



ESDU 07002

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Wing viscous drag coefficient in shock-free attached flow



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Mr S.F. Wood	—	BAE SYSTEMS, Woodford.

* Corresponding Member

The construction and subsequent development of the Data Item was undertaken by

Mr C.J. Batt — Engineer.

The person with overall responsibility for the work in this subject area is Mr R.W. Gilbey, Head of Aerodynamics Group.

WING VISCOUS DRAG COEFFICIENT IN SHOCK-FREE ATTACHED FLOW

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WING VISCOUS DRAG COEFFICIENT IN SHOCK-FREE ATTACHED FLOW

1. NOTATION

		<i>SI</i>	<i>British</i>
A	wing aspect ratio, b^2/S_w		
a_0	viscous lift-curve slope of streamwise aerofoil in two-dimensional flow	rad^{-1}	rad^{-1}
a_{0T}	inviscid lift-curve slope of streamwise aerofoil in two-dimensional flow	rad^{-1}	rad^{-1}
a_{1w}	viscous lift-curve slope of wing	rad^{-1}	rad^{-1}
b	wing span, 2s (Sketch 1.1)	m	ft
C_D	wing drag coefficient, based on S_w		
C_{Df0}	skin friction drag coefficient of streamwise aerofoil at zero lift, based on c , see ESDU 06001		
C_{Dmin}	minimum profile drag coefficient of streamwise aerofoil, based on c , see ESDU 00027		
C_{DP}	wing profile drag coefficient, based on S_w , see ESDU 07003		
C_{DPmin}	minimum value of C_{DP} (Sketch 3.1 and see ESDU 07003)		
C_{DPmin0}	value of C_{DPmin} for wing without twist		
$(\Delta C_{DPmin})_e$	increment in C_{DPmin} due to twist, see Section 3.3		
C_{dp}	local profile drag coefficient at spanwise station η , based on c , see ESDU 07003		
C_{dpm}	minimum value of C_{dp} (in a plot of C_{dp} verses C_l)		
C_{Dvisc}	wing viscous drag coefficient, based on S_w (Sketch 3.1)		
C_{DV}	wing inviscid trailing-vortex drag coefficient, based on S_w (Sketch 3.1)		
C_L	wing lift coefficient, based on S_w		
C_{LK}	value of C_L at end of linear range of force and moment characteristics, see ESDU 88030		

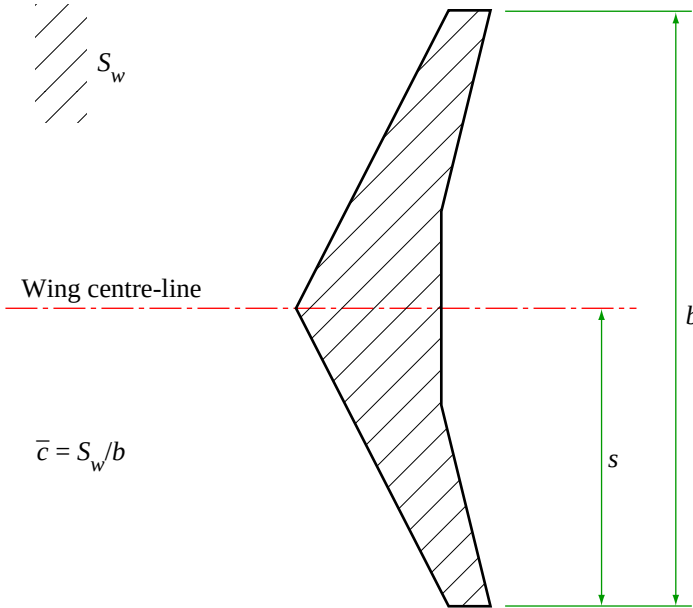
C_{Lmin}	value of C_L corresponding to the minimum value of C_{Dvisc} (Sketch 3.1)		
C_l	local lift coefficient based on c , at spanwise station η , see ESDU 07003		
c	local chord in streamwise direction	m	ft
c_k	value of c at crank station, η_k , (Sketches 1.3 and 3.2)	m	ft
c_t	value of c at wing tip (Sketches 1.2, 1.3 and 3.2)	m	ft
c_0	value of c at plane of symmetry (centre line) for straight-tapered wing (Sketches 1.2 and 3.2)	m	ft
$(c_0)_k$	value of c at plane of symmetry (centre line) for cranked wing (Sketches 1.3 and 3.2)	m	ft
c_η	value of c at spanwise station η	m	ft
$c_{\eta_{ref}}$	value of c at η_{ref} for straight-tapered planform (Sketch 1.2)	m	ft
$c_{\eta=0.5}$	value of c at $\eta = 0.5$ for straight-tapered planform (Sketch 1.2)	m	ft
$\bar{c}$	standard (or geometric) mean chord, S_w/b , (Sketch 1.1)	m	ft
K_{visc}	lift-dependent viscous drag factor, see Section 3.4		
k_G	factor accounting for aspect ratio and section thickness, see Section 3.3 and Figure 1		
$k_{R_{\bar{c}}}$	factor accounting for $R_{\bar{c}}$, see Equation (3.13)		
M	freestream Mach number		
R_c	Reynolds number, based on c and freestream properties		
$R_{\bar{c}}$	Reynolds number, based on $\bar{c}$ and freestream properties		
S_w	wing planform area (Sketch 1.1)	m ²	ft ²
s	wing semispan (Sketches 1.1, 1.2, 1.3 and 3.2)	m	ft
s_k	spanwise distance from centre-line to crank (Sketches 1.3 and 3.2)	m	ft
t	thickness of streamwise aerofoil at x/c	m	ft
t_m	maximum thickness of streamwise aerofoil	m	ft

