



**Full-potential method for
three-dimensional wings and wing-body
combinations – inviscid flow
Part 3: Method with improved estimates of
body lift and drag contributions (FPIBE)**

Endorsed by
The Royal Aeronautical Society

ESDU DATA ITEMS

Data Items provide validated information in engineering design and analysis for use by, or under the supervision of, professionally qualified engineers. The data are founded on an evaluation of all the relevant information, both published and unpublished, and are invariably supported by original work of ESDU staff engineers or consultants. The whole process is subject to independent review for which crucial support is provided by industrial companies, government research laboratories, universities and others from around the world through the participation of some of their leading experts on ESDU Technical Committees. This process ensures that the results of much valuable work (theoretical, experimental and operational), which may not be widely available or in a readily usable form, can be communicated concisely and accurately to the engineering community.

We are constantly striving to develop new work and review data already issued. Any comments arising out of your use of our data, or any suggestions for new topics or information that might lead to improvements, will help us to provide a better service.

THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Transonic Aerodynamics Committee. This Committee first met in 1960 and now has the following membership:

Chairman	
Mr G.R. Hargreaves	— BAE SYSTEMS, Woodford
Vice-Chairman	
Mr G.R. Richards	— BAE SYSTEMS, Farnborough
Members	
Mr C. Burke	— Aircraft Research Association
Mr K. Burton	— BAE SYSTEMS, Brough
Mr A. Dyke	— Airbus, Bristol
Mr M.C.P. Firmin	— Member Emeritus
Mr J.L. Fulker	— QinetiQ, Aerospace Division, Bedford
Mr M.D. Hodges	— Independent
Mr T. Pemberton	— BAE SYSTEMS, Warton
Dr D.R. Philpott	— University of Hertfordshire, Hatfield
Dr S. Prince	— The City University, London
Dr S. Shaw	— Cranfield University, Bedfordshire
Mr D.R. Stanniland	— Aircraft Research Association
Mr P.W.C. Wong	— Aircraft Research Association Limited, Bedford.

The construction and subsequent development of the Data Item was undertaken by

Dr M.M. Freestone	— Senior Engineer.
-------------------	--------------------

FULL-POTENTIAL METHOD FOR THREE-DIMENSIONAL WINGS AND WING-BODY COMBINATIONS – INVISCID FLOW PART 3: METHOD WITH IMPROVED ESTIMATES OF BODY LIFT AND DRAG CONTRIBUTIONS (FPIBE)

CONTENTS

	Page
1. INTRODUCTION	1
1.1 PROGRAM ACCESS	2
2. NOTATION	3
3. DESCRIPTION OF IMPROVED METHOD OF ESTIMATING BODY CONTRIBUTIONS TO LIFT AND DRAG	5
4. BASIS OF IMPROVED METHOD AND LIMITATIONS	6
4.1 Body Contribution to Overall Lift for a Finite Body	6
4.2 Body Contribution to Overall Lift where the Body is Infinitely Long	7
4.3 Body Contribution to Overall Drag	7
4.4 Tare Drag Corrections to Drag Coefficients Determined by FP and FPIBE	8
5. DIFFERENCES IN OUTPUT FILES RESULTING FROM USE OF FP AND FPIBE	8
6. COMPARISON OF RESULTS FROM ORIGINAL FP CODE, FPIBE AND POST-PROCESSING PROGRAMS	9
6.1 Example I: Wing-alone	9
6.2 Example II: Wing-body Configuration with Rectangular Wing and Infinite Body	10
6.3 Example III: Wing-body Configuration with Wing having Sweep, a Planform Crank, and Twist	10
6.4 Example IV: Wing-body Configuration Similar to that of Section 6.3, but having Supercritical type Control Sections	11
7. REMARKS CONCERNING RELATION BETWEEN INVISCID FLOWS AND REAL FLOWS	12
8. CONCLUDING REMARKS	13
9. REFERENCES	14
FIGURES 1 to 19	15 to 33

FULL-POTENTIAL METHOD FOR THREE-DIMENSIONAL WINGS AND WING-BODY COMBINATIONS – INVISCID FLOW

PART 3: METHOD WITH IMPROVED ESTIMATES OF BODY LIFT AND DRAG CONTRIBUTIONS (FPIBE)

