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## Noise transmission into aircraft cabins

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## THE PREPARATION OF THIS DATA ITEM

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## NOISE TRANSMISSION INTO AIRCRAFT CABINS

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## NOISE TRANSMISSION INTO AIRCRAFT CABINS

### 1. NOTATION

|                 |                                                                                              |
|-----------------|----------------------------------------------------------------------------------------------|
| $A$             | cylinder interior surface area                                                               |
| $\bar{A}$       | cylinder exterior surface area                                                               |
| $A_r, A_s$      | cross-sectional area of ring frames and stringers respectively                               |
| $A_t$           | transmitting area with trim                                                                  |
| $A'$            | transmitting area without trim                                                               |
| $a$             | radius of cylinder                                                                           |
| $a_{ij}$        | coefficients in trim transfer matrix                                                         |
| $a_z$           | pressure amplitude decay rate parameter                                                      |
| $C(j)$          | constant factor in equation for $\Phi_g(i)$ ; either 1 or 2                                  |
| $C_h$           | cosh $\gamma L$                                                                              |
| $C_w$           | trim parameter, derived from the trim transfer matrix                                        |
| $c_0$           | speed of sound in air                                                                        |
| $c_\omega$      | constant percentage bandwidth parameter $= \Delta\omega/\omega = 0.232$ for 1/3 octave bands |
| $Ci(z)$         | cosine integral; $Ci(z) = \int_0^z \frac{1 - \cos t}{t} dt$                                  |
| $D$             | bending rigidity of structure                                                                |
| $d$             | $(L_c - L)/2$ (see Sketch 5.1)                                                               |
| $E, E_r, E_s$   | Young's modulus for skin, ring frame and stringer respectively                               |
| $f$             | frequency                                                                                    |
| $f'(n, r)$      | Internal coupling factor                                                                     |
| $G_r, G_s$      | shear modulus of ring frame and stringer respectively                                        |
| $H_m^{(1)}(kr)$ | Hankel function of the first kind of order $m$                                               |
| $h$             | skin thickness                                                                               |
| $h_e$           | thickness of end wall                                                                        |

|                  |                                                                                                       |
|------------------|-------------------------------------------------------------------------------------------------------|
| $h_g$            | distance between grid points                                                                          |
| $I_r, I_s$       | 2nd moment of area of ring frame and stringer respectively                                            |
| $\text{Im}[ \ ]$ | denotes imaginary part of [ ]                                                                         |
| $i$              | acoustic mode number counter for fuselage cross-section modes, associated with mode $n \equiv (q, i)$ |
| $J_m$            | Bessel function of the first kind of order $m$                                                        |
| $J_r, J_s$       | torsional constant of ring frame and stringer respectively                                            |
| $j$              | circumferential location on fuselage wall                                                             |
| $j_r^2(\omega)$  | joint acceptance function                                                                             |
| $k$              | acoustic wavenumber, $k = 2\pi/\lambda$                                                               |
| $k_m, k_n$       | wavenumbers for the clamped-clamped boundary condition                                                |
| $k_x, k_z$       | wavenumbers                                                                                           |
| $L$              | fuselage structure length                                                                             |
| $L_a$            | depth of air gap                                                                                      |
| $L_c$            | cavity length                                                                                         |
| $L_i$            | insulation thickness                                                                                  |
| $L_y$            | panel length in circumferential direction = $2a(\pi - \theta_0)$                                      |
| $l_x$            | frame pitch                                                                                           |
| $l_y$            | stringer pitch                                                                                        |
| $M$              | number of axial $\frac{1}{2}$ wavelengths for structural mode $r \equiv (M, N)$                       |
| $M_r$            | generalized modal mass, for structural mode $r$                                                       |
| $m$              | average surface mass/unit area of cylinder                                                            |
| $m_t$            | mass/unit area of lining                                                                              |
| $N$              | structural mode counter, associated with mode $r \equiv (M, N)$                                       |
| $n$              | denotes acoustic mode $n \equiv (q, i)$                                                               |
| $n_b$            | number of boundary points on the fuselage shell at which the acoustic eigenvectors are defined        |
| $n_e$            | external points adjacent to boundary points                                                           |

|                               |                                                                                                  |
|-------------------------------|--------------------------------------------------------------------------------------------------|
| $n_j$                         | number of interior and boundary points                                                           |
| $NR$                          | noise reduction                                                                                  |
| $P_i$                         | amplitude of incident plane wave                                                                 |
| $p$                           | pressure amplitude at grid point $(m, n)$                                                        |
| $p_1^i$                       | pressure on the interior surface of the skin at $\bar{x}$                                        |
| $p_2^i$                       | pressure on the trim inside surface at $\bar{x}$ (cavity side)                                   |
| $\langle p_{bl}^2 \rangle$    | average, mean square blocked pressure in band                                                    |
| $\langle p_e^2 \rangle_{s,t}$ | mean square pressure over that space which will be occupied by the enclosure                     |
| $\langle p_i^2 \rangle_{s,t}$ | mean square pressure in the interior                                                             |
| $\langle p_{inc}^2 \rangle_t$ | band limited, time averaged, incident pressure                                                   |
| $\langle p_n^2 \rangle_{s,t}$ | band-limited, space averaged, mean square modal pressure, for $n$ th mode in interior volume $V$ |
| $q$                           | number of axial $\frac{1}{2}$ wavelengths for acoustic mode $n$                                  |
| $\text{Re}[ \ ]$              | denotes real part of $[ \ ]$                                                                     |
| $r$                           | denotes structural mode                                                                          |
| $S$                           | absorbing surface area of cabin wall                                                             |
| $S_e$                         | absorbing surface area on each end surface (end wall)                                            |
| $S_h$                         | $\sinh \gamma L$                                                                                 |
| $s$                           | acoustic mode order, <i>i.e.</i> $n \equiv q, n, s$                                              |
| $Si(z)$                       | sine integral; $Si(z) = \int_0^z \frac{\sin t}{t} dt$                                            |
| $V$                           | volume of enclosure                                                                              |
| $W$                           | wave impedance in the insulation material                                                        |
| $W_{diss}$                    | power dissipated on the interior wall                                                            |
| $W_{in}$                      | net power inflow                                                                                 |
| $w_1$                         | displacement of fuselage skin at $\bar{x}$                                                       |
| $w_2$                         | displacement of trim panel at $\bar{x}$                                                          |

