



ESDU 07003

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Modelling of wing viscous drag coefficient in shock-free attached flow

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MODELLING OF WING VISCOUS DRAG COEFFICIENT IN SHOCK-FREE ATTACHED FLOW

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MODELLING OF WING VISCOUS DRAG COEFFICIENT IN SHOCK-FREE ATTACHED FLOW

1. NOTATION

		SI	British
A	wing aspect ratio, b^2/S_w		
a_0	viscous lift-curve slope of streamwise aerofoil in two-dimensional flow, see Sketch 5.4	rad^{-1}	rad^{-1}
a_{0T}	inviscid lift-curve slope of streamwise aerofoil in two-dimensional flow, see Sketches 5.2a and 5.4	rad^{-1}	rad^{-1}
a_1	local viscous lift-curve slope at spanwise station η , see Sketches 5.2a, 5.4, 5.7a and 5.10a	rad^{-1}	rad^{-1}
a_{1T}	local inviscid lift-curve slope at spanwise station η , see Sketches 5.2a, 5.4, 5.7a and 5.10a	rad^{-1}	rad^{-1}
a_{1w}	viscous lift-curve slope of wing, see Sketch 5.9a	rad^{-1}	rad^{-1}
a_{1Tw}	inviscid lift-curve slope of wing, see Sketch 5.9a	rad^{-1}	rad^{-1}
b	wing span, $2s$, see Sketch 1.1	m	ft
C_D	wing drag coefficient, based on S_w , see Sketches 3.1 and 5.1		
C_{Df0}	skin friction drag coefficient of streamwise aerofoil at zero lift, based on c , see ESDU 06001		
C_{Dmin}	minimum profile drag coefficient of streamwise aerofoil, based on c , see ESDU 00027		
C_{DP}	wing profile drag coefficient, based on S_w , see Sketch 5.1		
ΔC_{DP}	wing lift-dependent profile drag coefficient, based on S_w , see Sketches 5.1, 5.9b and 5.14		
ΔC_{DP0}	lift-dependent profile drag coefficient of wing geometry without twist (Section 5.2.2), see Equation (5.23)		
C_{DPmin}	minimum value of C_{DP} , see Sketches 3.1 and 5.1		
C_{DPmin0}	value of C_{DPmin} for wing without twist		
$(\Delta C_{DPmin})_\epsilon$	increment in C_{DPmin} due to twist (Sections 5.2.2 and 6), see Sketches 5.14, 6.1 to 6.7 and 6.9 to 6.11		
C_{df}	local skin friction drag coefficient (Section 4.3), see Sketch 4.2		

C_{dpress}	local pressure drag coefficient (Section 4.2), see Sketch 4.2		
C_{dp}	local profile drag coefficient at spanwise station η , based on c , see Sketch 4.1		
C_{dpmin}	minimum value of C_{dp} (in a plot of C_{dp} versus C_l), see Sketch 4.1		
ΔC_{dp}	local lift-dependent profile drag coefficient, based on c , see Sketches 4.1, 5.2b, 5.3, 5.7b and 5.10b		
C_{Dtc}	tip-core drag coefficient, based on S_w , (Section 5) see Sketch 5.1		
C_{Dvisc}	wing viscous drag coefficient, based on S_w , see Sketches 3.1 and 5.1		
$(C_{Dvisc})_{min}$	minimum value of C_{Dvisc} in a plot of C_{Dvisc} against C_L		
ΔC_{Dvisc}	wing lift-dependent viscous drag coefficient, based on S_w , see Sketch 3.1		
C_{DV}	wing inviscid trailing-vortex drag coefficient, based on S_w , see Sketch 3.1		
C_L	wing lift coefficient, based on S_w		
$C_{L\alpha=0}$	value of C_L at $\alpha_w = 0$		
C_{Lmin}	value of C_L corresponding to the minimum value of C_{Dvisc} , see Sketches 3.1, 5.1, 5.9a, 7.2 and 7.3		
C_l	local lift coefficient based on c , at spanwise station η , see Sketches 4.1, 5.2a, 5.7a and 5.10a		
C_{lmin}	value of C_l corresponding to $\Delta C_{dp} = 0$, see Sketches 4.1 and 5.7a		
c	local chord in streamwise direction	m	ft
c_k	value of c at crank station, η_k , see Sketch 1.3	m	ft
c_t	value of c at wing tip, see Sketches 1.2 and 1.3	m	ft
c_0	value of c at plane of symmetry (centre line) for straight-tapered wing, see Sketch 1.2	m	ft
$(c_0)_k$	value of c at plane of symmetry (centre line) for cranked wing, see Sketch 1.3	m	ft
$\bar{c}$	standard (or geometric) mean chord, S_w/b , see Sketch 1.1	m	ft
K_p	lift-dependent profile drag factor, see Equation (5.22) and Sketch 5.13		

K_{visc}	lift-dependent viscous drag factor (Section 5.4), see Sketch 5.15		
k_G	factor accounting for aspect ratio and section thickness (Section 6.2), see Sketch 6.8		
k_{R_c}	factor accounting for R_c , see Equation (6.3)		
k_c	empirical correction factor (Section 7.1.1), see Sketch 7.2		
k_v	factor applied to ΔC_{DP} to account for tip-core drag coefficient, see Equation (5.26)		
$k_{v\eta}$	combined correction factor, k_v $k_{\eta a}$ $k_{\eta b}$, see Equation (5.32) and Sketch 5.15		
$k_{\eta a}$	correction factor used in Equation (5.13), see Section 5.1.1		
$k_{\eta b}$	correction factor used in Equation (5.14), see Section 5.1.1		
k_1	vortex drag factor, see ESDU 01007		
M	freestream Mach number, see Sketch 4.2		
$P_{\varepsilon, \lambda}$	parameter accounting for effects of geometric twist and taper ratio for datum conditions of $A = 6$, $t_m/c = 0.12$, $R_c = 2.1 \times 10^6$ and $\Lambda_{1/4} = 0$ (Section 6.2), see Sketch 6.7	deg ²	deg ²
R_c	Reynolds number, based on c and freestream properties		
$R_{\bar{c}}$	Reynolds number, based on $\bar{c}$ and freestream properties		
S_w	wing planform area, see Sketch 1.1	m ²	ft ²
s	wing semispan, see Sketches 1.1, 1.2 and 1.3	m	ft
s_k	spanwise distance from centre-line to crank, see Sketch 1.3	m	ft
t_m	maximum thickness of streamwise aerofoil, see ESDU 00027	m	ft
V	freestream velocity, see Sketches 1.4, 4.2 and 5.3	m/s	ft/s
V_T	velocity of two-dimensional inviscid flow local to wing section at spanwise station η (Sections 5.1.1 and 7.1), see Sketches 5.3 and 7.1	m/s	ft/s
V_v	velocity of two-dimensional viscous flow local to wing section at spanwise station η (Section 7.1), see Sketch 7.1	m/s	ft/s
x	chordwise distance aft of leading edge, measured in streamwise direction	m	ft

