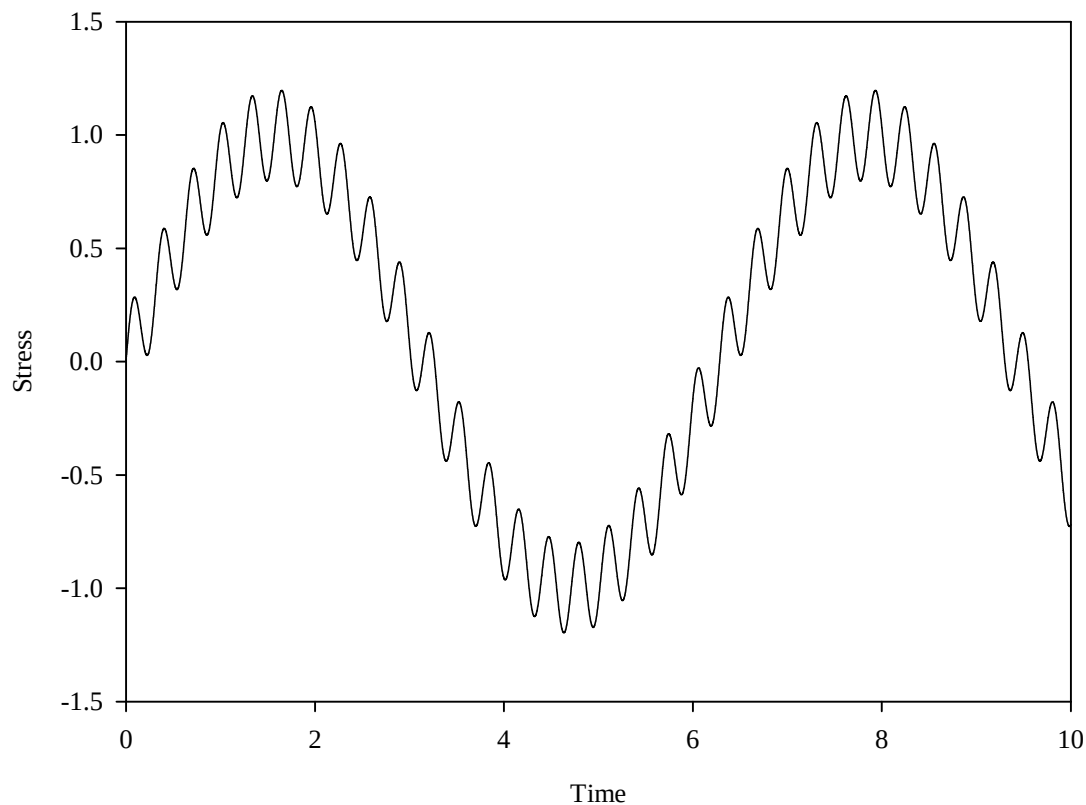
Hence, in fatigue analysis knowledge of the number of rainflow stress cycles in the stress-time history is

of fundamental importance in predicting the life of the structure. In the case when the stress response of the structure is a harmonic function of time, the stress cycle range or stress cycle amplitude can be established easily, see Sketch 2.1.



Sketch 2.1 (a) Fully reversed stress cycle with mean stress $S_m = 0$ and (b) stress cycle with $S_m \neq 0$

Similarly for a particular wide band process, such as that shown in Sketch 2.2, it can be seen that there are different classes of cycles, one with a high stress range and a series with lower stress ranges. However, for more complicated processes, especially wide band random processes, determination of the stress ranges for the cycles is not a straightforward procedure.



Sketch 2.2 Stress cycles of wide band process

The concept of rainflow cycles was introduced in 1968 by Matsuishi and Endo (Reference 2). Since this