|                                               |                                                                                                                        |
|-----------------------------------------------|------------------------------------------------------------------------------------------------------------------------|
| $x, y, z$                                     | Cartesian coordinates                                                                                                  |
| $\bar{x}$                                     | represents a point on the surface of the transmitting structure                                                        |
| $Z_1$                                         | structural impedance (see Section 11.2)                                                                                |
| $Z_r, Z_s$                                    | distance from skin middle surface to centroids of ring frame and stringer respectively                                 |
| $\alpha$                                      | attenuation constant of the insulation material                                                                        |
| $\beta$                                       | normalized wall surface admittance,                                                                                    |
| $\gamma$                                      | propagation constant of insulation, $\gamma = \alpha - i2\pi/\lambda_m$                                                |
| $\nabla$                                      | differential operator                                                                                                  |
| $\Delta p$                                    | pressure differential across fuselage wall                                                                             |
| $\Delta\omega$                                | frequency bandwidth                                                                                                    |
| $\varepsilon_n$                               | acoustic mode normalization factor                                                                                     |
| $\varepsilon_m, \varepsilon_N, \varepsilon_q$ | mode participation factors                                                                                             |
| $\zeta_m, \zeta_n$                            | ratio of bending wavelengths for clamped-clamped boundary conditions to those for simply supported boundary conditions |
| $\eta_n$                                      | acoustic mode loss factor                                                                                              |
| $\eta_r$                                      | structural mode loss factor                                                                                            |
| $\eta_r'$                                     | modified structural loss factor                                                                                        |
| $\eta_r''$                                    | internal radiation loss factor, due to closely coupled structural and acoustic modes                                   |
| $\eta_{rad}$                                  | radiation loss factor                                                                                                  |
| $\eta_t$                                      | trim loss factor                                                                                                       |
| $\theta_0$                                    | angle at which fuselage shell/floor joint is located                                                                   |
| $\theta_j$                                    | angle $\theta$ for point $j$ on fuselage wall, a boundary point for the acoustic eigenvectors                          |
| $\theta_{1j}, \theta_{2j}$                    | angles defining mid points between boundary points $j$ and adjacent boundary points.                                   |
| $\lambda_i$                                   | acoustic mode eigenvalue                                                                                               |

|                         |                                                                                                                     |
|-------------------------|---------------------------------------------------------------------------------------------------------------------|
| $\lambda_m$             | acoustic wavelength in insulation material                                                                          |
| $\mu$                   | Poisson's ratio                                                                                                     |
| $\xi$                   | normalized conductance of the wall (or trim)                                                                        |
| $\xi_e$                 | normalized conductance of the end wall                                                                              |
| $\rho$                  | air density in the cabin                                                                                            |
| $\rho_0$                | exterior air density                                                                                                |
| $\tau_{ML}$             | mass law sound transmission coefficient                                                                             |
| $\tau_t$                | trim transmission coefficient                                                                                       |
| $\Phi$                  | phase angle of $W$                                                                                                  |
| $\phi$                  | angle of incoming plane wave relative to cylinder axis                                                              |
| $\phi_n(\bar{\xi})$     | mode shape for cavity                                                                                               |
| $\phi_i(\theta_j)$      | mode shape of $i$ th acoustic mode of the fuselage cross section evaluated on the fuselage wall at angle $\theta_j$ |
| $\Phi_G(i)$             | generalized mass for 2-D acoustic mode $i$                                                                          |
| $\psi_r(\bar{x})$       | mode shape of structural mode $r$                                                                                   |
| $\omega$                | angular frequency                                                                                                   |
| $\omega_H$              | angular frequency of harmonic $H$                                                                                   |
| $\omega_n$              | acoustic mode resonance angular frequency                                                                           |
| $\omega_r$              | structural mode resonance angular frequency                                                                         |
| —                       | an overbar denotes band-averaged value                                                                              |
| $\langle \rangle$       | indicates an average over the modes in a band                                                                       |
| $\langle \rangle_t$     | band-limited, time averaged value                                                                                   |
| $\langle \rangle_{s,t}$ | band-limited, space averaged and time averaged value                                                                |
| $int$                   | denotes interior value                                                                                              |
| $ext$                   | denotes exterior value                                                                                              |
| $rev$                   | denotes value for diffuse field excitation                                                                          |

## 2. INTRODUCTION

This Item provides the user with a means to estimate the reduction in noise through a cylindrical fuselage wall with floor and cabin wall trim shown diagrammatically in Sketch 2.1. Intended for application to the radiation of tonal and broadband noise from various types of aircraft engines into the aircraft cabin, the prediction method is based on the power flow approach which was originally developed to predict noise exposure in the payload bay of the Space Shuttle Orbiter Vehicle.

The method can deal with the following forms of excitation:

- (i) single frequency plane waves incident from a specified direction relative to the cylinder axis
- and (ii) broadband noise in a diffuse (reverberant) field and therefore better suited to predicting results from test facilities.

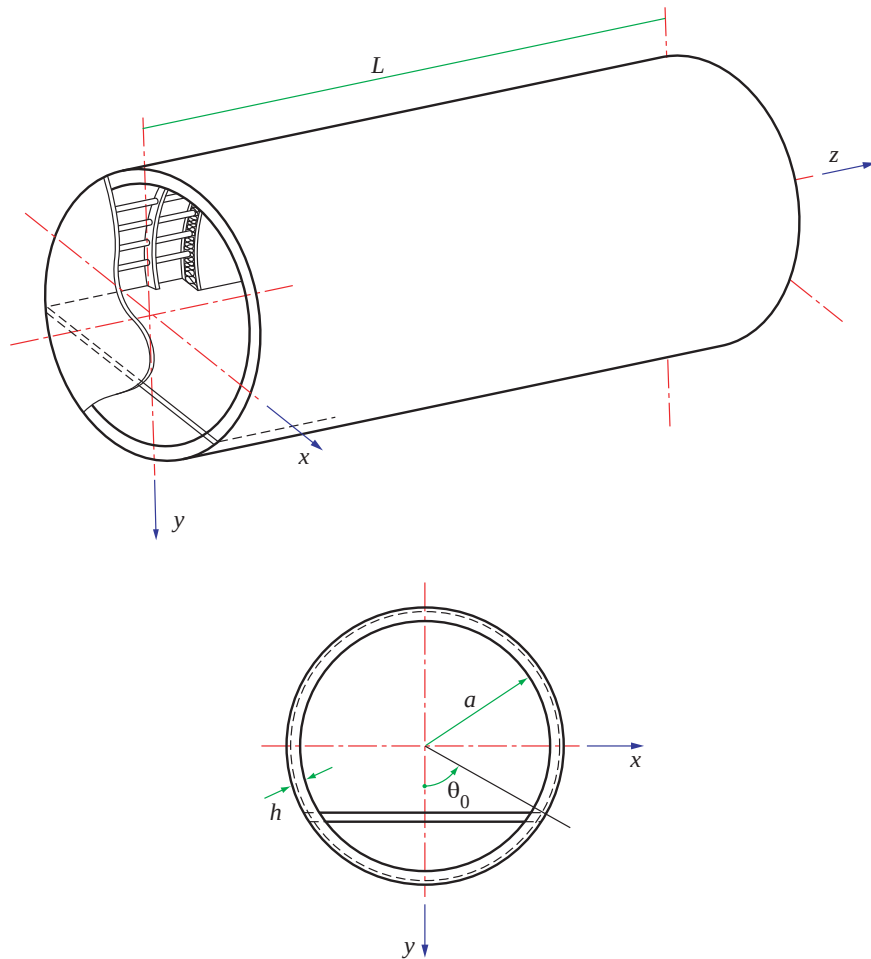
Note that the method in this Item does not deal with turbulent boundary layer noise.

