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Propeller slipstream modelling for incidence and sideslip

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* Corresponding Member

The construction and subsequent development of the Data Item was undertaken under contract to ESDU by Mr M.J. Green.

The person with overall responsibility for the work in this subject area is Mr R.W. Gilbey, Head of Aircraft Aerodynamics Group.

PROPELLER SLIPSTREAM MODELLING FOR INCIDENCE AND SIDESLIP

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PROPELLER SLIPSTREAM MODELLING FOR INCIDENCE AND SIDESLIP

1. NOTATION AND UNITS

		SI	British
B	number of blades on propeller		
C_N	vertical in-plane force coefficient, $N/(\frac{1}{2}\rho\pi R^2 V_s^2)$		
C_T	thrust coefficient, $T/(\frac{1}{2}\rho\pi R^2 V_s^2) \equiv T / (q_s S_p)$		
C_Y	lateral in-plane force coefficient, $Y/(\frac{1}{2}\rho\pi R^2 V_s^2)$		
D	propeller disc diameter	m	ft
F	resultant force acting on propeller, see Sketch 9.1	N	lbf
$f(x)$	$[1 + \cos(\xi)]/2$, function used in Section 9		
δL	elemental lift on propeller blade element, see Sketch 3.1	N	lbf
N	propeller vertical in-plane force	N	lbf
q_s	mean kinetic pressure in fully-developed slipstream, $\frac{1}{2}\rho V_s^2$	N/m ²	lbf/ft ²
R	propeller disc radius	m	ft
R_s	radius of fully-developed slipstream	m	ft
R_x	radius of slipstream at station x	m	ft
r	arbitrary radial position on propeller blade	m	ft
S_p	propeller disc area, πR^2	m ²	ft ²
S_s	cross-sectional area of fully-developed slipstream, πR_s^2	m ²	ft ²
T	thrust	N	lbf
δT	elemental thrust on propeller blade element, see Sketch 3.1	N	lbf
t	time	s	s
Δt	incremental time	s	s
V_a	axial velocity through propeller disc	m/s	ft/s
V_s	velocity in fully-developed slipstream	m/s	ft/s
V_w	rate of propagation of wake vortex tube	m/s	ft/s
V_x	velocity in slipstream at station x	m/s	ft/s

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V_0	freestream velocity	m/s	ft/s
$\bar{V}$	total velocity at propeller blade element, see Sketch 3.1	m/s	ft/s
x	position downstream from propeller along thrust axis	m	ft
Y	propeller lateral in-plane force	N	lbf
y	transverse displacement of wake laterally from propeller thrust axis	m	ft
z	transverse displacement of wake vertically down from propeller thrust axis	m	ft
α_P	angle of attack of propeller thrust axis, see Sketches 6.2 and 9.1	rad	rad
α_s	angle of fully-developed slipstream to propeller thrust axis, see Sketches 6.2 and 9.1	rad	rad
α_x	angle of slipstream to propeller thrust axis at station x , see Sketch 9.1	rad	rad
β_P	sideslip angle of propeller thrust axis, see Sketch 8.1	rad	rad
β_s	sideslip angle of fully-developed slipstream to propeller thrust axis, see Sketch 8.1	rad	rad
Γ	strength of bound vorticity on propeller blade, see Sketch 3.1	m ² /s	ft ² /s
ζ	$\cos^{-1}(\Omega r / \bar{V})$, see Sketch 3.1	rad	rad
θ_F	$\cos^{-1}(T/F)$, see Sketch 9.1, angle between thrust line and direction of resultant force on propeller	rad	rad
θ_P	fully-developed slipstream incidence relative to freestream, see Sketches 6.2 and 9.1	rad	rad
θ_x	slipstream incidence relative to freestream at station x , see Sketch 9.1	rad	rad
γ	strength of distributed annular vorticity, see Section 4 and Sketch 4.1	m/s	ft/s
γ_s	strength of distributed swirl vorticity, see Section 6 and Sketch 6.1	m/s	ft/s
γ_0	value of γ at $\phi = \pi/2$	m/s	ft/s
ξ	$\cos(\xi) = (x/D)/[1/4 + (x/D)^2]^{1/2}$, used in Section 9, see Sketch 9.2	rad	rad
σ_P	lateral angle of fully-developed slipstream relative to freestream, see Sketch 8.1	rad	rad