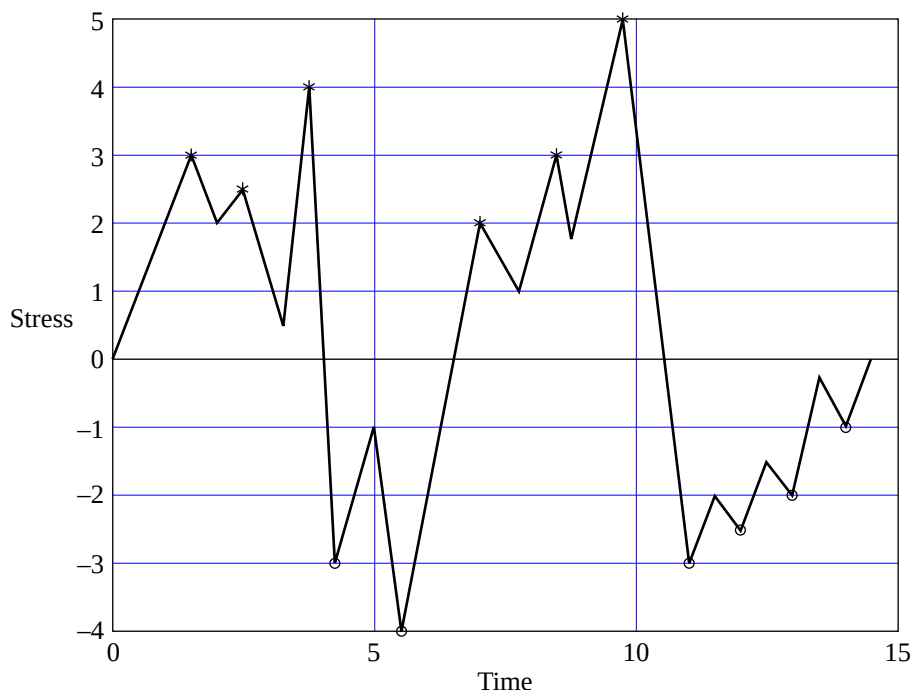
time rainflow counting has become widely used and is accepted as the most reliable method for dealing with random stress histories from structures with many modes of vibration (References 3 and 4). There are several methods, already mentioned, which were used before the adoption of rainflow cycle counting and, because of their inherent simplicity, some of them are still applied to the analysis of fatigue damage. The methods are level crossing cycle counting, peak-valley counting, simple range counting and simple mean-range counting (References 7, 16, 20 and 28). There are also different variations of each method used in practical applications.

Besides rainflow counting the most common method is the peak-valley counting and its variants, the peak method (Reference 27) and mean-crossing peak counting (Reference 20). The peak method is applied widely in experimental work dealing with the estimation of the acoustic fatigue life of structures with multi-modal response (References 19, 21 and 23). The concept of stress cycles associated with the occurrence of peaks is also applied to construct equivalent processes and is used in the design of spectral models (References 12, 27 and 30).

Simple range counting, numerically the simplest of the procedures, significantly underestimates the number of cycles with larger ranges, as shown in Sketch 5.9, and provides a nonconservative estimation of fatigue damage.

3. PEAK-VALLEY CYCLE COUNTING

The peak-valley counting approach is similar to level crossing cycle counting (Reference 20), except that maxima and minima in the process are considered. Only peaks above the mean stress value are counted as well as valleys below this reference level. Taking into account the peaks and valleys, the cycles in the stress-time history are extracted, beginning from the most damaging cycle which is characterised by the highest peak and the lowest valley, followed by the next most damaging cycle extracted from the remaining sample record, and so on. The procedure is illustrated in Sketch 3.1, Tables 3.1 and 3.2 and Sketch 3.2. In Sketch 3.1 the peaks and valleys are identified.



Sketch 3.1 Analysis of the process by peak-valley method (* positive peaks, o negative valleys)