In essence, time averaged power flowing into the cabin must equal power dissipated on the cabin walls. By considering power transmitted by both the resonant and non-resonant structure modes into acoustic modes, the ratio of external to internal sound power can be obtained. Hence, the ratio of external pressure to internal pressure can be calculated.

The computer program permits the user to calculate spatially-averaged interior sound pressure levels if exterior sound pressure levels are input using the standardized ESDU common data file. Exterior levels may either be harmonic or in one-third octave bands. Interior levels are obtained by subtracting the calculated Noise Reduction from exterior levels.

No account is taken of internal cabin noise sources such as air-conditioning.

The program requires as input the cabin geometry, material properties and details about trim properties. Aircraft bulkheads are modelled by specified end wall conditions. The user is given the option to have acoustic loss factors either estimated by the program or read from an input file. Results can be printed out in either standard or detailed form, as demonstrated in the Example section.



**Sketch 2.1 Fuselage Structure**

### 3. ANALYTICAL MODEL

The method to solve the sound transmission problem basically balances the band-limited time averaged power,  $W_{in}$ , flowing into the cabin against the net time-averaged power,  $W_{diss}$ , dissipated on the cabin walls and floor

$$W_{in} = W_{diss} \quad (3.1)$$

By expressing the in-flowing power in terms of the exterior exciting pressure and the dissipated power in terms of the interior response pressure, the equation can be solved for the interior pressure. The analytical model is described in detail through its stages of development in Derivations 1-8. This Item describes the components of the model used to perform the estimation of cabin noise reduction. The following sections provide the operating equations used by the computer program.

## 4. POWER FLOW EQUATIONS

### 4.1 Tonal Excitation

The equations giving  $W_{in}$  and  $W_{diss}$  for plane wave tonal excitation have been developed from those given in Derivations 1, 2 and 5 and are

$$W_{in} = \frac{32P_i^2 L^2}{\rho V} \left(\frac{mA}{4}\right)^2 \tau_{ML} \tau_t \omega_H^5 \sum_n \frac{\epsilon_n \eta_n}{\omega_n^2} \left[ \left(1 - \frac{\omega_H^2}{\omega_n^2}\right)^2 + \eta_n^2 \right]^{-1} \\ \times \sum_{r=MN} \frac{f'^2(n, r)}{M_r^2 \omega_r^4} \frac{j_M^2(\omega_H, a_z = \infty)}{|H_N^{(1)'}(k_x a)|^2} \left[ \left(1 - \frac{\omega_H^2}{\omega_r^2}\right)^2 + (\eta'_r + \eta''_r)^2 \right]^{-1}, \quad (4.1)$$

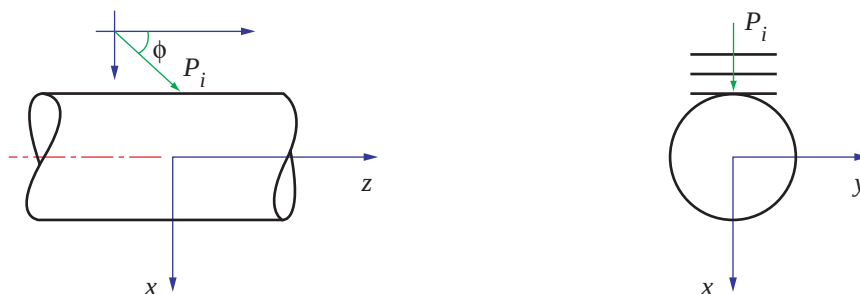
where  $H_m^{(1)'}(k_x a) = k_x H_{m-1}^{(1)}(k_x a) - \frac{m}{a} H_m^{(1)}(k_x a), \quad (4.2)$

$$k_x = \frac{\omega_H}{c_0} \sin \phi \quad (4.3)$$

and  $\tau_{ML} = \left(\frac{2\rho c_0}{\omega_H m}\right)^2. \quad (4.4)$

$$W_{diss} = \frac{V}{\rho c_0^2} \sum_n \frac{\eta_n \omega_n^2}{\omega_H} \langle p_n^2 \rangle_{s, t}. \quad (4.5)$$

The angle  $\phi$  required for calculating  $k_x$  and  $k_2$  is the angle of the incoming plane wave relative to the cylinder axis (see Sketch 4.1)



Sketch 4.1

Equating  $W_{in}$  to  $W_{diss}$  on a mode-by-mode basis and solving for the space averaged mean-square modal pressure for mode  $n$ , the space average interior levels are obtained from

$$\langle p_i^2 \rangle_{s, t} = \sum_n \langle p_n^2 \rangle_{s, t}. \quad (4.6)$$

Setting the mean square incident pressure

$$\langle p_{inc}^2 \rangle_t = P_i^2/2, \quad (4.7)$$

the ratio

$$\langle p_i^2 \rangle_{s,t} / \langle p_{inc}^2 \rangle_t$$

is calculated and, expressed in dB, is output by the program.