1. INTRODUCTION

In this third part of the series of Data Items dealing with the full-potential method for three-dimensional wings and wing-body combinations, a variant of the method, denoted by FPIBE, is presented. FPIBE provides improved estimates of the lift and drag contributions of the body, as compared with the original method (FP). (The letters IBE in FPIBE stand for 'improved body estimates'.) For a wing-alone configuration, results from a run of FPIBE are the same as those from a corresponding run of FP. The FPIBE code varies only in a very minor way from the original FP code, and Parts 1 and 2 of the series of Data Items describing the principles and use of FP (References 1 and 2) apply equally to the variant FPIBE, with the exception of those parts describing or relying on the procedure by which the body lift and drag are estimated. For cases where the lift and drag contributions of the body (or their influences) are of interest it is recommended that the FPIBE code should be run in preference to FP. In order to provide comparisons with pre-existing results from runs of the original FP code (should this be required), overall lift and drag coefficients as calculated by the original FP code are given in one of the output files of FPIBE.

In this Item, reference to the FP method generally means reference to both the original FP code and its variant FPIBE. When specific reference is required, this is done by referring to either the 'original FP method (or code)' or the 'FPIBE method (or code)'.

The FP method is a CFD (computational fluid dynamics) method coded in Fortran for calculating the flow field and aerodynamic forces of an isolated wing (denoted usually as a wing-alone) or a wing-body combination (denoted usually as a wing-body) in a subsonic freestream, including the effects of shock waves. The method uses a relaxation process to solve finite-difference forms of the full nonlinear velocity-potential equation for the flow around the three-dimensional geometry. The original FP program was developed over a period of years at ARA, Bedford and RAE (now QinetiQ), Farnborough, and FP and its variant FPIBE are made available by ESDU under the terms of an agreement with QinetiQ. Used originally on mainframe computers, FP has provided valuable data in the design of a number of aircraft. With advances in computers, it has become possible to perform runs of FP on a PC within a few minutes.

The results presented in this Item have all been produced from runs on a PC. The FP method takes no account of the boundary layer or viscous wake. (It is intended that an enhanced version that does take account of viscous effects, albeit in a simplified manner, will be issued subsequently.) Typical applications of the FP method to a number of wing-alone and wing-body geometries are presented in Part 2. Only axially symmetric bodies may be dealt with, and the body diameter must be neither too small nor too large relative to the span – in broad terms $0.05 \leq (R_{max} / s) \leq 0.4$.

In the FP method, for a wing-alone, local lift and drag coefficients are obtained around each grid section by appropriate integration of the calculated wing surface pressures. These local coefficients, weighted by the local chord, are then integrated across the span to yield values for the overall lift and drag coefficients. For a wing-body combination, the same procedure is used to determine the wing contributions to the overall coefficients. In the original FP code the body contributions are obtained by 'inboard extrapolation', i.e. by assuming constant values for the lift per unit span and drag per unit span between the most inboard grid section on the wing and the configuration centre line. In the FPIBE code the body lift contribution is not based on extrapolation, but on a correlation of a number of panel method results (as detailed in Sections 3 and 4). The body drag contribution in the FPIBE code is derived from the body lift by assuming

that the force on the body acts normal to its axis. The differences between the original FP code and the FPIBE code lie solely in the differences in the procedures for estimating the body contributions to the overall lift and drag. Since only the body lift and drag contributions are affected, the results for a wing-alone are the same whether the original FP code or FPIBE is used.

Input files required for FPIBE are exactly the same as for the original FP code, and the Sections in Part 2 (Reference 2) concerning procedures for obtaining reliable results from the original FP code apply equally to FPIBE. Section 5 of this Item lists the minor differences in the format of the output files produced by the original FP code and FPIBE. Section 6 presents a number of comments concerning the evaluation of overall lift and drag using FPIBE (and the original FP code), and illustrates these for a number of example geometries. The comparisons presented in Section 6 include results obtained by use of the 'post-processing' programs that are dealt with in detail in Part 4 (Reference 3).

Section 7 includes remarks concerning the relation between inviscid flows, which are considered in this Item, (and in Parts 1, 2 and 4, References 1, 2 and 3) and real (*i.e.* viscous) flows.