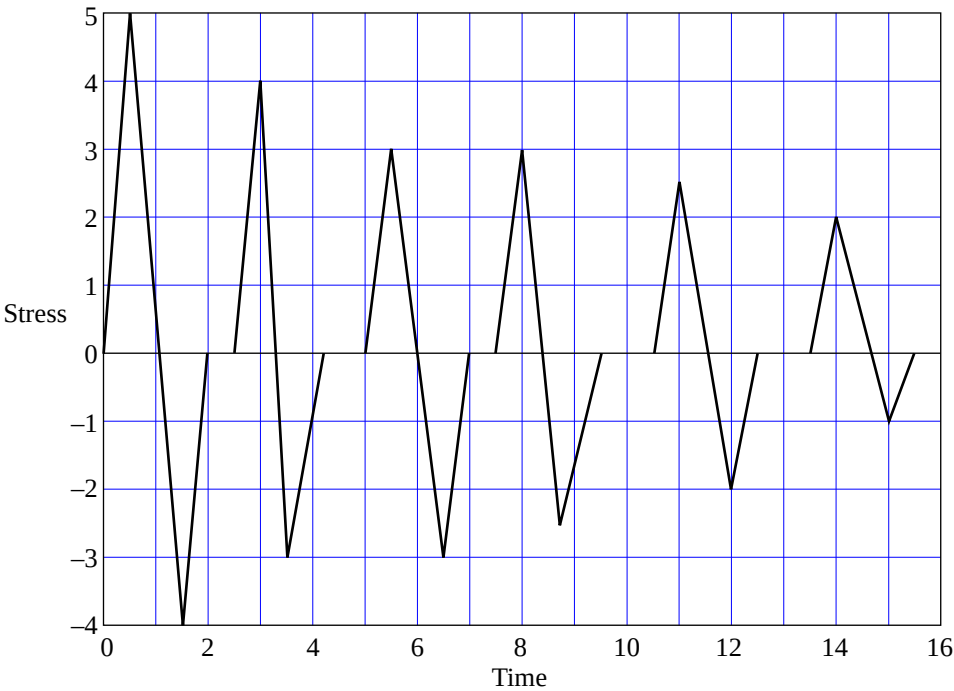
The level counts obtained from Sketch 3.1 are listed in Table 3.1. The cycle ranges with the corresponding number of cycle counts are shown in Table 3.2. The first cycle range of 9 units in Table 3.2 is formed from the largest peak, of value 5, and the lowest valley, of value -4. The second cycle range of 7 units is formed from the second largest peak (value 4) and the valley at -3, and so on. The extracted cycles indicated in Table 3.2 are laid out in Sketch 3.2.

TABLE 3.1

Level	No. of counts
5	1
4	1
3	2
2.5	1
2	1
1	0
0	0
-1	1
-2	1
-2.5	1
-3	2
-4	1
-5	0

TABLE 3.2

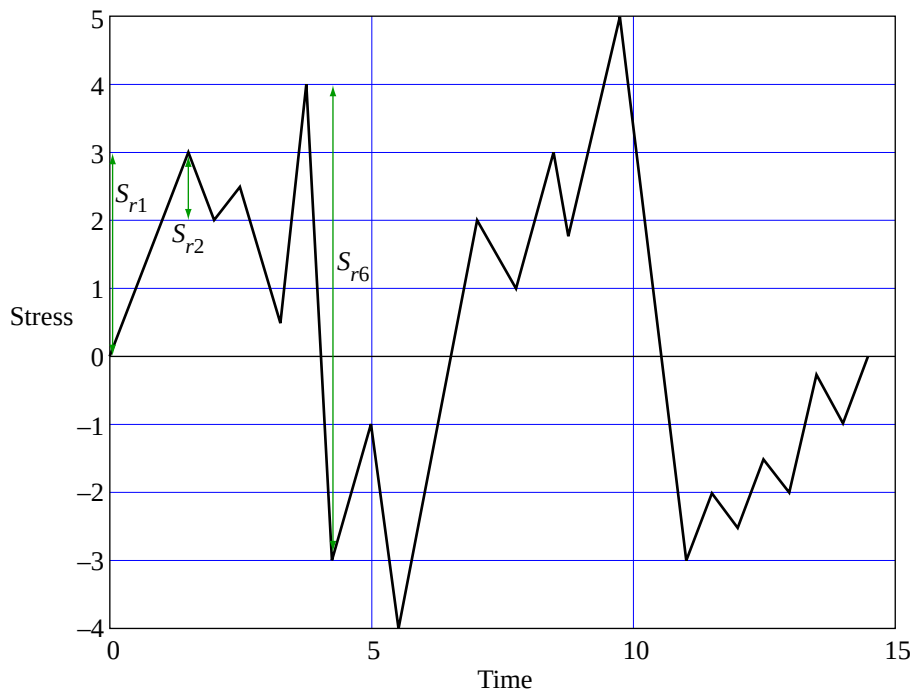
Cycle range	Cycle counts
9	1
7	1
6	1
5.5	1
4.5	1
3	1



Sketch 3.2 Cycles extracted from process in peak-valley approach

4. RANGE COUNTING METHOD

In the range counting method the difference between levels of stress for successive extremes (peak and valley) of the process define the range of a half cycle or reversal. Positive ranges and negative ranges (terms only used in the range counting method), called also the heights of rise and fall (References 3 and 6), can be defined for segments of the process with positive and negative slopes, respectively. The range counting method relies upon local information about the stress process, since each local extreme is associated only with the adjacent preceding and following extremes. Sketch 4.1 illustrates the procedure for range counting. The heights of rise and fall are tabulated in Table 4.1 and the count of cycle ranges in Table 4.2.