## 4.2 Diffuse Field Excitation

The equations giving  $W_{in}$  and  $W_{diss}$  for diffuse field excitation (Derivations 1, 2 and 5) are

$$W_{in} = \frac{\langle p_{bl}^2 \rangle}{c_\omega \omega} \frac{2\pi \bar{A}^2}{\rho V} \left( \frac{mA}{4} \right)^2 \tau_t \tau_{ML} \sum_n \epsilon_n \eta_n \sum_r \frac{j_r^2(\omega) f'^2(n, r)}{M_r^2} \\ \left\{ \frac{2\omega^2 \omega_n^2}{\pi D_{nr}} \left[ \left( \frac{c_r - c_n}{4} \right) \ln_n + \left( \frac{2c_n(b_r - b_n) - b_n(c_r - c_n)}{4\eta_n \omega_n^2} \right) \arctan_n + \left( \frac{c_n - c_r}{4} \right) \ln_r \right. \right. \\ \left. \left. + \left( \frac{2c_r(b_n - b_r) - b_r(c_n - c_r)}{4(\eta'_r + \eta''_r)\omega_r^2} \right) \arctan_r \right] \right\}, \quad (4.8)$$

where

$$\ln_j = \ln \left\{ \frac{\left| (1 + c_\omega/2)^4 \omega^4 + b_j(1 + c_\omega/2)^2 \omega^2 + c_j \right|}{\left| (1 - c_\omega/2)^4 \omega^4 + b_j(1 - c_\omega/2)^2 \omega^2 + c_j \right|} \right\}, \quad (4.9)$$

$$\arctan_j = \tan^{-1} \left[ \frac{(2 + c_\omega)^2 \omega^2 - 4\omega_j^2}{4\eta_j \omega_j^2} \right] - \tan^{-1} \left[ \frac{(2 - c_\omega)^2 \omega^2 - 4\omega_j^2}{4\eta_j \omega_j^2} \right], \quad (4.10)$$

$$D_{nr} = (c_r - c_n)^2 + (b_n - b_r)(b_n c_r - b_r c_n), \quad (4.11)$$

$$b_n = -2\omega_n^2, \quad (4.12)$$

$$b_r = -2\omega_r^2, \quad (4.13)$$

$$c_n = \omega_n^4(1 + \eta_n^2), \quad (4.14)$$

$$c_r = \omega_r^4(1 + (\eta'_r + \eta''_r)^2) \quad (4.15)$$

and

$$\tau_{ML} = \left( \frac{2\rho c_0}{\omega m} \right)^2 \quad (4.16)$$

$$W_{diss} = \frac{V}{\rho c_0^2} \sum \frac{\eta_n \omega_n^2}{\omega} \langle p_n^2 \rangle_{s,t} \quad (4.17)$$

$\omega$  is the band centre frequency.

Again, by equating  $W_{in}$  to  $W_{diss}$ , solving for the individual modal pressures,  $\langle p_n^2 \rangle_{s,t}$  one by one, and by adding the results according to the equation

$$\langle p_i^2 \rangle_{s,t} = \sum_n \langle p_n^2 \rangle_{s,t} \quad (4.18)$$

the interior level is obtained.

The exterior space-average mean square pressure,  $\langle p_e^2 \rangle_{s,t}$  is, for a diffuse field, related to the mean-square blocked pressure by

$$\langle p_e^2 \rangle_{s,t} = \frac{\langle p_{bl}^2 \rangle}{2} \quad (4.19)$$

The noise reduction,  $NR$ , is then given by

$$-10 \log_{10} \left[ \frac{\langle p_i^2 \rangle_{s,t}}{\langle p_e^2 \rangle_{s,t}} \right]$$

## 4.3 High Frequency Approximation

When the acoustic modal density is greater than a certain number of modes in an one-third octave band defined by the user, a high frequency approximation may be used. This approximation greatly reduces computer time\*. The development of this approximation is given in Derivations 1 and 2. The equations for  $W_{in}$  and  $W_{diss}$  are given below.

$$W_{in} = \frac{1}{4\rho c_0} [\tau_t(\tau_R + \tau_f)A_t + (\tau_R + \tau_f)A'] \langle p_e^2 \rangle_{s,t} - \frac{A'}{4\rho c_0} \left[ \frac{\bar{\eta}_r}{\eta_{rad}^{ext}} \tau_R + \tau_f \right] \langle p_i^2 \rangle_{s,t}, \quad (4.20)$$

where

$$\tau_R = \frac{2n_r \pi^2 c_0 \eta_{rad}^{ext} \eta_{rad}^{int}}{\rho_0 \omega \bar{\eta}_r} \frac{4\rho c_0}{\bar{A}}, \quad (4.21)$$

$$\tau_f = \frac{32}{\pi} \rho^2 \frac{\bar{A}}{m^2} \sum_{r < \Delta\omega} \left[ j_r^{rev}(\omega) \right]^2, \quad (4.22)$$

$$\bar{\alpha} = \frac{4\omega V}{c_0 S} \bar{\eta}_n, \quad (4.23)$$

\* A suggestion offered is a value of 30 modes as a starting point.

$$\eta_{rad}^{ext} = \frac{2\rho_0\omega\bar{A}}{\pi mc_0} \langle j_r^2(\omega) \rangle_r^{rev}, \quad (4.24)$$

and

$$\eta_{rad}^{int} = \frac{2\rho\omega\bar{A}}{\pi mc_0} \langle j_r^2(\omega) \rangle_r^{rev}. \quad (4.25)$$

$$W_{diss} = \frac{\bar{\alpha}S}{4\rho c_0} \langle p_i^2 \rangle_{s,t}. \quad (4.26)$$

## 5. INTERIOR COUPLING FACTOR

The interior coupling factor,  $f'(n, r)$ , which appears in both Equations (4.1) and (4.8) for the in-flowing power,  $W_{in}$ , determines the spatial coupling occurring between a fuselage structural mode and a cabin acoustic mode. It is defined as follows

$$f'(n, r) = \frac{1}{A} \int_A \phi_n(\bar{x}) \cdot \Psi_r(\bar{x}) d\bar{x}, \quad (5.1)$$

where  $\bar{x}$  represents a point on the transmitting structure,  $\phi_n(\bar{x})$  is the eigenvector for the  $n$ th cabin acoustic mode and  $\Psi_r(\bar{x})$  is the structural mode shape for the  $r$ th structural mode.

The mode shapes are given by

$$\phi_n(\bar{x}) = \phi_i(\theta_j) \cdot \cos \frac{q\pi z}{L} \quad (5.2)$$

and

$$\Psi_r(\bar{x}) = \sin \frac{M\pi z}{L} \cdot \sin \frac{N\pi y}{L_y}. \quad (5.3)$$

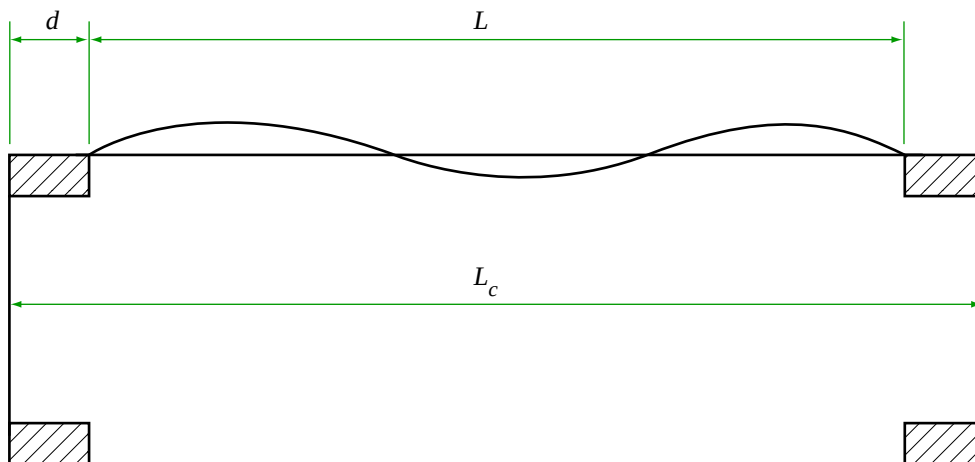
$\phi_i(\theta_j) = \phi_i(y)$  and is the  $i$ th eigenvector in two dimensions and is obtained from the finite difference method calculation (see Section 7) of the acoustic modal characteristics.