Section 8 summarizes the conclusions concerning procedures for obtaining the overall lift and drag coefficients to generally good accuracy. It is recommended that FPIBE be run in preference to FP, and that following any run of FPIBE, the post-processing programs of Part 4 (Reference 3) be run, since these provide the most reliable estimates for overall lift and trailing-vortex drag and alternative estimates of wave drag.

1.1 PROGRAM ACCESS

The files required to run FP and related programs on a PC are to be found in the folder (or directory) named FPv20, Executable forms of the programs are provided.

To run FPIBE, carry out the following five steps.

- (1) Open a 'C:\ Command Prompt' window or 'box'. (In some Windows versions the C:\ Command Prompt is an Accessory, in others it is a Program.)
- (2) Within the C:\ Command Prompt window, create a working directory.
- (3) Copy all the files in the directory FPV20 to this working directory.
- (4) Follow the instructions in Reference 2, replacing fp.exe with fpibe.exe.

It is recommended that the PC used should have at least 128Mbytes of RAM, a hard disk drive with at least 500 Mbytes of free space, and a processor speed of 1.5 GHz or more.

Although obtaining results using the original FP code and/or FPIBE is comparatively easy, care is required in interpreting them in relation to the task in hand. In general, this requires that all the Data Items concerned with FP be consulted in detail.

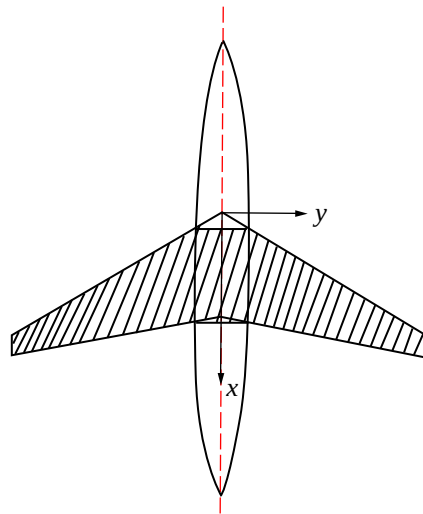
2. NOTATION

In the Notation used for FP and FPIBE, lengths are made non-dimensional by division by c_0 , the centre-line chord, see below. Quantities notionally of length (other than c_0) listed in this Section, such as c and s are thus non-dimensional, having been made so by division of the corresponding length quantities by c_0 . Similarly, the reference wing area S is also dimensionless, having been made so by dividing the corresponding actual area by c_0^2 . Note that alternative wing reference areas are defined and referred to in Parts 1 and 2, but in this Item only the reference area denoted in Parts 1 and 2 by S_1 is used, see Sketch 2.1. Here it is denoted simply by S .

Further notation, utilized in some FP and FPIBE input and output files, but not used specifically in this Item, is defined in Part 2 (Reference 2).

A	(gross) wing aspect ratio, $= 4s^2 / S$
C_D	overall drag coefficient, ($= C_{DTV} + C_{DW}$ in this Item)
C_{DL}	local drag coefficient, based on local chord
C_{DTV}	trailing-vortex drag coefficient
C_{DW}	overall wave drag coefficient
C_{DWL}	local wave drag coefficient, based on local chord
C_L	overall lift coefficient
C_{LL}	local lift coefficient, based on local chord
C_p	pressure coefficient
c	local wing chord
c_c	chord of wing at crank (where applicable)
c_t	chord of wing at tip
c_0	centre-line chord, formed (if necessary) by continuing leading edge and trailing edge of wing, near root, by straight lines to $y = 0$
D_B	drag contribution from body
D_C	drag contribution from cylindrical part of body
L_B	lift contribution from body
L_{Bfac}	factor applied to lift per unit span at wing root in estimation of lift contribution from body
L_C	lift contribution from cylindrical part of body