$t_{0.75}$	thickness of streamwise aerofoil at $x = 0.75c$	m	ft
$t_{0.9}$	thickness of streamwise aerofoil at $x = 0.9c$	m	ft
$t_{0.99}$	thickness of streamwise aerofoil at $x = 0.99c$	m	ft
x	chordwise distance aft of leading edge, measured in streamwise direction	m	ft
x_{tr}	mean of chordwise distances to upper and lower surface boundary-layer transition positions, measured aft of leading edge in streamwise direction	m	ft
y	spanwise distance from centre line (positive starboard) (Sketches 1.2 and 1.3)	m	ft
z_c	camber-line ordinate of streamwise aerofoil	m	ft
$z_{c0.9}$	value of z_c at $x = 0.9c$	m	ft
z_u	upper surface ordinate of streamwise aerofoil	m	ft
z_{um}	maximum value of z_u	m	ft
$z_{u0.9}$	value of z_u at $x = 0.9c$	m	ft
$z_{u0.99}$	value of z_u at $x = 0.99c$	m	ft
α_{0v}	zero-lift angle of attack of streamwise aerofoil in viscous flow (Sketch 1.4), see ESDU 98011	rad	rad
α_{0vw}	wing zero-lift angle of attack in viscous flow, see ESDU 07003	rad	rad
$(\alpha_{0vw})_c$	component of α_{0vw} due to camber, see Section 3.5.2	rad	rad
β	$\sqrt{1 - M^2}$, Prandtl-Glauert compressibility factor		
ε	local angle of twist measured from datum, positive nose up (Sketch 1.4)	deg	deg
$ \varepsilon $	modulus of ε (Sketch 1.5)	deg	deg
$ \varepsilon _m$	maximum value of $ \varepsilon $ across semispan (Sketch 1.5)	deg	deg
$ \varepsilon _{0.2}$	value of $ \varepsilon $ at $\eta = 0.2$ (Sketch 1.5)	deg	deg
$ \varepsilon _{0.8}$	value of $ \varepsilon $ at $\eta = 0.8$ (Sketch 1.5)	deg	deg
η	spanwise distance from centre-line as fraction of semispan, y/s		
η_k	value of η at crank station, s_k/s (Sketches 1.3 and 3.2)		

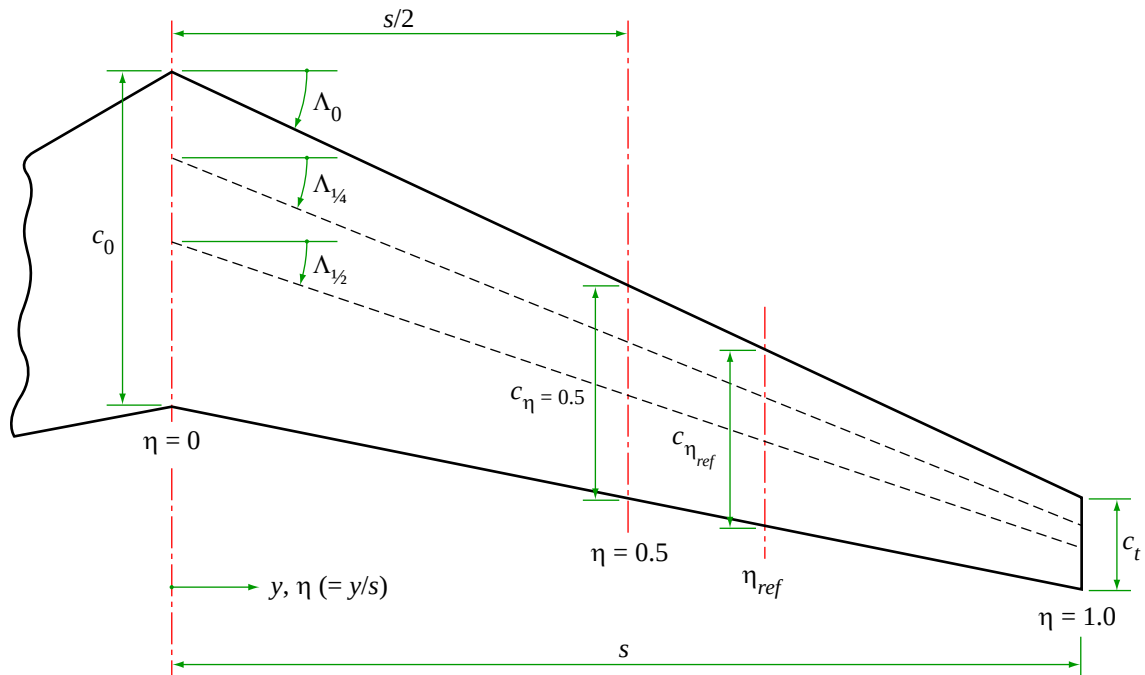
η_{ref}	spanwise reference station (Sketch 1.2), see Section 3.4 and Figures 2 to 7		
$\eta_{ \epsilon _m}$	value of η at which $ \epsilon _m$ occurs (Sketch 1.5)		
Λ_0	leading-edge sweep angle (Sketches 1.2, 1.3 and 3.2)	deg	deg
$\Lambda_{1/4}$	quarter-chord sweep angle of straight-tapered wing (Sketch 1.2)	deg	deg
$\Lambda_{1/2}$	mid-chord sweep angle of straight-tapered wing (Sketch 1.2)	deg	deg
λ	taper ratio, c_t / c_0		