Structural mode shapes are complicated by factors like variation in skin thickness, presence of stiffeners, windows etc. and a simplifying assumption of representing them by sinusoidal functions has been made.

Then,

$$f'(n, r) \equiv f'(qi, MN) = \frac{1}{L \cdot L_y} \int_d^{d+L} \cos \frac{q\pi z}{L} \sin \frac{M\pi z}{L} dz \int_0^{L_y} \sin \frac{N\pi y}{L_y} \phi_i(y) dy. \quad (5.4)$$

When  $L = L_c$ ,  $d = 0$  (see Sketch 5.1).



Sketch 5.1

## 6. JOINT ACCEPTANCE FUNCTION

The joint acceptance function describes, on a scale between zero and unity, the coupling between the excitation field and the structure. It will, therefore, be different for free-field tonal excitation and diffuse field excitation.

### 6.1 Tonal Excitation

The joint acceptance function for tonal excitation corresponds to the  $j_M^2(\omega_H)$  term in Equation (4.1) and is obtained from the equation below.

$$j_M^2(\omega_H) = \frac{2}{M\pi^2 \Delta_M^2} \left[ P_M \left\{ 1 - (-1)^M \cos k_z L \right\} \right], \quad (6.1)$$

where

$$\Delta_M = \left[ 1 + \left( \frac{k_z L}{M\pi} \right)^2 \right] - 4 \left( \frac{k_z L}{M\pi} \right)^2, \quad (6.2)$$

$$P_M = \left[ 1 - \left( \frac{k_z L}{M\pi} \right)^2 \right]^2, \quad (6.3)$$

and

$$k_z = \frac{\omega_H}{c_0} \cos \phi. \quad (6.4)$$

## 6.2 Diffuse Field Excitation

The joint acceptance function for diffuse field excitation, corresponds to the  $j_r^{2^{rev}}(\omega)$  term in Equation (4.8), and can be expressed in the form

$$j_r^{2^{rev}}(\omega) = j_{MN}^{2^{rev}}(\omega) = j_M^{2^{rev}}(\omega) \cdot j_N^{2^{rev}}(\omega). \quad (6.5)$$

The axial component,  $j_M^{2^{rev}}(\omega)$ , is given by

$$j_M^{2^{rev}}(\omega) = I_1(M) + I_2(M) + I_3(M), \quad (6.6)$$

where

$$I_1(M) = \frac{1}{2\pi M k L} [Ci(kL + M\pi) - Ci(|M\pi - kL|)], \quad (6.7)$$

$$I_2(M) = \frac{1}{2kL} [Si(kL + M\pi) - Si(M\pi - kL)] \quad (6.8)$$

and

$$I_3(M) = \frac{1 - (-1)^M \cos kL}{(M\pi)^2 - (kL)^2}. \quad (6.9)$$

The circumferential component,  $j_N^{2^{rev}}(\omega)$ , is given by

$$j_N^{2^{rev}}(\omega) = I_1(N) + I_2(N) + I_3(N), \quad (6.10)$$

where

$$I_1(N) = \frac{1}{2\pi N k L_y} [Ci(kL_y + N\pi) - Ci(|N\pi - kL_y|)], \quad (6.11)$$

$$I_2(N) = \frac{1}{2kL_y} [Si(kL_y + N\pi) - Si(N\pi - kL_y)] \quad (6.12)$$

and

$$I_3(N) = \frac{1 - (-1)^N \cos kL_y}{(N\pi)^2 - (kL_y)^2}. \quad (6.13)$$

## 7. FUSELAGE ACOUSTICAL MODAL CHARACTERISTICS

The cross-section of an aeroplane fuselage is modelled by a cylinder with a floor partition as shown in Sketch 2.1.

The acoustic modal characteristics of the cabin are defined by the resonance frequencies, the mode shapes and the mode normalization given by the constant  $\epsilon_n$ .

Because the floor is present, it is not possible to derive the modal characteristics analytically since the boundary conditions are irregular. It is therefore necessary to resort to numerical methods and the finite difference technique has been selected. The problem is solved initially in two dimensions. The axial information can then be utilized to give a three-dimensional solution.

For this Item, this finite difference technique was validated by comparison with a finite element method and excellent agreement was obtained (Reference 6).

In the cabin, the Helmholtz equation applies. For the two dimensional situation, this is