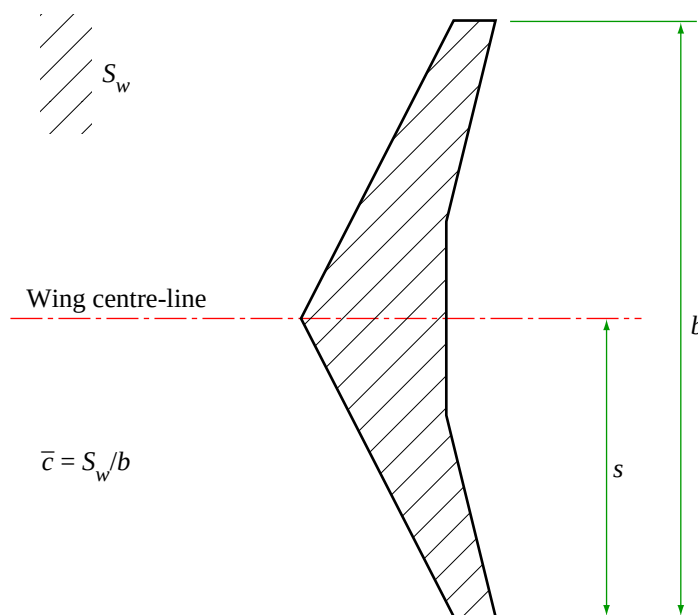
x_{tr}	mean of chordwise distances to upper and lower surface boundary-layer transition positions, measured aft of leading edge in streamwise direction	m	ft
y	spanwise distance from centre line (positive starboard), see Sketches 1.2 and 1.3	m	ft
y_{ref}	value of y corresponding to spanwise reference station, see Sketch 1.2	m	ft
z_u	upper surface ordinate of streamwise aerofoil, see ESDU 00027	m	ft
$z_{u0.9}$	value of z_u at $x = 0.9c$, see ESDU 97020	m	ft
α_{eT}	effective angle of attack of aerofoil, at spanwise station η , in two-dimensional inviscid flow (Section 5.1.1), see Sketches 5.2a, 5.3, 5.7a and 5.8	rad	rad
α_{ev}	effective angle of attack of aerofoil, at spanwise station η , in two-dimensional viscous flow (Section 7.1), see Sketch 7.1	rad	rad
α_i	inviscid component of induced incidence due to trailing vortices at spanwise station η (Section 5.1.1), see Sketches 5.2a, 5.3, 5.7a and 5.8	rad	rad
α_{iv}	induced incidence in three-dimensional viscous flow, see Sketch 7.1	rad	rad
α_{iw}	inviscid component of induced incidence of a wing due to trailing vortices (Section 7.1)	rad	rad
α_s	geometric angle of attack measured between chord of streamwise section and freestream direction in three-dimensional viscous flow (Section 5.1.1), see Sketches 5.2a, 5.3, 5.7a, 5.8 and 7.1	rad	rad
α_T	geometric angle of attack measured between chord of streamwise section and freestream direction in three-dimensional inviscid flow, see Sketches 5.2a, 5.3 and 5.7a	rad	rad
α_w	wing angle of attack measured from datum, see Sketches 1.4 and 5.9a	rad	rad
α_{wT}	value of α_w in inviscid three-dimensional flow, see Sketches 5.10a, 5.12 and 5.13	rad	rad
α_{wv}	value of α_w in viscous three-dimensional flow, see Sketch 5.10a	rad	rad
α_{w0}	value of α_w where $C_l = 0$ at spanwise station η , see Sketch 5.10a	rad	rad
α'	$\alpha_T + \Delta\alpha_{vc}$, angle of attack of streamwise section used in construction of Sketch 5.7a	rad	rad
α_{0v}	zero-lift angle of attack of streamwise aerofoil in viscous flow see Sketch 1.4 and ESDU 98011	rad	rad

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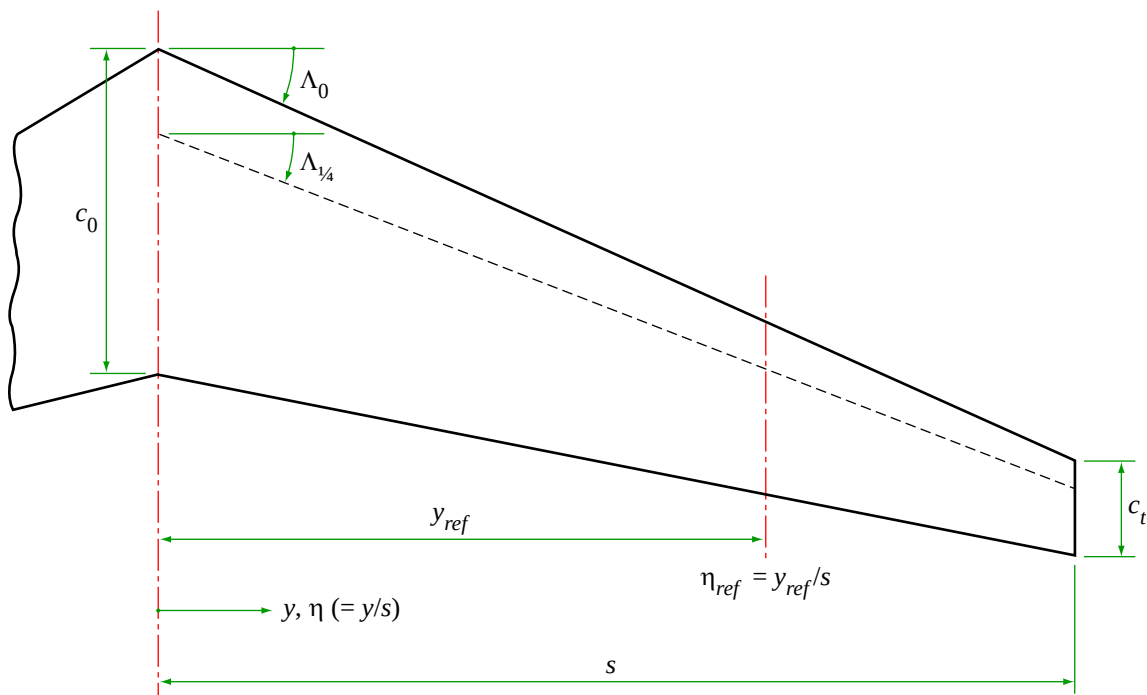
α_{0vw}	wing zero-lift angle of attack in viscous flow	rad	rad
$(\alpha_{0vw})_c$	component of α_{0vw} due to camber (Section 7.2.2)	rad	rad
$\Delta\alpha_{vc}$	increment in angle of attack of streamwise section due to camber in viscous flow (Section 5.1.2), see Sketches 5.7a and 5.8	rad	rad
$\Delta\alpha_{vl}$	increment in induced incidence, at spanwise station η , due to lift-dependent viscous effects in three-dimensional flow (Section 5.1.1), see Sketches 5.2a, 5.3, 5.7a, 5.8 and 5.10a	rad	rad
ε	local angle of twist measured from datum, positive nose up, see Sketches 1.4 and 1.5	deg	deg
$ \varepsilon $	modulus of ε , see Sketch 1.6	deg	deg
$ \varepsilon _m$	maximum value of $ \varepsilon $ across semispan, see Sketches 1.6 and 6.6	deg	deg
$ \varepsilon _{0.2}$	value of $ \varepsilon $ at $\eta = 0.2$, see Sketch 1.6	deg	deg
$ \varepsilon _{0.8}$	value of $ \varepsilon $ at $\eta = 0.8$, see Sketch 1.6	deg	deg
η	spanwise distance from centre-line as fraction of semispan, y/s , see Sketch 1.2		
η_k	value of η at crank station, s_k/s , see Sketches 1.3 and 4.4		
η_{ref}	spanwise reference station, y_{ref}/s , (Section 5.1.1), see Sketch 1.2		
$\eta_{ \varepsilon _m}$	value of η at which $ \varepsilon _m$ occurs, see Sketch 1.6		
$\bar{\eta}$	spanwise centre of pressure		
Λ_0	leading-edge sweep angle, see Sketches 1.2 and 1.3	deg	deg
$\Lambda_{1/4}$	quarter-chord sweep angle of straight-tapered wing, see Sketch 1.2	deg	deg
$(\Lambda_{1/4})_l$	local quarter-chord sweep angle at spanwise station η (Section 4.4)	deg	deg
λ	taper ratio, c_t/c_0		