Subscripts

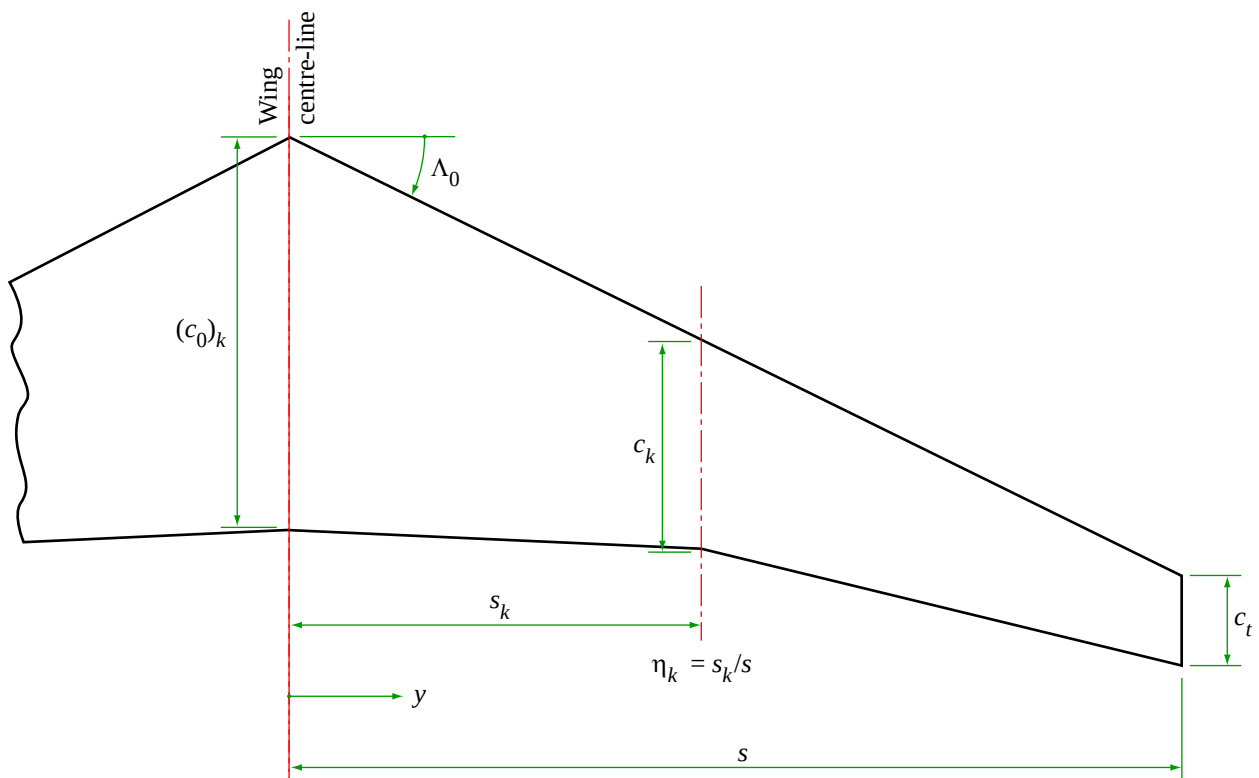
$()_\eta$	value at spanwise station η
$()_{\eta = 0.5}$	value at mid-semispan ($\eta = 0.5$)
$()_{\eta = 1.0}$	value at wing tip ($\eta = 1.0$)
$()_{\eta_{ref}}$	value at η_{ref}



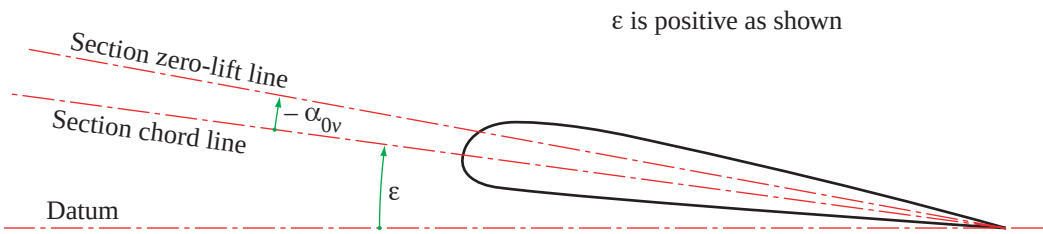
Sketch 1.1 Wing planform area



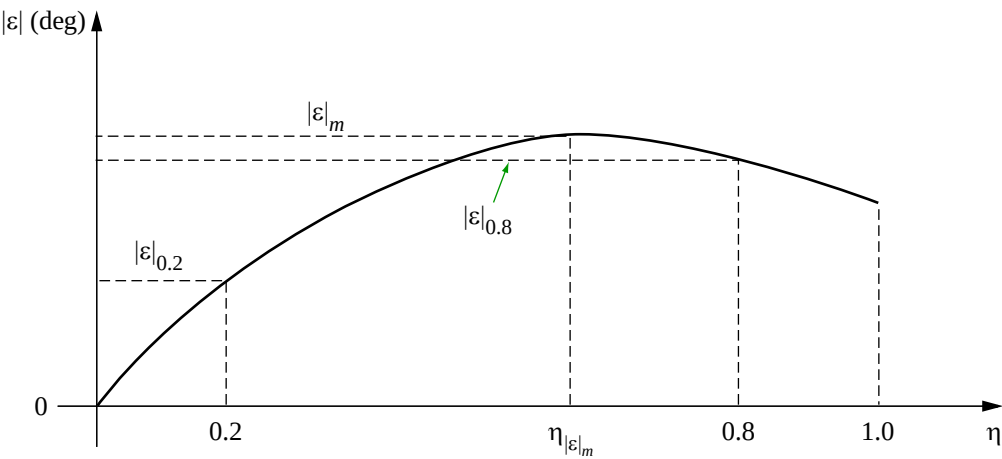
Sketch 1.2 Planform geometry of straight-tapered wing