M_{∞}	freestream Mach number
R	body radius (may vary with x)
S	reference wing area (see Sketch 2.1)
s	wing semispan, measured from configuration centre-line (<i>i.e.</i> not from wing root in case of wing-body)
x, y, z	Cartesian coordinates in physical space (nondimensional)
α	incidence angle, angle of inclination of x axis to freestream direction (positive nose up), deg (see Reference 1)
Λ	sweep angle (positive for sweepback), deg
λ	taper ratio, = c_t
λ_i	taper ratio of inboard panel of cranked wing, = c_c
η_F	ratio of R_{max} to s , effectively value of η at wing root, <i>i.e.</i> at junction of wing and body
η	relative spanwise position, = y/s
η_c	value of η of crank section
<i>Suffices</i>	
i	denotes inner panel of cranked wing
le	denotes value at leading edge
max	denotes maximum value
o	denotes value at outer panel of cranked wing
1	denotes first-order method of wave drag estimation
2	denotes second-order method of wave drag estimation



Sketch 2.1 Illustration of reference wing area S (denoted by S_1 in References 1 and 2)

(Note that S is made non-dimensional by division by c_0^2 .)

3. DESCRIPTION OF IMPROVED METHOD OF ESTIMATING BODY CONTRIBUTIONS TO LIFT AND DRAG

The FP method was devised primarily as a means of studying wing flows, and although the velocity potential of the complete flow is computed, including (in the case of a wing-body combination) the effects due to the presence of an axially symmetric body, no details pertaining to the body, such as the body surface pressures or the body contributions to lift and drag, are determined within the FP program. It is intended to devise a means for determining some body flow parameters from the computed velocity potential field for presentation in a forthcoming Data Item, but the amount of work entailed in so doing is substantial. Partly because of this, but also because the body contributions to the overall lift and drag are generally small, the original FP method adopts a very simple approach for their estimation. It assumes that these contributions are the same as if the lift and drag per unit span were to remain (respectively) constant over the lateral extent of the body and equal to their values at the wing root (or, more precisely, were to remain equal to their respective values computed at the most inboard wing grid section). The wing contributions to the overall lift and drag coefficients are found from integrations of the calculated wing pressures around each wing grid section (see Part 1, Reference 1) resolved in the lift and drag directions. This Item deals with the variant of the FP code, denoted by FPIBE, which provides an improved estimation method for the body contributions to the overall lift and drag coefficients of a wing-body combination relative to that of the original FP method. Again it assumes that the body contribution to the lift may be obtained from the lift per unit span computed at the most inboard wing grid section, but multiplies the result used in the original FP method by the factor L_{Bfac} as given in Figure 1. The body contribution to the drag coefficient is obtained from the improved body lift estimate by assuming that the total aerodynamic force on the body (obtained by vector addition of the body lift and drag) acts at right angles to the body axis. Obtaining the improved body lift contribution, L_B , is a simple procedure since the value of L_{Bfac} is taken to depend only on the value of η_F , the ratio of the maximum body radius, R_{max} , to the semispan, s . Similarly, the improved body drag contribution, D_B , follows simply from

$$D_B = L_B \tan \alpha. \quad (3.1)$$

4. BASIS OF IMPROVED METHOD AND LIMITATIONS

4.1 Body Contribution to Overall Lift for a Finite Body

The factor L_{Bfac} , given in Figure 1, which yields the improved body lift contribution, is based on numerous results obtained from a panel method. This panel method is described in Reference 4, where a number of results obtained from the method are also presented. The panel method utilized represents an extension of the vortex-lattice method for wings (see, for example Reference 5) and is valid for small-disturbance subsonic flows about a wing-body combination which comprises an axially symmetric body and an infinitely thin wing. The wing boundary conditions are satisfied on a plane, $z = \text{constant}$, that may be displaced from the body axis. The part of the body to which the wing adjoins is required to be cylindrical. The effects of finite wing thickness are not represented in the panel method. A typical layout of panels is shown in Figure 2. Each panel is assigned a horseshoe vortex-singularity comprising a bound element and two trailing elements. The trailing elements of the horseshoe vortex associated with a body panel are taken to lie in the body surface (each at a constant radial position). The trailing elements associated with a wing panel are assumed to be straight lines parallel to the body axis, with the exception of the most inboard ones, which again are taken to lie in the body surface, each at a constant radial position. A number of checks on the panel method have shown that with suitable numbers and arrangements of the panels it provides a consistent and reliable way of estimating lift distributions in small-disturbance subsonic flow for a range of wing-body geometries. The range of relative body diameters for which the panel method gives reliable results more than covers the range for which the FP method provides successfully converged results.