Subscripts

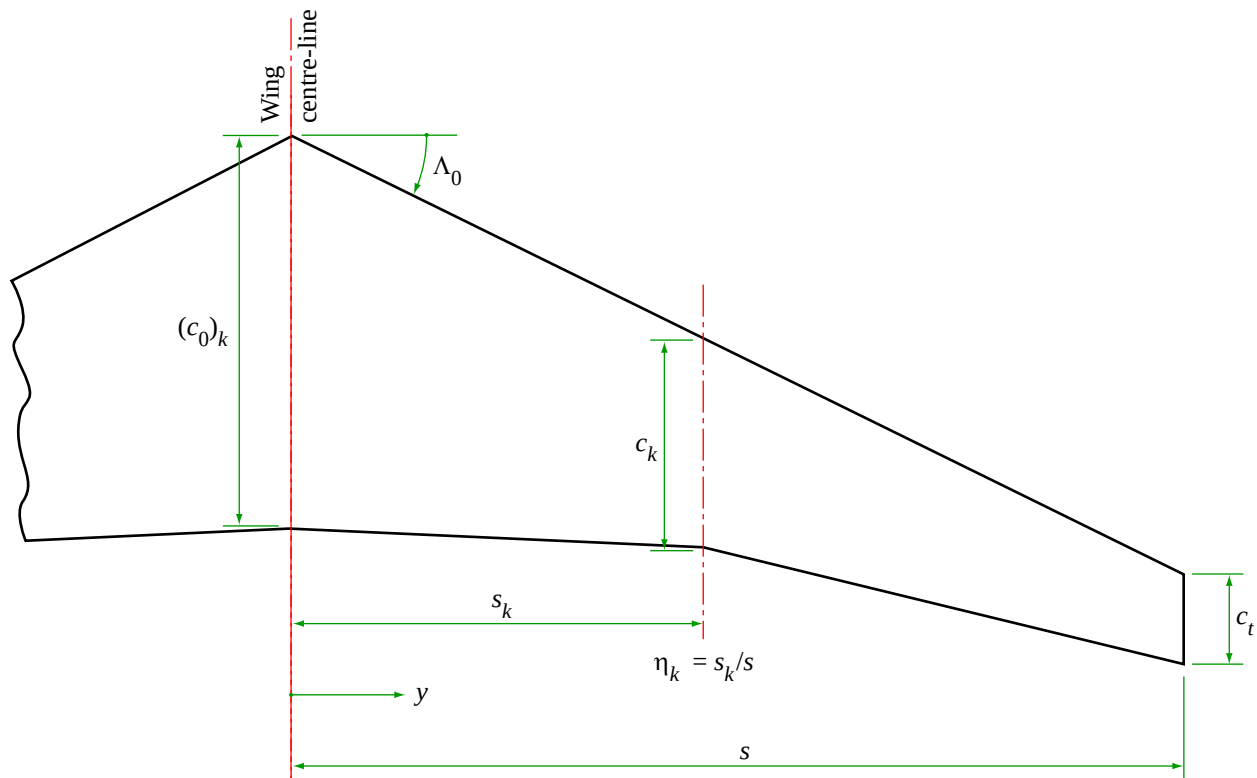
$()_\eta$	value at spanwise station η
$()_{\eta=0.5}$	value at mid-semispan ($\eta = 0.5$)
$()_{\eta_{ref}}$	value at η_{ref}