ϕ	angular coordinate around propeller disc	rad	rad
ρ	density of air	kg/m ³	slug/ft ³
ψ_P	yaw angle of propeller thrust line ($= -\beta_P$), see Section 8	rad	rad
Ω	propeller rotational speed	rad/s	rad/s

2. INTRODUCTION

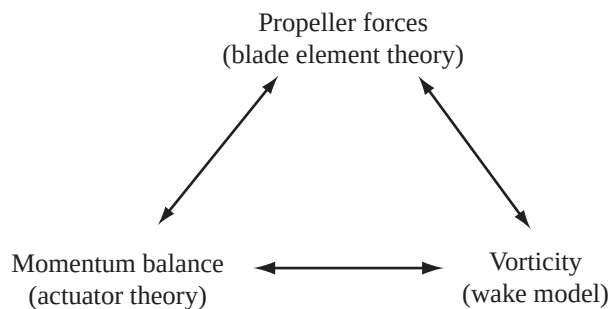
Ribner (Reference 1) considers the case of a propeller in yaw, and develops equations (*e.g.* Equation (12) of that reference) from which the propeller wake deflection can be calculated. However, Ribner does not give an analytical derivation of his equations; rather they are presented from the standpoint of physical reasoning.

This Data Item relates the forces acting on a propeller to the characteristics of the far wake (*i.e.* fully-developed slipstream), both for the axial case and for a propeller at incidence. Note that Ribner is strictly incorrect in describing the lateral case as “yaw”; properly the term “sideslip” should be used, although this introduces a slight complexity of notation. The relationship between these cases is discussed.

Certain simplifying assumptions are made:

- i) The propeller has B blades, each carrying a constant bound vorticity, Γ , shed at the tips (this does not imply constant elemental lift or thrust along the blade – see below).
- ii) The swirl component of rotational velocity is neglected, being small compared to the propeller rotational velocity.
- iii) Initially, only thrust is considered. The model is then extended to incorporate an in-plane transverse force.

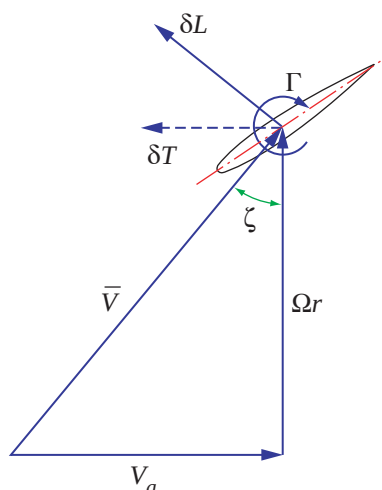
This Data Item shows compatibility between different approaches to propeller modelling.



Expressions for the development of the slipstream at intermediate stations are related to the conditions in the far wake.

3. BLADE LIFT AND THRUST

Take a blade element at radius r , rotational speed Ω , with a (constant) axial velocity V_a through the propeller disc, see Sketch 3.1. The blade is considered to have constant bound vorticity of strength Γ along its span, which is shed into the wake at the tip. This is clearly a simplification of the actual case, but is used here in order to illustrate the principles involved.



Sketch 3.1 A blade element at radius r

It follows that the elemental lift on the blade element is

$$\delta L = \rho \bar{V} \Gamma dr, \quad (3.1)$$

producing an elemental thrust

$$\delta T = \rho \bar{V} \Gamma \cos(\zeta) dr = \rho \Omega r \Gamma dr. \quad (3.2)$$

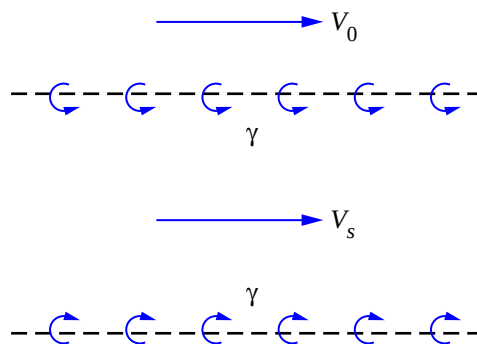
By integration across the blade, radius R , and for a number of blades B , the total thrust is

$$T = B \Gamma^{1/2} \rho \Omega R^2. \quad (3.3)$$

4. VORTICITY IN THE FAR WAKE

The basis of this model is to assume that the discrete bound vorticity described above is transformed in the wake onto the surface of a vortex tube. There will be two components of this vorticity – a tube of elemental annular (toroidal) vortices that account for the thrust, and lines of elemental axial vorticity along the tube that account for swirl. The relationship between these depends on the helix angle ($= \zeta$) of the original discrete wake vortices.