Analysis of the results from the panel method of Reference 4 has shown that for configurations satisfying the condition $\eta_F < 0.6$ a good approximation to the body lift is generally obtained if the factor L_{Bfac} is taken to depend on η_F in the way shown in Figure 1. The geometric parameters investigated with the panel method in this analysis include a number of variations in the body geometry, namely the shapes of the forebody and afterbody, and the lengths of the forebody, cylindrical mid-body, and afterbody in relation to the maximum wing chord. Variations in the wing planform such as aspect ratio, sweep and taper have also been considered, as have changes in wing twist and the setting of the wing relative to the body axis. Clearly the range of possible configurations that could be considered is very large and in order to make the investigation manageable most of the changes have been considered in relation to a baseline configuration having a value of η_F equal to $1/8$, consisting of an untwisted rectangular wing mounted centrally on a relatively long body. It has been found from the panel method calculations that most of the effects of these geometric variations on L_{Bfac} are very small, and all are within plus or minus 0.05. With an error of 0.05 in L_{Bfac} the corresponding error in the overall lift would generally be less than 1 per cent (unless the overall lift were itself small), for configurations with η_F of the order of 0.1, and generally no more than 3 per cent for configurations with η_F of the order of 0.5. One geometric change that is found to have a relatively large influence is the displacement of the wing plane from the $z = 0$ plane. Such displacement is found to decrease the value of L_{Bfac} . However, provided this displacement does not exceed 0.3 of the maximum body radius, the decrease in L_{Bfac} is found to be limited to about 0.05. For a displacement of about 0.4 of the maximum body radius the decrease in L_{Bfac} rises to about 0.1.

Wing-body combinations with non-zero wing thickness must be expected to give rise to different values of L_{Bfac} from those corresponding to zero wing thickness, and where the wing maximum thickness is as large as (say) half the maximum body radius the differences would probably be substantial, particularly where the wing mean plane is displaced by a significant amount from the $z = 0$ plane. The improved method of estimating the body contribution to the overall lift of a wing-body combination should give reliable results provided the wing thickness is considerably less than the maximum body radius and the wing plane displacement is less than 0.3 of the maximum body radius. Its use for wing-body combinations not conforming to these limits is subject to possibly substantial errors. Even then, however, it would generally be expected to provide an improvement on the estimate of body lift incorporated in the original FP code.

4.2 Body Contribution to Overall Lift where the Body is Infinitely Long

On occasion, the inviscid flow about a wing-body combination where the body extends to infinity upstream and downstream is of theoretical interest. The FP method permits such configurations to be considered (for bodies of circular section), see Reference 1, Section 3.1 and Reference 2, Section 4.1.2. So far as the FP method is concerned, there is no difference between the aerodynamic effects of infinitely long bodies that are closed (*i.e.* bodies having a forebody and an afterbody) or partially closed (*i.e.* bodies with either a forebody or an afterbody) or open (*i.e.* bodies with neither a forebody nor an afterbody). The panel method of Reference 4 does not allow direct representation of an infinite body, but results for a closed infinite body may be inferred from those for a wing-body combination with a long (but finite) cylindrical part, since the values of lift carried on the various portions of the body in such cases are found to be nearly independent of the geometry of the remote forebody and afterbody. The lift on the forebody for such a case of a long body is effectively the same as that on the forebody of the body in isolation, but the lift on the afterbody is generally not the same as when the isolated body is considered because the vortex wake of the wing induces a disturbance flow about the body which (in inviscid theory) does not decay to zero far downstream. However, the lift on the afterbody (according to the panel method) rapidly approaches a constant value as the length of the cylindrical portion of the body extending aft of the wing is increased. For an isolated body the lift on the forebody (which is positive at positive incidence) is equal in magnitude but opposite in sign to the lift on the afterbody, and there is then zero overall lift. The difference in the overall lift of a wing-body combination with a closed infinite body and that of the corresponding open wing-body combination is therefore due to the effect of the wing vortex wake on the lift on the afterbody. Generally the effect of the vortex wake is to decrease the magnitude of the afterbody lift, so that for wing-body combinations the body lift contribution of a closed body (at positive incidence) is greater than that of the corresponding open body. For configurations where η_F is less than about 0.2, the differences in the body contribution to lift for closed and open body cases are found to be small and may reasonably be neglected, see Reference 4. For configurations having values of η_F of the order of 0.5, however, the differences correspond to a change in L_{Bfac} of about 0.2. *In such cases it is therefore important to recognize that the results given by FPIBE for an infinitely long body correspond to a body which is closed fore and aft.*