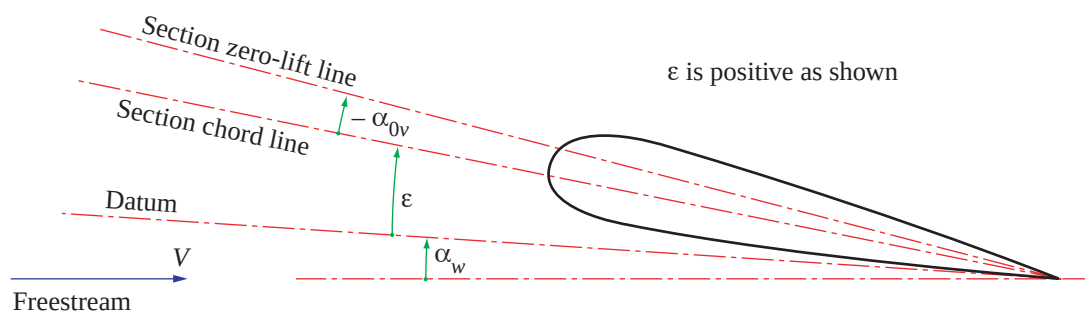
Sketch 1.1 Wing planform area



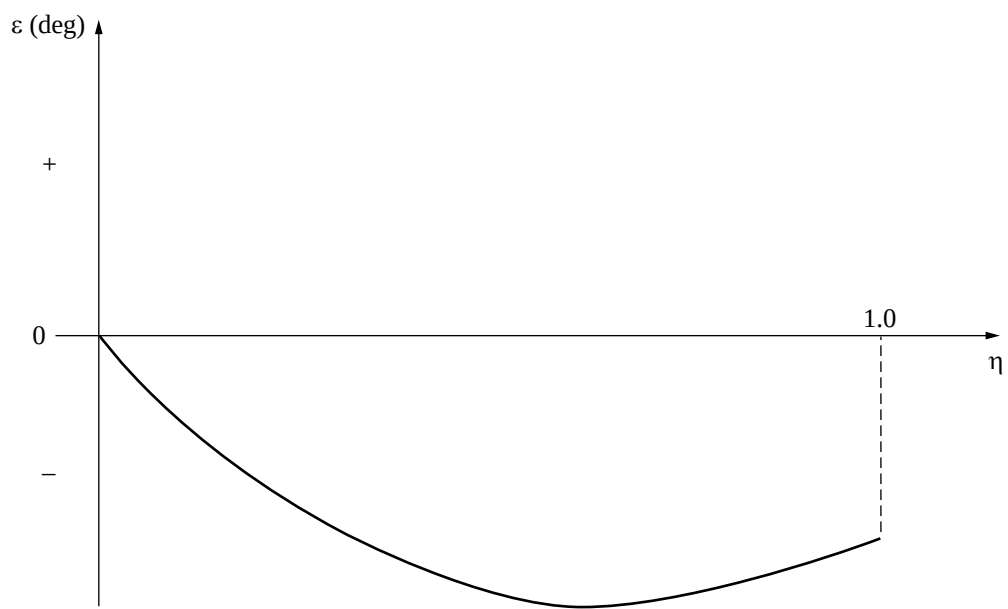
Sketch 1.2 Planform geometry of straight-tapered wing



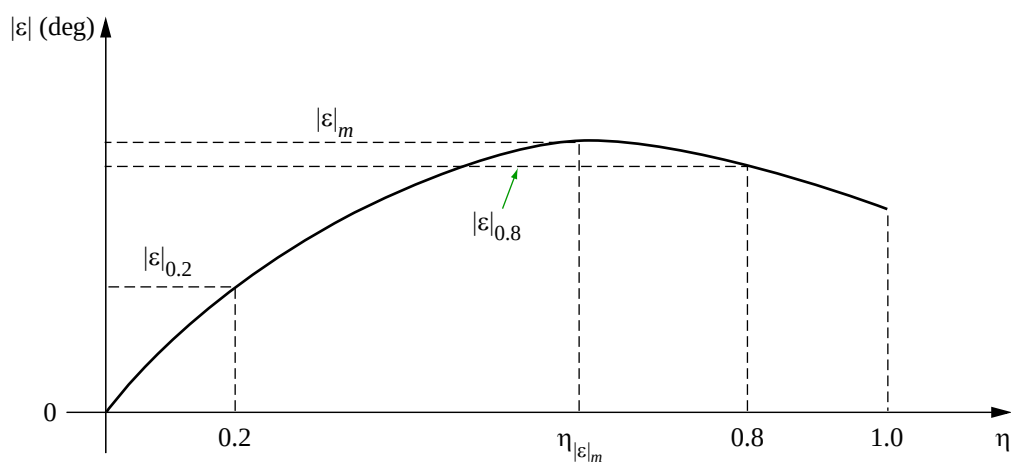
Sketch 1.3 Planform geometry of cranked wing



Sketch 1.4 Datum used to define wing angle of attack



Sketch 1.5 Twist distribution across wing semispan



Sketch 1.6 Distribution of modulus of twist, $|\epsilon|$, across wing semispan

2. INTRODUCTION

The total drag of a wing in shock-free flow can be considered as two components; one due to viscous effects and a second due to inviscid effects. The viscous drag coefficient is defined in this Data Item as that arising from all the effects of viscosity. This Item presents a semi-empirical investigation of the viscous drag coefficient of wing geometries in the absence of a body, and forms the basis for ESDU 07002 (Reference 24), which gives a method of predicting viscous drag coefficient for wings in shock-free attached flow.

Strip theory has been used throughout the investigation to obtain equations from which wing viscous drag coefficient may be predicted. These equations are applicable to those wing geometries and values of lift coefficient where both upper and lower surface boundary layers are fully attached, and there are no shockwaves present. The effects of cranks, section geometry and spanwise variations of camber and geometric twist on wing viscous drag coefficient are all investigated.

Section 3 discusses the various components of drag coefficient that make up the total drag coefficient of a wing, and an equation for viscous drag coefficient is given in terms of four parameters. Equations giving each of these parameters were obtained from work described in Sections 4 to 7. The Derivation and References relevant to that work are given in Section 8. Appendix A evaluates a number of local profile drag prediction methods that exist in the available literature. Appendix B gives a method of obtaining viscous drag coefficient from wind-tunnel data. Appendices C and D contain material supplementary to Sections 7.1 and 7.2.2, respectively.

3. COMPONENTS OF WING DRAG COEFFICIENT

In shock-free flow, total wing drag coefficient C_D may be considered to comprise two components, one of which is the viscous drag coefficient C_{Dvisc} and the other is the inviscid trailing-vortex drag coefficient C_{DV} , so that

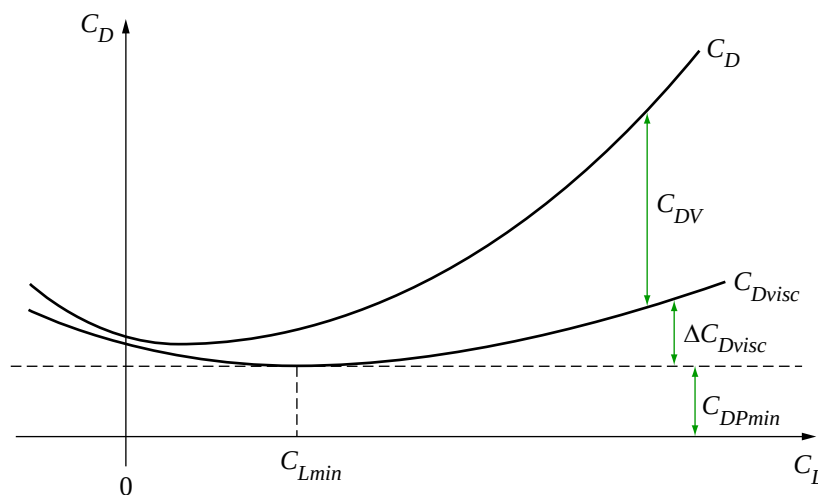
$$C_D = C_{Dvisc} + C_{DV} . \quad (3.1)$$

The inviscid trailing-vortex drag coefficient is the inviscid component of C_D and is associated primarily with the wing downwash field.