Sketch 1.3 Planform geometry of cranked wing



Sketch 1.4 Datum used to define wing angle of attack



Sketch 1.5 Distribution of modulus of twist, $|\epsilon|$, across wing semispan

2. INTRODUCTION

Viscous drag coefficient is defined in this Data Item as the drag coefficient arising from all the effects of viscosity. This Item provides a means of predicting the viscous drag coefficient of wings with sweep angles not exceeding 45° , aspect ratios not exceeding 12, maximum section thickness to chord ratios within the range 0.1 to 0.2 and up to 10° wash-in or wash-out. ESDU 07002 is applicable to values of lift coefficient at which both upper and lower surface boundary layers are fully attached, in flow fields where shockwaves are not present. The method is applicable to both straight-tapered and cranked wing planforms and where the section geometry, including camber, either remains unaltered or varies with spanwise location.

The method, described in Section 3, predicts the viscous drag coefficient of a given wing geometry at a chosen value of lift coefficient, where the location of boundary-layer transition is known and its chordwise position is constant across the span. The method is based on a semi-empirical investigation using strip theory presented in ESDU 07003 (Derivation 10). The method employs ESDU 70011, 76003, 87031, 97020, 98011, 00027 and 06001 (Derivations 1 to 3, 5 to 7 and 9).

The applicability and accuracy is discussed in Section 4 and the ranges of applicability of various geometric and flow parameters are given. As a means of testing the validity of the method, polars of predicted and measured total drag coefficient for five wing geometries are compared. The predicted total drag coefficient was obtained by adding values of inviscid trailing-vortex drag coefficient (predicted using ESDU 10022; Derivation 11) to values of viscous drag coefficient obtained by the method of this Item. The value of C_L giving an approximate upper limit of applicability of the method may be obtained using ESDU 88030 (Derivation 4). However, viscous drag coefficient may be predicted with reasonable confidence over the range $0 \leq C_L \leq 0.5$; in most cases, lift coefficients at the cruise condition lie within this range.

The Derivation and References are listed in Section 5. Two worked examples illustrating the use of the method for a straight-tapered wing with constant spanwise camber and a wing with a single crank and spanwise varying camber, are contained in Section 6.

3. METHOD

In shock free flow, the total wing drag coefficient C_D comprises the sum of viscous drag coefficient C_{Dvisc} and inviscid trailing-vortex drag coefficient C_{DV} , so that

$$C_D = C_{Dvisc} + C_{DV}. \quad (3.1)$$

The inviscid trailing-vortex drag coefficient is associated primarily with the wing downwash field.

The viscous drag coefficient is defined in this Data Item as the drag coefficient due to all viscous effects. It comprises profile drag and a component due to viscous losses in the tip vortex cores (see Section 5 of ESDU 07003). The latter component becomes more significant at large values of lift coefficient, C_L . The viscous drag coefficient for values of C_L , where both upper and lower surface boundary layers remain fully attached, may be obtained using

$$C_{Dvisc} = C_{DPmin} + K_{visc}(C_L - C_{Lmin})^2, \quad (3.2)$$

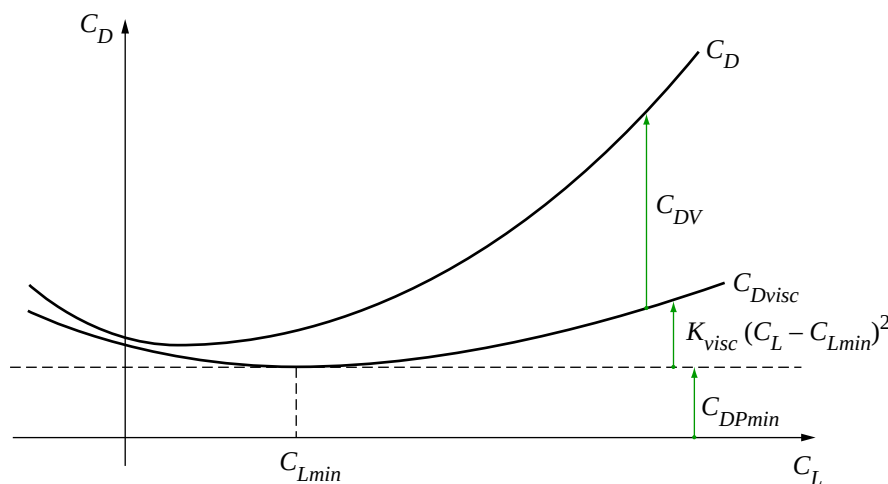
see Section 3 of ESDU 07003. In Equation (3.2) C_{DPmin} is the minimum profile drag coefficient of the wing, K_{visc} is a lift-dependent viscous drag factor and C_{Lmin} is the lift coefficient corresponding to the minimum value of C_{Dvisc} . In Equation (3.2) C_{DPmin} can be subdivided to allow for twist, so that

$$C_{Dvisc} = C_{DPmin0} + (\Delta C_{DPmin})_\epsilon + K_{visc}(C_L - C_{Lmin})^2, \quad (3.3)$$

where C_{DPmin0} is the minimum profile drag coefficient of the wing without twist, and $(\Delta C_{DPmin})_\epsilon$ is an increment in wing profile drag coefficient due to geometric twist, see Section 3 of ESDU 07003.

In most cases viscous drag coefficient may be predicted using Equation (3.3) over the range $0 \leq C_L \leq 0.5$ for those wing geometries for which this Item is applicable, see Section 4.