Only the annular vorticity is considered initially. This has strength γ , and has to meet certain compatibility conditions. The outer flow has velocity V_0 , and the velocity inside the fully-developed slipstream is V_s , see Sketch 4.1.



Sketch 4.1 Section through wake

- i) *Continuity of mass.* The radius of the fully-developed slipstream, R_s , is related to that of the propeller, R , by

$$V_a R^2 = V_s R_s^2. \quad (4.1)$$

- ii) *Vorticity.* By considering the velocity jump across the wake boundary,

$$\gamma = V_s - V_0. \quad (4.2)$$

- iii) *Pressure.* The total pressure inside the wake is taken to be higher than the freestream by the loading on the propeller $T / \pi R^2$, where πR^2 is the disc area. However, the inner and outer flows must have the same static pressure, which yields the condition:

$$\frac{1}{2} \rho V_s^2 = \frac{1}{2} \rho V_0^2 + T / (\pi R^2). \quad (4.3)$$

This can be re-written

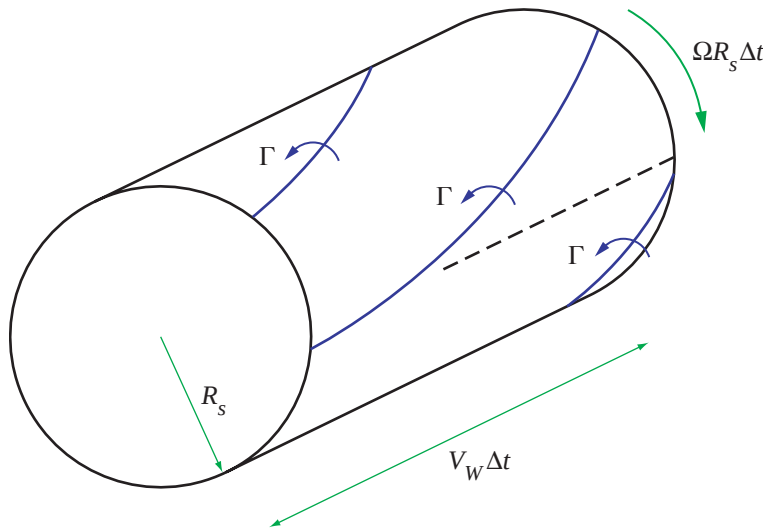
$$T / (\pi R^2) = \frac{1}{2} \rho (V_s^2 - V_0^2),$$

or
$$T / (\pi R^2) = \frac{1}{2} \rho (V_s - V_0)(V_s + V_0),$$

whence
$$T / (\pi R^2) = \rho \gamma (V_s + V_0) / 2. \quad (4.4)$$

5. VORTICITY COMPATIBILITY

In order to relate the bound vorticity to that distributed in the wake, it is necessary to visualise the vorticity shed from the blade tips per unit time. At this stage it is not specified how fast the vortex tube is propagating downstream as it is “sandwiched” between the inner and outer flows. Let this velocity be V_w .



Sketch 5.1 Section of wake

The annular component of the bound vorticity shed per unit time is related to the elemental annular vorticity ‘smeared’ over the surface of the tube by:

$$B\Gamma\Omega R_s \Delta t = \gamma 2\pi R_s V_w \Delta t, \quad (5.1)$$

$$\text{or} \quad \gamma 2\pi V_w = B\Gamma\Omega. \quad (5.2)$$

But from Equation (3.3),

$$B\Gamma\Omega = 2T / (\rho R^2), \quad (5.3)$$

$$\text{so that} \quad \rho\gamma V_w = T / (\pi R^2). \quad (5.4)$$

Comparison of Equation (5.4) with Equation (4.4) leads to the conclusion,

$$V_w = (V_s + V_0) / 2. \quad (5.5)$$

Thus the vortex tube is seen to be propagating at the mean of the inner and outer flows, which seems physically reasonable as it suggests it is acting as “roller-bearings” between the two.

Also, from actuator theory,

$$(V_s + V_0) / 2 = V_a (= V_w), \quad (5.6)$$

where V_a is the axial velocity at the propeller disc.