4.3 Body Contribution to Overall Drag

The resultant force on the cylindrical portion of a body due to any distribution of pressure on its surface must act at right angles to the axis of the body. Thus the lift, L_C , acting on the cylindrical section between the forebody and the afterbody implies a corresponding drag, D_C , given by

$$D_C = L_C \tan \alpha. \quad (4.1)$$

This is true whether or not the body considered is isolated or forms part of a wing-body combination.

The resultant force on the forebody and afterbody portions of a body (taken together) does not, in general, act normal to the axis of the body. For an isolated body in inviscid subsonic flow the resultant force is zero, and as implied in Section 4.2, at least for a relatively long body, the forebody lift and afterbody lift again almost cancel for wing-body combinations provided η_F is less than about 0.2. For such cases it is reasonable to assume

$$D_B = L_B \tan \alpha, \quad (4.2)$$

where the suffix B denotes the complete body. Equation (4.2) is taken to apply without restriction in the FPIBE code. In Section 7 some comparisons between the overall lift and drag coefficients evaluated by use of FPIBE with the corresponding coefficients determined by the post-processing programs presented in Part 4 (see Reference 3) are given, and these comparisons suggest that utilisation of Equation (4.2) is reasonably accurate.

4.4 Tare Drag Corrections to Drag Coefficients Determined by FP and FPIBE

In some inviscid flow cases the drag is known to be zero. This is true for configurations symmetric about the plane $z = 0$ (*i.e.* those with vertical symmetry) at zero incidence provided the flow generates no shock waves. The configurations then have zero overall lift and zero local lift across the span, and therefore have zero trailing-vortex (or induced) drag, as well as zero wave drag. The same is not true for configurations lacking vertical symmetry at zero overall lift, since these will not in general have zero local lift across the span. However, for configurations which have only a few degrees of twist or a moderate variation of section camber, it is found that the error in neglecting the trailing-vortex drag at zero overall lift is very small. Thus, in cases of vertical symmetry and also cases of moderate twist and camber, it is possible to adjust the values of drag coefficient given by FP or FPIBE for shock-free flows by a constant amount so as to make the curve of drag coefficient against incidence (at a particular freestream Mach number) have a minimum value of zero. In this Item, this adjustment is termed the tare drag correction. It is found that the tare drag correction may be positive or negative, and that it is generally non-zero for both wing-alone and wing-body configurations. Examples of applying a tare drag correction are given in Section 6

5. DIFFERENCES IN OUTPUT FILES RESULTING FROM USE OF FP AND FPIBE

This Section deals with differences in the names and format of the output files resulting from use of the original FP code and FPIBE. (As stated in Section 1, the input files required for the original FP code and for FPIBE are identical, and Part 2 (Reference 2) should be consulted on the actions required to produce input files suited to a particular investigation.)

As stated in Section 6 of Part 2 (Reference 2), a run of the original FP code generates a number of annotated output files, the principal ones, such as 18WRITE.DAT, CP3D.DAT and 19WRITE.DAT being listed and described. In order to make it obvious which of the programs, the original FP code or FPIBE, has been run to generate output, the names of all output files produced by FPIBE are different from those from the original FP code, with the files from FPIBE commencing with 'B'. Thus the file named 18WRITE.DAT is produced by the original FP code, whilst the corresponding file produced by FPIBE is named B18WRITE.DAT. The only other obvious difference between files produced by the original FP code and FPIBE is that whenever a reference is made in the former set of files to 'FP' this is replaced in the latter set of files by 'FPIBE'. The changed filenames and changed annotation in the files removes any confusion as to which program has been run to generate any particular output file. The annotated output files also include a run title and the date and time that the first data in the file were written, so for these files there is a further means of avoiding confusion.