Viscous drag coefficient is defined in this Item as the drag due to viscous effects and can be considered as the sum of two components,

$$C_{Dvisc} = C_{DPmin} + \Delta C_{Dvisc} . \quad (3.2)$$

In this equation C_{DPmin} is the minimum profile drag coefficient of the wing and ΔC_{Dvisc} is the lift-dependent viscous drag coefficient of the wing. The various components of total drag coefficient are illustrated in Sketch 3.1.



Sketch 3.1 Components of total drag coefficient, C_D

The wing lift-dependent viscous drag coefficient may be obtained at a given value of lift coefficient C_L using an equation of the form

$$\Delta C_{Dvisc} = K_{visc} (C_L - C_{Lmin})^2 . \quad (3.3)$$

In this equation K_{visc} is a lift-dependent viscous drag factor and C_{Lmin} is the lift coefficient corresponding to the minimum value of C_{Dvisc} . Combining Equations (3.2) and (3.3), gives

$$C_{Dvisc} = C_{DPmin} + K_{visc}(C_L - C_{Lmin})^2. \quad (3.4)$$

From the work detailed in Section 5.2.2 it is apparent that there is a component of C_{DPmin} due to the effect of geometric twist. Minimum profile drag coefficient may therefore be obtained using

$$C_{DPmin} = C_{DPmin0} + (\Delta C_{DPmin})_\epsilon, \quad (3.5)$$

where C_{DPmin0} is the minimum profile drag coefficient of the wing without twist, and $(\Delta C_{DPmin})_\epsilon$ is an increment in wing profile drag coefficient due to geometric twist, which is assumed to be independent of lift.

Substituting Equation (3.5) into Equation (3.4), gives

$$C_{Dvisc} = C_{DPmin0} + (\Delta C_{DPmin})_\epsilon + K_{visc}(C_L - C_{Lmin})^2. \quad (3.6)$$

Equations giving C_{DPmin0} , $(\Delta C_{DPmin})_\epsilon$, K_{visc} and C_{Lmin} have been derived from investigations presented in Sections 4 to 7.

The various coefficients in Equation (3.6) are all based on wing planform area, S_w (Sketch 1.1). Sketches 1.2 and 1.3 show the planform geometries of a straight-tapered wing and a cranked wing respectively.

4. MINIMUM PROFILE DRAG COEFFICIENT OF UNTWISTED WING, C_{DPmin0}

Profile drag comprises skin friction drag and pressure drag. It is generally regarded as the drag arising due to the presence of a boundary layer on any object moving through a fluid. There is a change in profile drag with lift coefficient due to changes in surface pressure distribution and boundary-layer transition position (if transition occurs naturally). For a typical wing the variation of profile drag coefficient, C_{DP} , with lift coefficient may be modelled by a parabola. The minimum value of wing profile drag coefficient is C_{DPmin} .

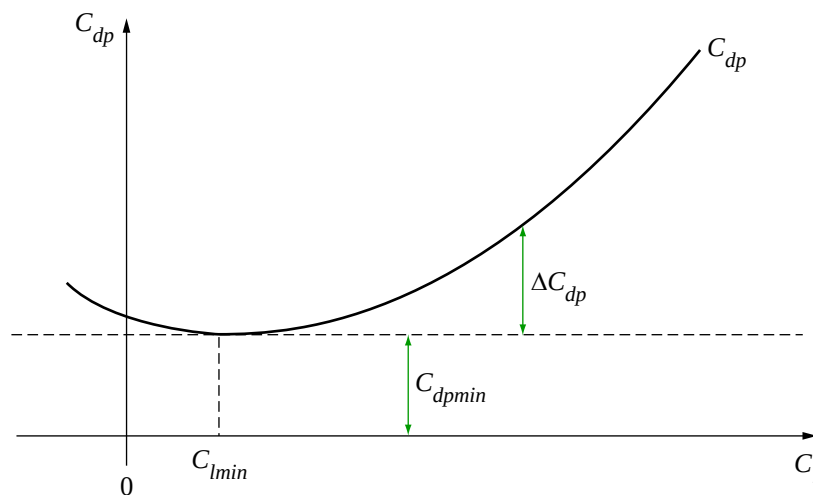
4.1 Local Profile Drag Coefficient, C_{dp}

The local profile drag is the force acting in the freestream direction at a specific spanwise station of the wing.

The variation of local profile drag coefficient C_{dp} with local lift coefficient C_l , at spanwise station η , may be modelled by a parabola as shown in Sketch 4.1. The sketch shows that C_{dp} may be considered as the sum of two components,

$$C_{dp} = C_{dpmin} + \Delta C_{dp}, \quad (4.1)$$

where C_{dpmin} is the minimum value of C_{dp} , at spanwise station η , which occurs at local lift coefficient C_{lmin} . The increment in profile drag coefficient ΔC_{dp} , at other values of local lift coefficient, is shown in Sketch 4.1.



Sketch 4.1 Variation of local profile drag coefficient with local lift coefficient at spanwise station η

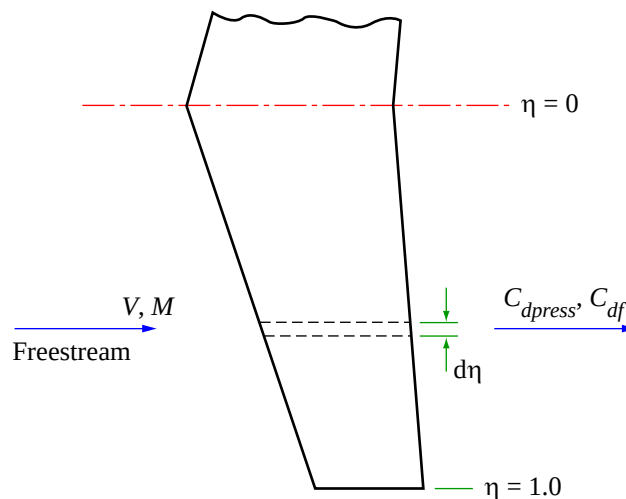
The local profile drag coefficient arises from the sum of two physical components, both of which vary with C_l , i.e.

$$C_{dp} = C_{dpress} + C_{df} \quad , \quad (4.2)$$

where C_{dpress} is the local pressure drag coefficient (see Section 4.2) and C_{df} is the local skin friction drag coefficient (see Section 4.3).