The various components of wing total drag coefficient are illustrated in Sketch 3.1.



Sketch 3.1 Components of total drag coefficient

The method is applicable to both straight-tapered and cranked wing geometries. Sketches 1.2 and 1.3 show the planform geometries of a straight-tapered wing and a cranked wing respectively. If the wing is cranked, an equivalent straight-tapered planform is required, see Section 3.1.

The methods in Sections 3.2, 3.3, 3.4 and 3.5 are used to evaluate C_{DPmin0} , $(\Delta C_{DPmin})_\varepsilon$, K_{visc} and C_{Lmin} respectively. The viscous drag coefficient may then be evaluated using Equation (3.3).

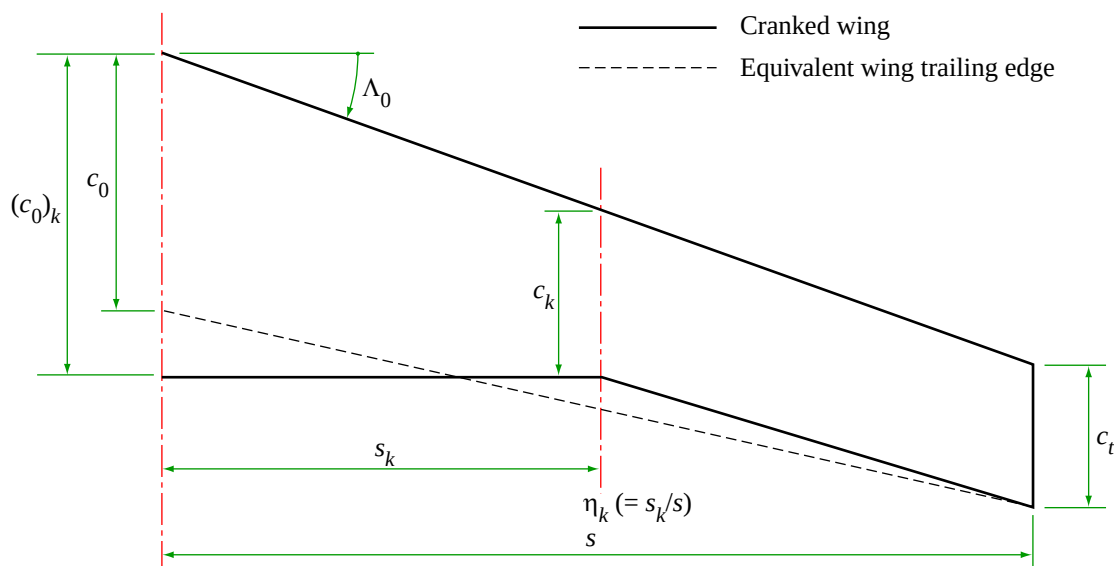
3.1 Cranked Planforms and the Equivalent Straight-tapered Wing

The method of this section is only required if the wing planform is cranked.

To determine the overall aerodynamic characteristics of cranked wing geometries it is a useful artifice to create an 'equivalent straight-tapered' wing planform which is suitably representative of the true wing planform. The method of obtaining an equivalent straight-tapered wing, for the common case of a wing with a single crank and a straight leading edge, is given below. For wings with more than one crank, or where the leading edge is not straight, the equivalent wing planform is obtained using ESDU 76003 (Derivation 2). The equivalent wing maintains the same tip chord fixed in space relative to the leading edge of the centre-line chord of the cranked wing. It also maintains the same planform area and span as the cranked wing it represents, and is suitable for the estimation of gross wing parameters such as lift-curve slope from methods constructed by parametric analysis of straight-tapered wings.

From the cranked wing planform shown in Sketch 1.3 the spanwise location of the crank is $\eta_k = \frac{s_k}{s}$, where s is the wing semi-span, and s_k is the distance between the centre-line station and the crank station. Also, the centre-line chord of the cranked wing is $(c_0)_k$, the chord at the crank is c_k , and the streamwise tip chord is c_t .

Provided that the true wing planform has a straight leading edge and a straight streamwise wing tip, the equivalent wing has the same leading-edge sweep Λ_0 , span b , streamwise tip chord c_t , and the **same planform area S_w** (Sketch 1.1) as the true planform. The trailing edge of the equivalent straight-tapered wing is shown dashed in Sketch 3.2 and the true wing planform is shown by the continuous line. The spanwise sections of the equivalent wing are assumed to be the same as for the true wing.



Sketch 3.2 Equivalent straight-tapered wing planform

From Sketch 1.3 the planform area of the true wing is

$$S_w = [(c_0)_k + c_k]s_k + (c_k + c_t)(s - s_k), \quad (3.4)$$

which is also equal to the planform area of the equivalent wing. Therefore, the centre-line chord of the equivalent wing, c_0 , is

$$c_0 = \frac{S_w}{s} - c_t, \quad (3.5)$$

where S_w , c_t and s have the same values for both true and equivalent wing geometries. The equivalent wing geometry for a wing with a straight leading edge and a single crank is obtained as part of a worked example in Section 6.2.1.