In addition to the files of similar format that are produced by both FP and FPIBE, there is an additional file, named B25WRITE.DAT that is output by FPIBE. This file includes the run title, date and time in the same way as the other annotated files, together with the values of the overall lift and drag coefficients corresponding to both the original FP code and to FPIBE. This file is useful when it is required to know the magnitude of the changes introduced by FPIBE, since this may be done simply with a run of FPIBE, rather than by requiring runs of both the original FP code and FPIBE.

6. COMPARISON OF RESULTS FROM ORIGINAL FP CODE, FPIBE AND POST-PROCESSING PROGRAMS

In this Section a number of comparisons are presented of results for overall lift and drag coefficients as calculated from FP, FPIBE and the post-processing procedures of Part 4 (Reference 3). Comparisons coefficients dealing with the M6 wing and the W4 wing-body are given in Reference 3, and a number of further examples dealing with other wing-body geometries are given in Reference 6.

6.1 Example I: Wing-alone

Although this Item relates principally to wing-body configurations, in considering lift and drag comparisons it is instructive first to consider a number of flows about a wing-alone. The wing selected has a rectangular planform, an aspect ratio of 8, zero twist, and the NACA 0012 aerofoil section across the whole span. The Mach number selected is 0.5. Fully converged runs of FPIBE were obtained for a number of angles of incidence from 0 to 8 deg. The post-processing programs yielding the lift and drag coefficients (including the breakdown into trailing-vortex and wave drag components) described in Part 4 (Reference 3) were also run for each flow case. As noted in Section 1, there is no difference between the original FP code results and the corresponding FPIBE results for a wing-alone. As stated in Part 4, the results for lift and drag from the post-processing programs should generally be of good accuracy. For simplicity, results from using the post-processing programs described in Part 4 are referred to here as 'Part 4 results'. Also, for simplicity, generally only results from one of the two wave drag calculation methods in Part 4 are presented here, namely those from the 'second-order method'. A discussion of the two wave drag calculation methods termed respectively 'first-order' and 'second-order' is given in Reference 7. Generally, for cases where the shock waves are relatively weak, the first- and second-order methods of wave drag calculation give quite similar results. Further comparisons are given in Reference 3.

Figure 3 gives the results from FPIBE and from the post-processing programs of Part 4 for lift and drag coefficients for this wing-alone configuration. Figure 3 shows that there is good agreement between the results obtained directly from FPIBE and those from Part 4 for the lift coefficient, but not for the drag coefficient. The flows are shock-free for incidence angles up to about 6 deg. The flow at 8 deg incidence yields a wave drag coefficient of 0.00064 according to the second-order wave drag method of Part 4.

In Figure 4 the drag results of Figure 3 are repeated, except that a tare drag correction (see Section 4.4) equal to 0.00175 has been applied to the FPIBE results. Also shown in Figure 4 is the curve of the theoretical minimum trailing-vortex drag for given span and lift, obtained from

$$C_D = (1/A)C_L^2.$$

The agreement with the Part 4 (post-processing) values is much improved by the tare drag correction, but it should be noted that the corrected drag values are still underestimates. The trailing-vortex drag given by the post-processing programs is very close to (and slightly greater than) the theoretical minimum (which corresponds to an elliptic distribution of loading across the span). Figure 5 shows the spanwise variations of local lift and local drag calculated by FPIBE for $\alpha = 0, 4$ deg and 8 deg. The rather rapid variations in local drag coefficient near the tip suggest that this area might be a source of error in the direct determination of overall drag in FP and FPIBE. It is to be noted that there is little practical significance in the spanwise distribution of drag coefficient. For example, for a given lift, one cannot expect to devise a means of lowering the local drag in one part of the span without raising it in another part. In shock-free flow the overall drag is entirely trailing-vortex drag, and the trailing-vortex drag is determined, for a given span, by the distribution of loading over that span, and therefore, for a given planform, by the spanwise distribution of local lift coefficient. As seen in Figure 5, the distributions of loading calculated at $\alpha = 4$ deg and $\alpha = 8$ deg reveal a near-elliptic form, and therefore give rise to trailing-vortex drag values only very little greater than the theoretical minimum values.