4.2 Local Pressure Drag Coefficient, C_{dpress}

The pressure drag of the surface is the summation of the normal pressure components resolved in the freestream direction. For a wing, the local pressure drag coefficient, C_{dpress} , is defined in this Item as the pressure drag coefficient acting on the surface of a strip of width $d\eta$ shown in Sketch 4.2, and is dependent on wing sweep. The direction of the freestream flow velocity, V , and freestream Mach number, M , are also shown on the sketch.



Sketch 4.2 Local drag coefficients at spanwise station η

4.3 Local Skin Friction Drag Coefficient, C_{df}

Skin friction drag is the resultant force acting in the streamwise direction due to shear stresses acting on a surface. The local skin friction drag coefficient, C_{df} , is defined in this Item as the skin friction drag coefficient acting on the surface of a strip of width $d\eta$, see Sketch 4.2. The boundary-layer flow direction, viewed from above or below the wing, is assumed to roughly approximate that of the freestream. Therefore for the same streamwise aerofoil section (profile and chord) and freestream flow conditions, local skin friction drag may be considered independent of sweep angle.

4.4 Prediction of Minimum Local Profile Drag Coefficient, C_{dpmin}

At a given spanwise location the minimum local profile drag coefficient may be predicted using aerofoil data. To account for the effect of sweep on the normal pressure drag coefficient component various prediction methods are available. Three such methods are assessed in Appendix A by comparing predicted data with test data. The closest prediction of the test data given in Appendix A was achieved using

$$C_{dpmin} = (C_{Dmin} - C_{Df0}) \cos^2(\Lambda_{1/4})_l + C_{Df0} , \quad (4.3)$$

where C_{Dmin} is the minimum profile drag coefficient of the streamwise aerofoil section in two-dimensional flow. Also, in this equation C_{Df0} is the value of C_{df} at zero lift of the streamwise aerofoil section in two-dimensional flow, and $(\Lambda_{1/4})_l$ is the angle of the local quarter-chord sweep. The first term in Equation (4.3) is the minimum value of the local pressure drag coefficient C_{dpress} . It is the pressure drag coefficient of the streamwise aerofoil section, corrected to account for the effect of sweep using the sweep factor $\cos^2(\Lambda_{1/4})_l$. For spanwise stations η where $(\Lambda_{1/4})_l = 0$, Equation (4.3) gives the minimum local profile drag coefficient as equal to the minimum profile drag coefficient of the streamwise aerofoil.

Equation (4.3) may be used to calculate C_{dpmin} at selected spanwise stations using values of C_{Dmin} and C_{Df0} obtained from ESDU 00027 and 06001 respectively (Derivations 7 and 10). The values of C_{Dmin} and C_{Df0} may be obtained for the streamwise aerofoil at a selected spanwise station using the freestream Mach number M , and Reynolds number R_c based on streamwise chord length at that station. The value of R_c may be obtained using

$$R_c = R_{\bar{c}} \left(\frac{c}{\bar{c}} \right) , \quad (4.4)$$

where c is the local chord length at the selected spanwise station, $\bar{c}$ is the standard (or geometric) mean chord (see Sketch 1.1) and $R_{\bar{c}}$ is the Reynolds number based on $\bar{c}$.

4.5 Prediction of Minimum Profile Drag Coefficient of Untwisted Wing, C_{DPmin0}

At any given spanwise station of an untwisted wing at C_{Lmin} the local drag coefficient is not greatly influenced by C_l so that $C_{dp} \approx C_{dpmin}$. The minimum profile drag coefficient of an untwisted wing is therefore given by

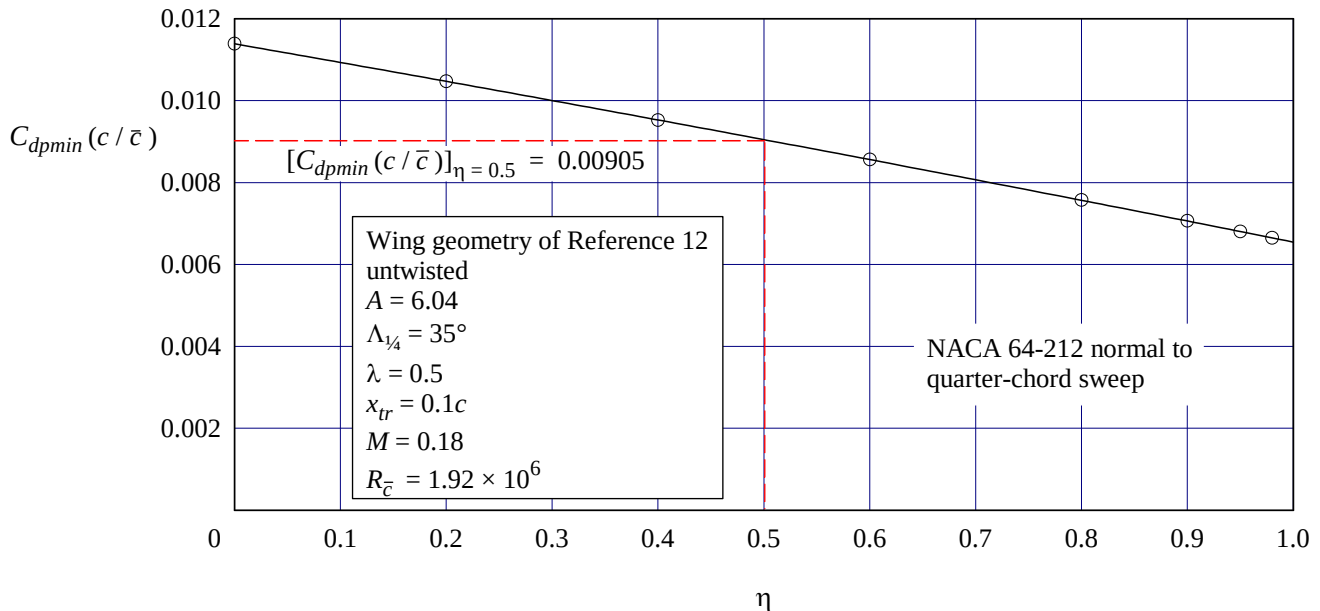
$$C_{DPmin0} = \int_0^1 C_{dpmin} \left(\frac{c}{\bar{c}} \right) d\eta , \quad (4.5)$$

where the integrand in this equation is the local minimum profile drag coefficient based on $\bar{c}$.