$$\nabla^2 \phi + k^2 \phi = 0, \quad (7.1)$$

where 
$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad (7.2)$$

and 
$$k = \frac{\omega}{c_0}. \quad (7.3)$$

The central differences for the grid shown in Sketch 7.1 are

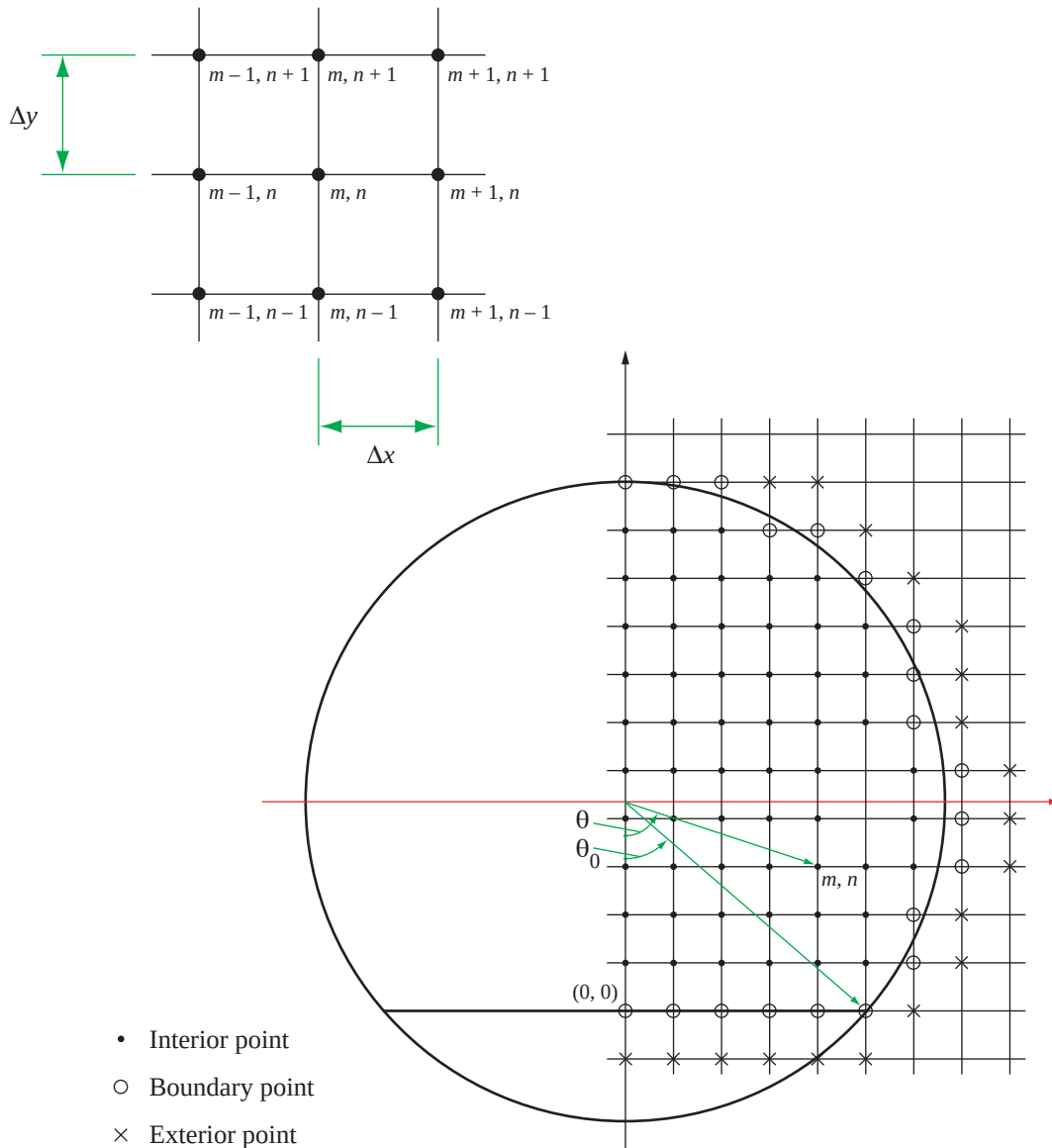
$$\frac{\partial \phi_{mn}}{\partial x} = \frac{P_{m+1,n} - P_{m-1,n}}{2\Delta x}, \quad (7.4)$$

$$\frac{\partial \phi_{mn}}{\partial y} = \frac{P_{m,n+1} - P_{m,n-1}}{2\Delta y}, \quad (7.5)$$

$$\frac{\partial^2 \phi_{mn}}{\partial x^2} = \frac{P_{m+1,n} - 2P_{m,n} + P_{m-1,n}}{(\Delta x)^2}, \quad (7.6)$$

and 
$$\frac{\partial^2 \phi_{mn}}{\partial y^2} = \frac{P_{m,n+1} - 2P_{m,n} + P_{m,n-1}}{(\Delta y)^2}, \quad (7.7)$$

where  $P$  and  $\phi$  are taken to be synonymous.



**Sketch 7.1 Finite difference grid**

If  $\Delta x = \Delta y = h_g,$  (7.8)

the Helmholtz equation becomes

$$4P_{m,n} - P_{m+1,n} - P_{m-1,n} - P_{m,n+1} - P_{m,n-1} = k^2 h_g^2 P_{m,n}. \quad (7.9)$$

The boundary condition is that the outward normal gradient is zero if the wall admittance is assumed to be sufficiently small. Referring to Sketch 7.1, in the region  $\theta_0 < \theta < 2\pi - \theta_0$ ,

$$(P_{m,n+1} - P_{m,n-1}) \cos \theta_{mn} = (P_{m+1,n} - P_{m-1,n}) \sin \theta_{mn}. \quad (7.10)$$

If  $-\theta_0 < \theta < \theta_0$ , the boundary condition above can be applied by setting  $\theta_{mn} = 0$  wherever  $\theta_{mn} < \theta_0$ , i.e.

$$P_{m, n+1} - P_{m, n-1} = 0 \text{ when } -\theta_0 < \theta < \theta_0. \quad (7.11)$$

Since the cabin is symmetrical about  $x = 0$ , only half of the cylinder is considered and the following grid points are identified.

- (i) cylinder interior and boundary points (total =  $n_j$ )
- (ii) cylinder boundary points (total =  $n_b$ )
- (iii) external points adjacent to boundary points (total =  $n_e$ )

Letting

$$\left\{ \mathbf{P}_I \right\}_{n_j \times 1} = \text{pressure at interior and boundary points}$$

$$\left\{ \mathbf{P}_E \right\}_{n_e \times 1} = \text{pressure at exterior points,}$$

Equation (7.9) can be expressed in matrix notation as

$$\begin{bmatrix} \mathbf{R}_I \end{bmatrix}_{(n_j \times n_j)} \begin{Bmatrix} \mathbf{P}_I \end{Bmatrix}_{(n_j \times 1)} + \begin{bmatrix} \mathbf{R}_E \end{bmatrix}_{(n_j \times n_e)} \begin{Bmatrix} \mathbf{P}_E \end{Bmatrix}_{(n_e \times 1)} = \lambda \begin{Bmatrix} \mathbf{P}_I \end{Bmatrix}_{(n_j \times 1)}, \quad (7.12)$$

where  $\lambda = k^2 h_g^2$ .