Sketch 4.1 Range counting method

For example, S_{r2} is the unit stress range, formed by the second pair of extremes (peak, valley), shown in Sketch 4.1. Similarly, S_{r6} denotes the stress range of value 7 units, defined by the sixth pair of extremes, also peak and valley.

Both the peak-valley and range counting methods can be used to obtain a probability density function (PDF) of the process.

TABLE 4.1

Ranges (heights of rise and fall)	Counts
6.0	1
3.5	1
3.25	1
3.0	1
2.0	2
1.75	1
1.0	3
0.5	1
-0.5	2
-0.75	1
-1.0	2
-1.25	1
-2.0	1
-3.0	1
-7.0	1
-8.0	1

TABLE 4.2

Cycle range	Cycle counts
8.0	0.5
7.0	0.5
6.0	0.5
3.5	0.5
3.25	0.5
3.0	1
2.0	1.5
1.75	0.5
1.25	0.5
1.0	2.5
0.75	0.5
0.5	1.5

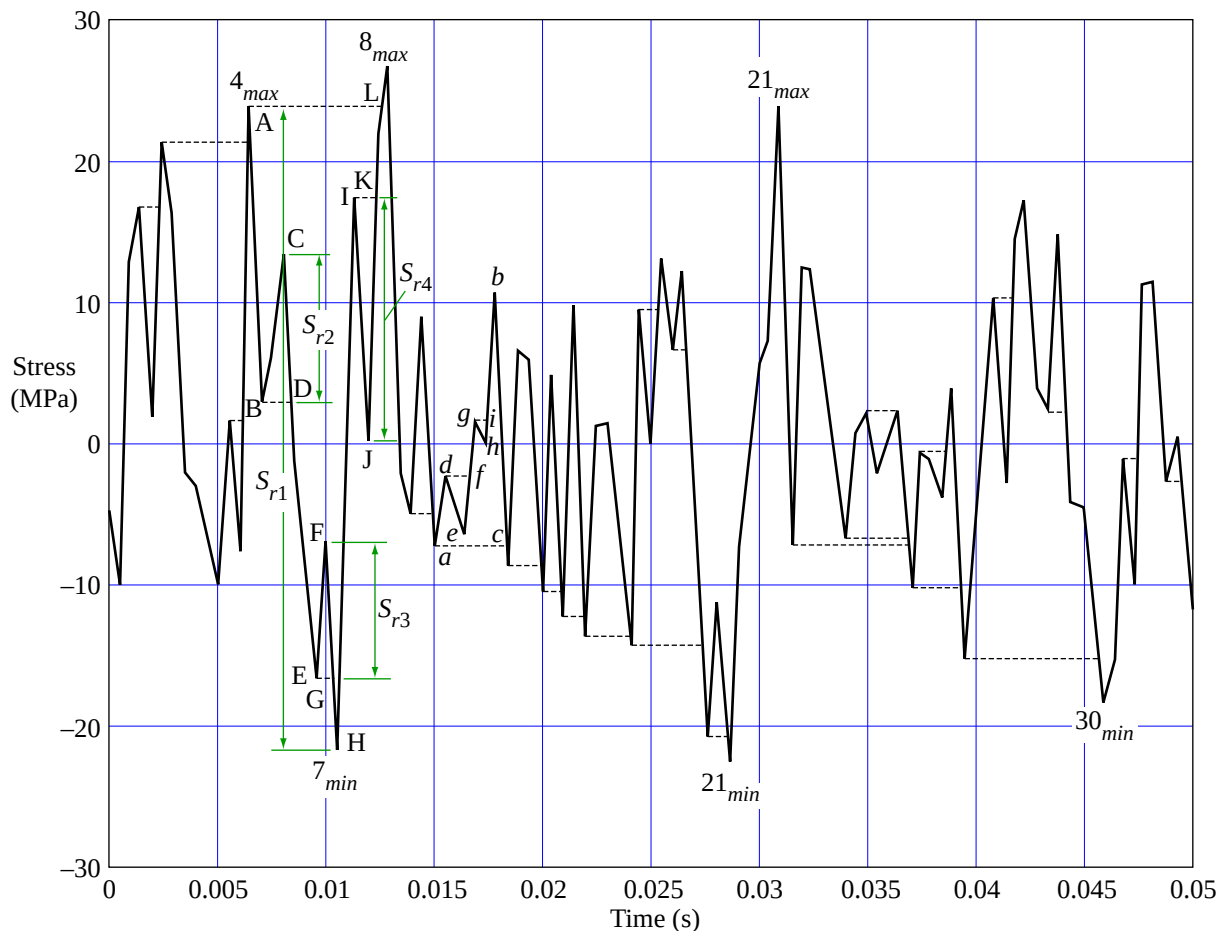
It is worth mentioning that PDFs of positive ranges (height of rise) as well as negative ranges (height of fall) have been investigated in the context of metal fatigue and problems associated with oceanography. The exact analytical PDFs of these ranges are unknown, although several approximate models exist. However, for the special case of a stationary, Gaussian sample record, $S(t)$, it is known that the mean value of the height, H , is:

$$H = \sqrt{2\pi} \gamma \sigma_s. \quad (4.1)$$

Hence, the expected range obtained by the range counting method should be proportional to the irregularity factor, γ (References 1, 7 and 16). The irregularity factor, see ESDU 06009 and 06011, as well as the directly related spectral width parameter, $\varepsilon = (1 - \gamma^2)^{1/2}$, provide a measure of the bandwidth of a random process. The factor γ is defined for a stochastic process as the ratio of the occurrence of the mean level upcrossings to the rate of occurrence of peaks. Values of irregularity factor vary from 0 to 1. Wide band processes are characterised by low value of γ , whilst for narrow band processes values of γ tend to unity.

5. RAINFLOW CYCLE COUNTING

A short sample of a stress time series of 0.05 s is shown in Sketch 5.1. In this diagram the individual rainflow cycles are identified. It may sometimes be useful to imagine Sketch 5.1 rotated through 90°, when it takes on the pagoda form and the dotted lines indicate the paths of the raindrops from pagoda roofs. A cycle is associated with each dotted line.



Sketch 5.1 Rainflow cycles

In Sketch 5.1 some maxima and minima are indicated, for example, 4_{max} (the fourth maximum which is also point A), 8_{max} (the eighth maximum, which has the highest coordinate), 7_{min} and 21_{min} (which has the smallest coordinate). The extremum 8_{max} (the highest maximum) precedes the extremum, 21_{min} (the lowest minimum), hence the analysis of the sample record shown in Sketch 5.1 starts from the first minimum and proceeds to the maximum, 8_{max} , creating half a cycle. Within this half cycle several full cycles are extracted. For example, the fourth cycle is ABCDEFGHIJKL with stress cycle range S_{r1} . Associated with the cycle ABCDEFGHIJKL are smaller cycles BCD, EFG, IJK with stress cycle ranges S_{r2} , S_{r3} and S_{r4} , respectively. The extraction of the smaller cycles may be appreciated by considering the rain drop analogy. If the diagram in Sketch 5.1 is rotated 90° clockwise the rain flows along the path not only from A to L, but also from A to B, drops to D, runs to E, drops to G and runs to H. In dropping from B to D, the cycle BCD is extracted within range S_{r2} . In the sample shown in Sketch 5.1 point 8_{max} has the greatest y-coordinate and 21_{min} is the point with the smallest y coordinate. The path between 8_{max} and 21_{min} forms also a half cycle. Within this half cycle there are smaller cycles associated with, for example, abc . Furthermore, in cycle abc , there are smaller cycles, def and ghi . After 21_{min} the next most pronounced extremes that determine rainflow (dotted lines) in the sample of the process are 21_{max} and 30_{min} , as shown in Sketch 5.1. They define the stress cycles which can be extracted from the final part of the process.

There are different ways of extracting numerically the rainflow cycles from the process. The most common are the four-point and the three-point counting methods (Reference 5, 15 and 28).