4.5.1 Prediction of C_{DPmin0} for simple wing geometries

For a simple wing geometry, the planform is straight tapered (see Sketch 1.2) so that the quarter-chord sweep angle $\Lambda_{1/4}$, is constant across the span and the chord length c , is constant or varies linearly across the span. The camber and chordwise position of boundary-layer transition x_{tr}/c , are constant across the span. Also the section thickness distribution is constant or varies linearly across the span.

The spanwise variation of local minimum profile drag coefficient (based on $\bar{c}$) for the untwisted wing geometry given in Reference 12 was obtained using the method outlined in Section 4.4 and is shown in Sketch 4.3. Values of aspect ratio A , taper ratio λ , M , $\Lambda_{1/4}$, $R_{\bar{c}}$ and x_{tr}/c for the chosen test conditions are given on the sketch. It is seen that, for simple wing geometries, there is a near linear spanwise variation of local minimum profile drag coefficient when based on $\bar{c}$.



Sketch 4.3 Spanwise variation of local minimum profile drag coefficient, based on $\bar{c}$

From Equation (4.5) C_{DPmin0} is equal to the area under the curve in Sketch 4.3 and for the example wing $C_{DPmin0} = 0.00902$. It is evident that C_{DPmin0} has, to a good approximation, the same value as the local minimum profile drag coefficient (based on $\bar{c}$) at the mid-semispan station, so that

$$C_{DPmin0} = \left[C_{dpmin} \left(\frac{c}{\bar{c}} \right) \right]_{\eta=0.5} \quad (4.6)$$

Because the wing is straight tapered, $\left(\frac{c}{\bar{c}} \right)_{\eta=0.5} = 1.0$, so that Equation (4.6) becomes

$$C_{DPmin0} = \left[C_{dpmin} \right]_{\eta=0.5} \quad (4.7)$$

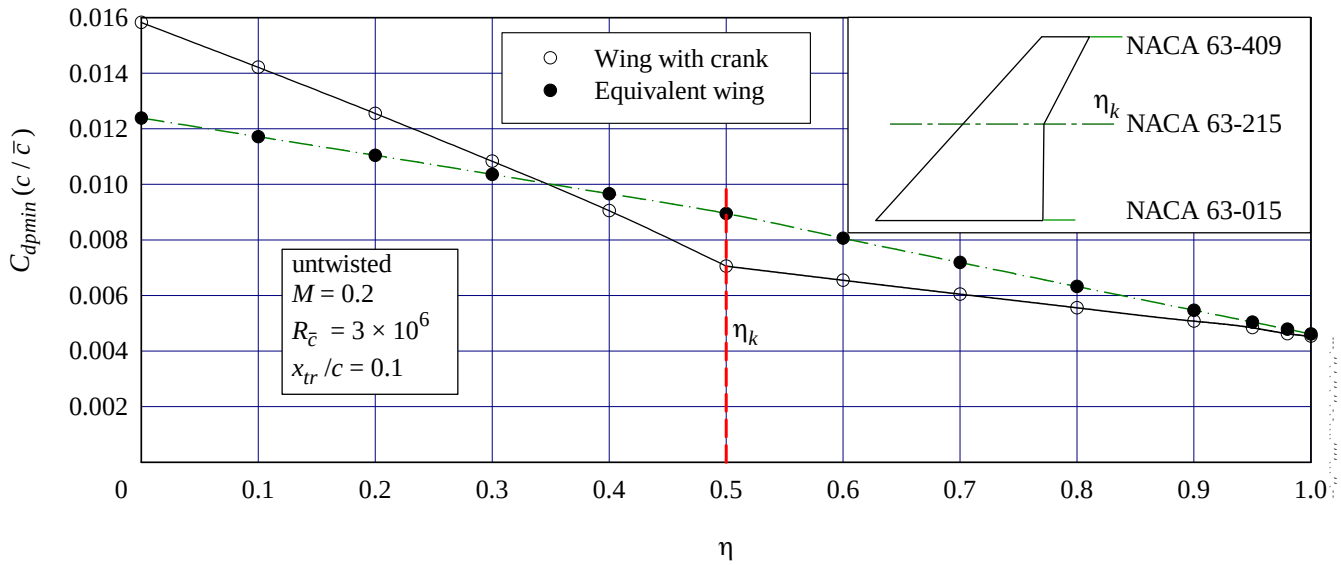
For a simple wing geometry, therefore, C_{DPmin0} is equal to the value of local minimum profile drag coefficient at the mid-semispan station. Using Equation (4.7) with the example wing geometry the value of C_{DPmin0} obtained is 0.00905, which is nearly the same value as that obtained using Equation (4.5).

4.5.2 Prediction of C_{DPmin0} for complex wing geometries

Most practical wing geometries can not be described adequately using the simple definition in Section 4.5.1. Cranks may exist and there may be a complex variation of section geometry across the span. Such wings usually have spanwise variation of camber. A cranked wing planform is illustrated in Sketch 1.3 and η_k is the spanwise location of the crank.

The spanwise variation of local minimum profile drag coefficient (based on $\bar{c}$) for an untwisted example wing geometry exhibiting a mid-semispan crank was obtained using the method outlined in Section 4.4. The example wing has different spanwise variations of section geometry inboard and outboard of the crank.

The camber varies linearly both inboard and outboard of the crank as shown by the inset planform in Sketch 4.4. The calculated values of local minimum profile drag coefficient are plotted in Sketch 4.4 and are joined by a continuous line.



Sketch 4.4 Spanwise variation of $C_{dpmin}\left(\frac{c}{\bar{c}}\right)$ for cranked and equivalent wings

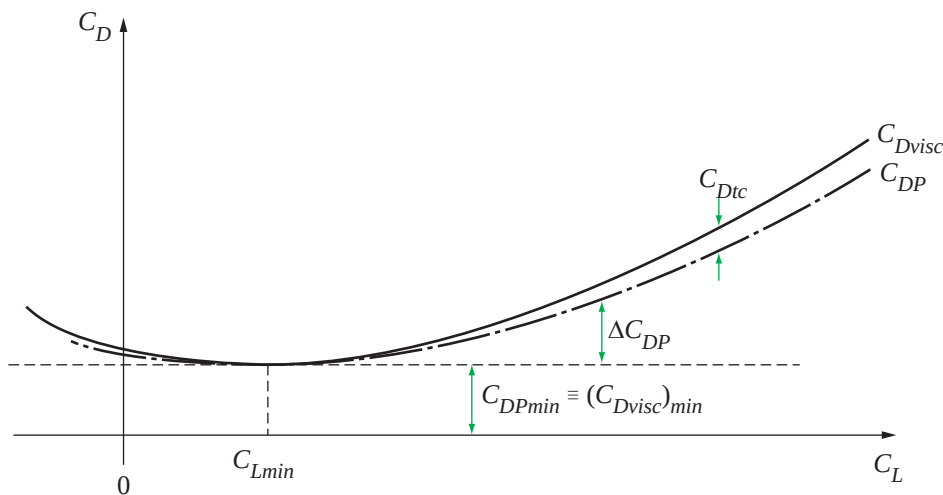
A wing planform geometry with a crank may be represented by an equivalent straight-tapered planform see ESDU 76003 (Derivation 2). The spanwise variation of local minimum profile drag coefficient (based on $\bar{c}$) for the equivalent wing representing the example geometry, and with the same spanwise variation of section geometry, is shown by the dashed line in Sketch 4.4. Note that the equivalent wing data indicate that the effect of the different variations of inboard and outboard section geometries is small compared to the effect of taper.