The boundary condition, Equations (7.10) and (7.11), give  $(n_e - 2)$  equations. Two more equations are required and are obtained by assuming the tangential gradient to be zero at two boundary points close to  $\theta = \pi/4$  and  $\theta = 3\pi/4$ , i.e. at these points it is assumed that

$$P_{m+1, n} - P_{m-1, n} = 0. \quad (7.13)$$

This results in a total of  $n_e$  equations giving

$$\begin{bmatrix} \mathbf{B}_I \end{bmatrix}_{n_e \times n_j} \begin{Bmatrix} \mathbf{P}_I \end{Bmatrix}_{n_j \times 1} + \begin{bmatrix} \mathbf{B}_E \end{bmatrix}_{n_e \times n_e} \begin{Bmatrix} \mathbf{P}_E \end{Bmatrix}_{n_e \times 1} = 0. \quad (7.14)$$

Combining Equations (7.12) and (7.14) gives the eigenvalue problem

$$\left[ \begin{bmatrix} \mathbf{R}_I \end{bmatrix} - \begin{bmatrix} \mathbf{R}_E \end{bmatrix} \begin{bmatrix} \mathbf{B}_E \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{B}_I \end{bmatrix} \right] \begin{Bmatrix} \mathbf{P}_I \end{Bmatrix} = \lambda \begin{Bmatrix} \mathbf{P}_I \end{Bmatrix}. \quad (7.15)$$

The eigenvalues and eigenvectors are calculated for symmetric and antisymmetric modes separately. The modes are then normalized by making the maximum value achieved at any coordinate position unity and adjusting the other values to retain the computed ratios from point to point.

For symmetric modes

$$P_{-1, n} = P_{1, n} \quad (7.16)$$

and for antisymmetric modes

$$P_{-1, n} = -P_{1, n}, \quad (7.17)$$

The modes are combined and ranked in ascending order of frequency by the computer program. Each mode must be identified as symmetric or antisymmetric because the modes,  $n_j$ , represent only half the cylinder.

It should be noted that the first symmetric mode is a translation mode with zero frequency.

The vector  $\{\mathbf{P}_i\}$  corresponding to  $\lambda_i$  is the mode shape  $\{\phi_i(j)\}$  where  $i$  is the 2-D mode counter and  $j$  defines the position in the fuselage cross section. The eigenvalues  $\lambda_i$  are used to calculate the resonance frequencies using the equation

$$f_i = \frac{c_0}{2\pi h_g} \sqrt{\lambda_i}. \quad (7.18)$$

These data are calculated for the cross section of a cylinder of unit radius.

## 7.1 Extension to three-dimensional cabin of arbitrary radius

The resonance frequencies for the three-dimensional cabin of arbitrary radius are calculated using the equation below

$$f_n = f_{qi} = \frac{c_0}{2\pi} \left[ \left( \frac{q\pi}{L} \right)^2 + \left( \frac{2\pi f_i}{c_0 a} \right)^2 \right]^{1/2}, \quad (7.19)$$

where  $a$  is the radius of the cylinder,  
 $L$  is the length of the cylinder,  
 $f_i$  is the frequency for unity radius calculated according to Equation (7.18),  
 and  $q$  is the number of axial half-wavelengths for acoustic mode  $n \equiv (q, i)$ .

The acoustic modes,  $\phi_n$ , for the three-dimensional cabin are given by

$$\phi_n(X, Y, z) = \cos \frac{q\pi z}{L} \phi_i(j), \quad (7.20)$$

where  $\phi_i(j)$  is the eigenvector for the two dimensional mode as calculated by the finite difference technique.

## 7.2 Normalization of Acoustic Modes

Normalization of the acoustic modes is carried into the transmission prediction with the parameter  $\varepsilon_n$  given by

$$\varepsilon_n = \frac{V}{\iiint_V \phi_n^2(\bar{\xi}) d\bar{V}}, \quad (7.21)$$

where

$$V = a^2 L [\pi - \theta_0 + \cos \theta_0 \sin \theta_0] \quad (7.22)$$

and

$$\int_V \phi_{qi}^2(\bar{\xi}) d\bar{V} = \frac{a^2 L}{2} \varepsilon_q \Phi_G(i), \quad (7.23)$$

where the generalized mass,  $\Phi_G(i)$ , is given by

$$\Phi_G(i) = \sum_{j=1}^{n_j} \phi_i^2(j) h_g^2 C(j), \quad (7.24)$$

where  $h_g$  = distance between grid points,  
 $C(j)$  = 1 for all locations on the centreline and on the cylinder boundary  
 2 for all other locations

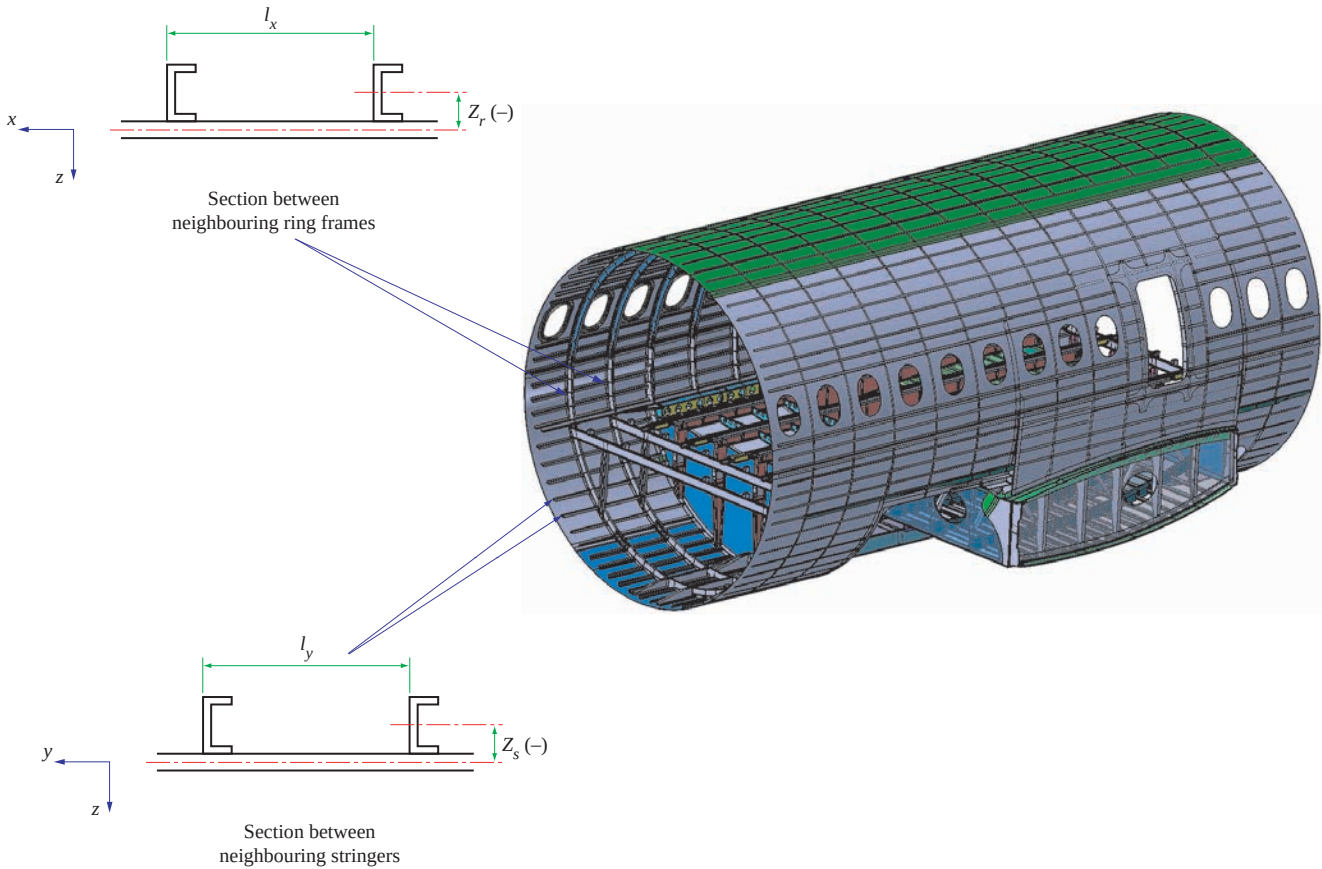
and  $\varepsilon_q = 2$  for  $q = 0$   
 $= 1$  for  $q > 0$ . (7.25)