For a straight-tapered wing geometry, the quarter-chord sweep angle, $\Lambda_{1/4}$, is given by

$$\Lambda_{1/4} = \tan^{-1} \left[\tan \Lambda_0 - \frac{1}{A} \left(\frac{1 - \lambda}{1 + \lambda} \right) \right], \quad (3.6)$$

see ESDU 76003 (Derivation 2), where the wing aspect ratio A is given by

$$A = \frac{b^2}{S_w}. \quad (3.7)$$

Also in Equation (3.6) the wing taper ratio, λ , is given by

$$\lambda = \frac{c_t}{c_0}. \quad (3.8)$$

3.2 Minimum Profile Drag Coefficient of Untwisted Wing, C_{DPmin0}

The streamwise chord length at the spanwise station $\eta = 0.5$ is $c_{\eta=0.5}$ and for a straight-tapered wing (see Sketch 1.2), $c_{\eta=0.5} = \bar{c}$, where $\bar{c}$ is the wing standard (or geometric) mean chord. With this as the characteristic section, Section 4.5.1 of ESDU 07003 gives

$$C_{DPmin0} = (C_{dpmin})_{\eta=0.5}. \quad (3.9)$$

In this equation $(C_{dpmin})_{\eta=0.5}$ is the local profile drag coefficient at the spanwise station $\eta = 0.5$, evaluated at the local chord Reynolds number $(R_c)_{\eta=0.5}$. Since $c_{\eta=0.5} = \bar{c}$,

$$(R_c)_{\eta=0.5} = R_{\bar{c}}, \quad (3.10)$$

where $R_{\bar{c}}$ is the Reynolds number based on $\bar{c}$. Section 4.5.2 of ESDU 07003 indicates that Equation (3.9) accounts adequately for wing geometries where the section maximum thickness-to-chord ratio is constant or has a linear (or near linear) variation across the span.

3.2.1 Minimum local profile drag coefficient at $\eta = 0.5$, $(C_{dpmin})_{\eta = 0.5}$

The local profile drag at $\eta = 0.5$ is the force acting in the streamwise direction at that spanwise wing station. Section 4.4 of ESDU 07003 shows that the minimum local profile drag coefficient $(C_{dpmin})_{\eta = 0.5}$ in Equation (3.9) can be represented in terms of two-dimensional section values as

$$(C_{dpmin})_{\eta = 0.5} = \left[(C_{Dmin})_{\eta = 0.5} - (C_{Df0})_{\eta = 0.5} \right] \cos^2 \Lambda_{1/4} + (C_{Df0})_{\eta = 0.5} \quad (3.11)$$

In Equation (3.11) $(C_{Dmin})_{\eta = 0.5}$ is the minimum profile drag coefficient of the streamwise aerofoil section at $\eta = 0.5$ in two-dimensional flow, and $(C_{Df0})_{\eta = 0.5}$ is the skin friction drag coefficient of the same section. For unswept wings $\Lambda_{1/4} = 0$, and Equation (3.11) gives $(C_{dpmin})_{\eta = 0.5} = (C_{Dmin})_{\eta = 0.5}$.

3.2.1.1 Aerofoil minimum profile drag coefficient at $\eta = 0.5$, $(C_{Dmin})_{\eta = 0.5}$

In Equation (3.11), the value of $(C_{Dmin})_{\eta = 0.5}$ for the streamwise aerofoil section at $\eta = 0.5$ in shock-free compressible flow, is obtained from ESDU 00027 (Derivation 7) using $R_{\bar{c}}$ (i.e. $(R_c)_{\eta = 0.5}$ for a straight-tapered wing). The method of that Item requires knowledge of the mean upper and lower surface boundary-layer transition positions as a fraction of the chord length, x_{tr} / c .

3.2.1.2 Aerofoil skin friction drag coefficient at $\eta = 0.5$, $(C_{Df0})_{\eta = 0.5}$

In Equation (3.11), the value of $(C_{Df0})_{\eta = 0.5}$ for the streamwise aerofoil section in shock-free compressible flow is obtained from ESDU 06001 (Derivation 9) using $R_{\bar{c}}$ (i.e. $(R_c)_{\eta = 0.5}$ for a straight-tapered wing). The method of that Item also requires knowledge of x_{tr} / c .

3.3 Increment in Wing Minimum Profile Drag Coefficient Due to Geometric Twist, $(\Delta C_{DPmin})_{\epsilon}$

Geometric twist was found in Section 5.2.2 of ESDU 07003 to increase the minimum wing profile drag coefficient above that of a similar untwisted wing, see Equation (3.3), where the increment is denoted by $(\Delta C_{DPmin})_{\epsilon}$.

Local angle of twist, ϵ , is the angle between a chosen datum and the chord of the local section, see Sketch 1.4. This angle is measured positive in the nose-up direction. The local lift-dependent profile drag coefficient at spanwise station η was found in Section 6 of ESDU 07003 to be dependent on the amount of twist relative to the datum, and was independent of the direction of twist, nose-up (wash-in) or nose-down (wash-out). The method therefore employs the modulus of the local twist angle $|\epsilon|$. The datum for all of the wing geometries included in the database was taken as the centre-line chord, so that $\epsilon = 0$ at $\eta = 0$. In addition to geometric twist, $(\Delta C_{DPmin})_{\epsilon}$ was also found in Section 6 of ESDU 07003 to be dependent on taper ratio, aspect ratio, section thickness and Reynolds number.

For wing geometries exhibiting relatively simple twist distributions such as those illustrated later in Table 4.2, Section 6.2 of ESDU 07003 gives

$$(\Delta C_{DPmin})_\epsilon = 1.2 \times 10^{-5} k_G k_{R_{\bar{c}}} \eta_{|\epsilon|_m} |\epsilon|_m \left[\frac{1 - \eta_{|\epsilon|_m} (1 - \lambda)}{(1 + \lambda)^2} \right] (|\epsilon|_{0.2} + |\epsilon|_{0.8}), \quad (3.12)$$

where $|\epsilon|_m$ is the maximum value of $|\epsilon|$ (in degrees) occurring across the semispan and $\eta_{|\epsilon|_m}$ is the spanwise station at which it occurs, see Sketch 1.5. Also in Equation (3.12), $|\epsilon|_{0.2}$ is the value of $|\epsilon|$ at $\eta = 0.2$ and $|\epsilon|_{0.8}$ is the value of $|\epsilon|$ at $\eta = 0.8$, see Sketch 1.5.