## 8. FUSELAGE STRUCTURAL MODAL CHARACTERISTICS

The fuselage structural modal characteristics are defined by the eigenvectors,  $\Psi_r(\bar{x})$ , resonance frequencies,  $\omega_r$ , and the mode normalization given by a constant, the modal mass,  $M_r$ .

The analytical technique used to predict the resonance frequencies of the frame-stringer stiffened shell or curved panel is described in Reference 1. The method approximates a cabin fuselage by a stiffened cylinder when no floor is present or by a stiffened, curved, orthotropic panel running from floor to floor when a floor is present.

The equation to estimate resonance frequencies for a simply-supported stiffened cylinder (see Sketch 8.1) is given here. When applied to a curved panel,  $N$  is replaced by  $N/2$  as  $N$  then refers to the number of circumferential half-wavelengths.



**Sketch 8.1 Geometry of eccentrically stiffened cylinder**

From Reference 1,

$$\frac{\omega^2 mL}{\pi^4 D} = M^4(1 + \beta^2)^2 + M^4 \left[ \frac{E_s I_s}{D \cdot l_y} + \beta^2 \left( \frac{G_s J_s}{D \cdot l_y} + \frac{G_r J_r}{D \cdot l_x} \right) + \beta^4 \frac{E_r I_r}{D \cdot l_x} \right] + \frac{12L^4(1 - \mu^2)}{\pi^4 h^2 a^2} \left[ \frac{1 + \bar{S}\Omega s + \bar{R}\Omega r + \bar{S} \cdot \bar{R}\Omega r s}{\Omega} \right] + \Delta p \frac{a}{2} \frac{M^2 L^2}{\pi^2 D} (1 + 2\beta^2), \quad (8.1)$$

where  $M$  = No. of axial  $\frac{1}{2}$  wavelengths, (8.2)

$N$  = No. of circumferential full wavelengths,

$$\bar{R} = \frac{E_r A_r}{E \cdot h \cdot l_x}, \quad \bar{S} = \frac{E_s A_s}{E \cdot h \cdot l_y} = \text{ring frame \& stringer membrane parameters}, \quad (8.3)$$

$$\Omega s = 1 + 2\alpha^2 (Z_s/a)(\beta^2 - \mu) + \alpha^4 (Z_s/a)^2 (1 + \beta^2)^2, \quad (8.4)$$

$$\Omega r = 1 + 2N^2 (Z_r/a)(1 - \mu\beta^2) + N^4 (Z_r/a)^2 (1 + \beta^2)^2, \quad (8.5)$$

$$\Omega = (1 + \beta^2)^2 + 2\beta^2(1 + \mu)(\bar{R} + \bar{S}) + (1 - \mu^2)[\bar{S} + \beta^4\bar{R} + 2\beta^2\bar{R}\bar{S}(1 + \mu)], \quad (8.6)$$

$$\begin{aligned} \Omega rs = N^2 \alpha^2 [\beta^2(1 - \mu^2) + 2(1 + \mu)](Z_s/a)^2 + N^4 [1 - \mu^2 + 2\beta^2(1 + \mu)](Z_r/a)^2 \\ + 2N^2(1 - \mu^2)[Z_s/a + Z_r/a] + 2N^4(1 + \mu)^2(Z_s/a)(Z_r/a) + (1 - \mu^2), \end{aligned} \quad (8.7)$$

$$\alpha = \frac{M\pi a}{L}, \quad (8.8)$$

$$\beta = \frac{2NL}{ML_y}, \quad (8.9)$$

and 
$$D = \frac{Eh^3}{12(1 - \mu^2)}. \quad (8.10)$$

The terms  $\frac{E_r I_r}{D \cdot l_x}$  and  $\frac{E_s I_s}{D \cdot l_y}$  are the ring frame and stringer bending parameters respectively.

As frequency increases the influence of frames and stringers varies. This effect is described in Reference 3, where the structural parameters used to describe the panel change with wavelength and frequency of the modes of vibration. The user is given the option of calculating the resonance frequencies without 'smearing out' the frames and stringers at intermediate and high frequencies as described below.

At low frequencies, where the structural modal wavelengths, both axially and circumferentially, are greater than the frame and stringer separation distances, the frames and stringers are assumed to be 'smeared out' orthotropically over the skin in both cases, and the treatment is the same.

As frequency increases the axial half-wavelength of the modes becomes less than the spacing between adjacent frames and the frame motion becomes small compared to the motion of the panels between the frames. Inclusion of the frame stiffness now results in overestimates of the panel resonance frequencies. Thus, in this mid-frequency region, the frame stiffness and mass are omitted from the analytical model, and only the stringers are smeared out over the panel.

Finally, in the high frequency regime, the axial and circumferential half-wavelengths of the structural models are less than the spacings between frames and stringers, respectively, and subpanel resonances occur. Only the skin stiffness and mass are now included in the analytical model.

The boundaries are defined by:

- (a) Low - Intermediate:  
The minimum value of the resonance frequency of the sub-element (see Sketch 8.2) of size  $l_x \cdot l_y$ , clamped along two adjacent frames, for any circumferential mode order  $N$  with half wavelengths > stringer pitch, using (stringer + skin) structural parameters
- (b) Intermediate - High:  
The fundamental frequency of subpanel with clamped edges, size  $(l_x \cdot l_y)$  using structural parameters only.

For clamped-clamped boundary conditions the bending wavelengths are smaller than those of a simply