The factor k_G is obtained using Figure 1 and accounts for the combined effects of aspect ratio and section thickness, see Sections 6.1.2 and 6.1.3 of ESDU 07003. The parameter used to represent section thickness in Figure 1 is $(t_m / c)_{\eta=0.5}$, the maximum section thickness-to-chord ratio at $\eta = 0.5$.

The factor $k_{R_{\bar{c}}}$ accounts for the effect of Reynolds number and is obtained using

$$k_{R_{\bar{c}}} = 6.62 + 0.0743 (\log_{10} R_{\bar{c}})^2 - 1.36 \log_{10} R_{\bar{c}}, \quad (3.13)$$

see Section 6.2 of ESDU 07003.

The two worked examples given in Section 6 illustrate the use of Equation (3.12). Equation (3.12) is not applicable to wing geometries having more complex twist distributions than those illustrated in Table 4.2. In such cases its use could result in poorer accuracy and it is recommended that a method similar to that outlined in Section 5.2.2 of ESDU 07003 be used to obtain $(\Delta C_{DPmin})_\epsilon$.

3.4 Wing Lift-dependent Viscous Drag Factor, K_{visc}

The wing lift-dependent viscous drag factor is obtained from

$$K_{visc} = \frac{1.3}{a_{1w}} \left(1 - \left(\frac{a_0}{a_{0T}} \right)_{\eta_{ref}} \right), \quad (3.14)$$

(see Section 5.4 of ESDU 07003) where a_{1w} is the lift-curve slope of the wing in viscous flow, which **must be obtained using ESDU 70011*** (Derivation 1), and $(a_0/a_{0T})_{\eta_{ref}}$ is the ratio of viscous to inviscid lift-curve slopes of the streamwise section, which **must be obtained using ESDU 97020** (Derivation 5), at the reference spanwise station η_{ref} introduced in Section 5.1.1 of ESDU 07003.

Figures 2 to 7, obtained from an investigation using the FP program (ESDU 02014), provide values of η_{ref} for $\lambda = 0.1, 0.2, 0.4, 0.6, 0.8$ and 1.0 , for straight-tapered planform geometries. Values of η_{ref} at intermediate taper ratios may be obtained to a very good approximation using linear interpolation.

* The theory from which ESDU 70011 is derived is limited to thin wings in inviscid flow, although good agreement has always been found between data predicted using the method of that Item and wind-tunnel data, see Appendix A of ESDU 70011. By inference, the effect of neglecting thickness and viscosity in determining lift-curve slope is small.

The Reynolds number based on the local chord at η_{ref} is calculated using

$$(R_c)_{\eta_{ref}} = R_{\bar{c}} \left(\frac{c_{\eta_{ref}}}{\bar{c}} \right), \quad (3.15)$$

where the local chord at η_{ref} for a straight-tapered wing is given by

$$c_{\eta_{ref}} = c_0 - \eta_{ref}(c_0 - c_t). \quad (3.16)$$

3.5 Wing Lift Coefficient for Minimum Viscous Drag Coefficient, C_{Lmin}

Effect of twist

An investigation into the effect of geometric twist on wing profile drag coefficient is summarised in Section 5.2.2 of ESDU 07003. The effect of geometric twist on wing lift coefficient for minimum profile drag coefficient, C_{Lmin} , is sufficiently small to be neglected.

Effect of camber

The wing lift coefficient C_{Lmin} , at which the minimum value of C_{Dvisc} occurs on a plot of C_L against C_{Dvisc} , is greatly dependent on camber. For a wing where the sections have the same camber line across the span, the method of Section 3.5.1 may be used. For a wing where the camber line varies with spanwise location, the method of Section 3.5.2 must be used.

3.5.1 Wing geometry with camber constant across the span

In Sections 7.1.1 and 7.2.1 of ESDU 07003 it is demonstrated that for twisted or untwisted wings with camber constant across the span, the lift coefficient at which the viscous drag coefficient is a minimum may be obtained using

$$C_{Lmin} = \frac{-1.21 a_{1w} (\alpha_{0v})_{\eta_{ref}}}{1 - \left(\frac{a_{0T}}{a_0} \right)_{\eta_{ref}} \frac{a_{1w}}{\pi A}}. \quad (3.17)$$

In this equation a_{1w} is the lift-curve slope of the wing in viscous flow, from ESDU 70011, and $(\alpha_{0v})_{\eta_{ref}}$ is the zero-lift angle of attack of the streamwise aerofoil geometry at the reference spanwise station η_{ref} . The local Reynolds number at the reference spanwise station, $(R_c)_{\eta_{ref}}$, is obtained using Equation (3.15). The value of $(\alpha_{0v})_{\eta_{ref}}$ is obtained at $(R_c)_{\eta_{ref}}$ with the camber-line geometry of the streamwise aerofoil using ESDU 98011 (Derivation 6). An example calculation of C_{Lmin} for a wing geometry with constant camber across the span is given in Section 6.1.5.

3.5.2 Wing geometry with camber varying across the span

In Sections 7.1.2 and 7.2.2 of ESDU 07003 it is demonstrated that for twisted or untwisted wings with camber varying across the span, the lift coefficient at which viscous drag coefficient is a minimum may be obtained using

$$C_{Lmin} = \frac{-1.21a_{1w}(\alpha_{0vw})_c}{1 - \left(\frac{a_{0T}}{a_0}\right)_{\eta_{ref}} \frac{a_{1w}}{\pi A}}. \quad (3.18)$$

In this equation a_{1w} is the lift-curve slope of the wing in viscous flow, from ESDU 70011, and $(\alpha_{0vw})_c$ is the component of wing zero-lift angle of attack due to the effect of camber.

To obtain a value for $(\alpha_{0vw})_c$ the spanwise variation of local zero-lift angle of attack, α_{0v} , for streamwise sections is required. Values of α_{0v} may be obtained using ESDU 98011 (Derivation 6) at selected spanwise stations. At each spanwise station, α_{0v} is obtained using the camber-line geometry of the streamwise aerofoil and the Reynolds number based on local chord at that station.

The Reynolds number based on local chord, $(R_c)_\eta$, at a spanwise station η may be obtained using

$$(R_c)_\eta = R_{\bar{c}} \left(\frac{c_\eta}{\bar{c}} \right), \quad (3.19)$$

where c_η is the local streamwise chord at η for a straight-tapered wing, which may be obtained using

$$c_\eta = c_0 - \eta(c_0 - c_t). \quad (3.20)$$

For an untwisted wing, $(\alpha_{0vw})_c$ is the zero-lift angle of attack of that wing and may be obtained using ESDU 87031 (Derivation 3). For a twisted wing, $(\alpha_{0vw})_c$ is the value of zero-lift angle of attack of that wing obtained using ESDU 87031 **neglecting any geometric twist**. ESDU 87031 is applicable for most spanwise variations of camber. However, a value for $(\alpha_{0vw})_c$ may also be obtained using a spanwise integration, see Appendix A. The spanwise integration may prove beneficial if the spanwise variation of camber is complex.

An example calculation of C_{Lmin} for a wing geometry with camber varying across the span is given in Section 6.2.5.

4. APPLICABILITY AND ACCURACY

Values of total drag coefficient, C_D , were predicted by adding values of $C_{D_{visc}}$ obtained by the method of this Item to values of inviscid trailing-vortex drag coefficient obtained using ESDU 10022 (Derivation 11). Sketches 4.1 to 4.5 compare parabolae fitted through predicted values of C_D with test data obtained from References 12 to 16. These references contain drag coefficient data obtained from wind-tunnel tests where the location of boundary-layer transition is known on both upper and lower wing surfaces. The availability of such data is, however, severely limited.

4.1 Applicability

This Data Item is applicable to wings with smooth surfaces where the boundary layer on both upper and lower surfaces remains fully attached. For wings operating at high lift or having parameter values outside the ranges of applicability given in Table 4.1, ESDU 95025 or 96025 (References 17 and 18) may be used instead of this Item.

For some aerofoil geometries exhibiting a small leading-edge radius, a small region of boundary-layer separation near the leading edge is possible, even at moderate angles of attack. As discussed at the beginning of Section 5 of ESDU 07003, this causes an increase in drag due to a loss of leading-edge suction. The leading-edge radius is likely to be small enough to cause a region of localised separation near the leading edge when $t_m / c < 0.10$. Boundary-layer separation near the trailing edge of an aerofoil begins to influence drag significantly at small angles of attack when $t_m / c > 0.20$. Due to the influence of both leading-edge and trailing-edge separation, the range of applicability for streamwise section thickness is considered to be $0.10 \leq t_m / c \leq 0.20$.

The method is applicable to wings where the chordwise location of boundary-layer transition is known on both upper and lower surfaces, and its position in the streamwise direction, x_{tr} / c , remains unchanged across the span. The range of applicability for R_c and x_{tr} / c is determined by the use of the aerofoil prediction methods in ESDU 97020, 98011, 00027 and 06001. The lowest value of x_{tr} / c used to obtain the database on which these Data Items are based is 0.01, however, extrapolation to $x_{tr} / c = 0$ is reasonable. Hence the method of this Item is restricted to the ranges $10^6 \leq R_c \leq 10^8$ and $0 \leq x_{tr} / c \leq 0.5$.

During the development of the method, the flow in the wing spanwise direction was assumed to be small. This is a reasonable assumption for wings having small sweep angles at low angles of attack. For wings with large sweep angles there is a noticeable component of spanwise flow even at small angles of attack. Also, for wings with $0.10 \leq t_m / c \leq 0.20$ at moderate to high angles of attack, the flow over the upper surface is complex for sweep angles greater than about 45° , and is likely to include leading-edge vortices and localised regions of separated flow. Sketches 4.2 and 4.3 indicate that the method provides a good prediction of test data for $\Lambda_{1/4} = 45^\circ$, at least for relatively simple planforms. Due to the complex nature of the flow field over highly-swept wings, 45° should be taken as the maximum quarter-chord sweep angle for which this method applies.

The effect of boundary-layer separation is to cause viscous drag to be greater than the predicted value where fully-attached boundary layers are assumed. At the larger values of C_L in Sketches 4.2 and 4.3, boundary-layer separation is indicated by the divergence of the test data from the curve through the predicted total drag coefficient, C_D , data.

For those positive values of C_L where the boundary layers are indicated as being fully attached, there is good agreement between the test data and the predicted total drag coefficient, C_D , data. Sketches 4.1 to 4.5 indicate that $C_{D_{visc}}$ may be predicted with reasonable confidence over the range $0 \leq C_L \leq 0.5$, for those wings and flow conditions applicable to this Item. In most cases lift coefficients at the cruise condition lie within this range. The value of C_L giving an approximate upper limit of applicability of the method, C_{LK} ,