Integration of the cranked wing data gives $C_{DPmin0} = 0.00871$. Integration of the equivalent wing data gives the nearly identical result $C_{DPmin0} = 0.00872$, which is close to the local mid-semispan value of C_{dpmin} for that wing.

Analysis of a number of untwisted wing geometries, with cranks varying in position from $\eta = 0.2$ to $\eta = 0.7$, indicates that their integrated values of C_{DPmin0} are generally within about 0.0002 of the local mid-semispan values of C_{dpmin} for their equivalent wings. Hence, within that level of accuracy, Equation (4.7) may be used for complex wing geometries, with cranked wings represented by their equivalent straight-tapered geometries.

5. WING LIFT-DEPENDENT DRAG DUE TO VISCOUS EFFECTS

As angle of attack increases so does the thickness of the wing boundary layer. There is a greater increase of boundary-layer thickness on the upper surface than on the lower surface, and this is even more pronounced on swept wing geometries (Reference 13). The drag of a wing increases with an increase of boundary-layer thickness at the trailing edge. Since lift increases with angle of attack, there is a lift-dependent component of drag due to the boundary layer known as lift-dependent profile drag, ΔC_{DP} . Further, Reference 20 identifies a lift-dependent drag component due to diffusion and turbulent mixing in the wing-tip vortex cores referred to in this Item as tip-core drag coefficient, C_{Dtc} . The results of a wake survey across the vortex cores given in Reference 16 show that the tip-core drag component is small at low positive values of C_L but becomes larger as C_L increases. Lift-dependent viscous drag therefore comprises components due to lift-dependent profile drag and tip-core drag. Since the tip-core drag is small at low positive values of C_L , the minimum values of viscous drag coefficient, $(C_{Dvisc})_{min}$, and profile drag coefficient, C_{DPmin} , are assumed to be the same and both to occur at C_{Lmin} . The variation of C_{Dtc} and ΔC_{DP} , with C_L is shown in Sketch 5.1.



Sketch 5.1 Components of lift-dependent viscous drag coefficient, C_{Dvisc}

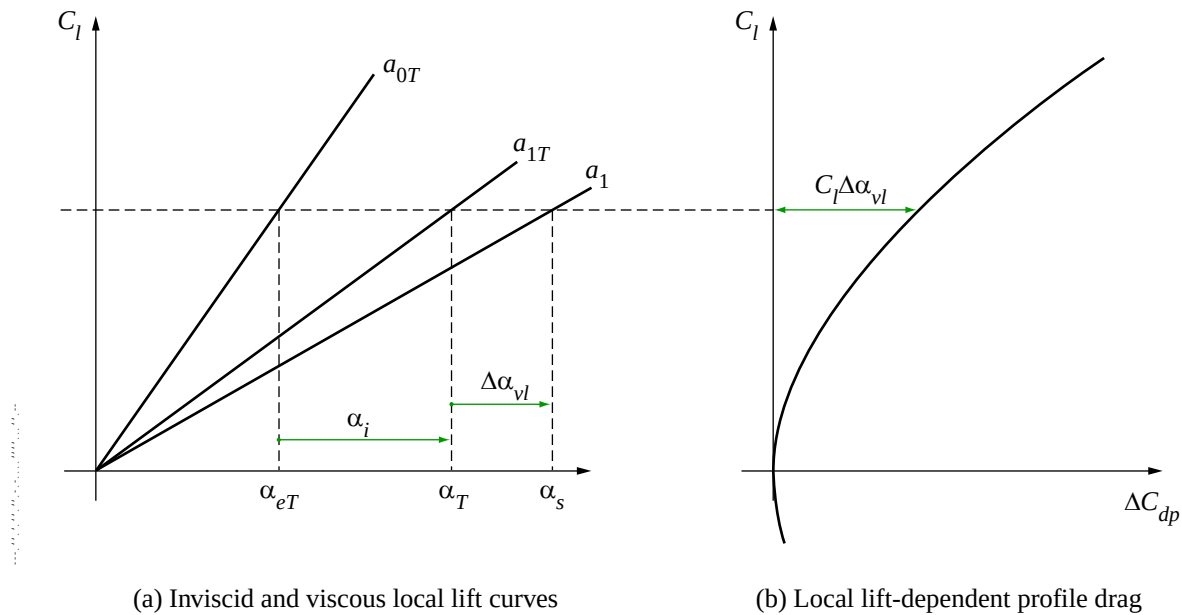
There is a region of low pressure near the leading edge on the upper surface of a wing at moderate angles of attack when the boundary layers are fully attached. There is therefore a localised thrust component near the leading edge in the direction of flight. For some section geometries exhibiting small leading-edge radius a small region of boundary layer separation is possible even at moderate angles of attack. This results in a loss of leading-edge suction and therefore the localised thrust component, see ESDU 95025 (Reference 23). The overall result is an increase in drag coefficient. For highly swept wings at moderate to high angles of attack the flow over the upper surface is complex and is likely to include leading-edge vortices and localised regions of separated flow, which have the effect of greatly increasing drag. The method developed in this section accounts for fully attached boundary layers and therefore does not account for the drag component due to loss of leading-edge suction and the increase of drag associated with leading-edge vortices.

5.1 Lift-dependent Profile Drag Coefficient of Untwisted Wing, ΔC_{DP}

The lift-dependent profile drag coefficient of a wing is greatly influenced by wing geometry. The specific case of an untwisted wing with symmetrical aerofoil sections is considered in Section 5.1.1 and the general case of an untwisted wing with cambered aerofoil sections is considered in Section 5.1.2.

5.1.1 Wing with symmetrical sections and without twist

For an untwisted wing with symmetrical sections the local lift-curves at a spanwise station η are illustrated in Sketch 5.2a, in which a_{0T} is the lift-curve slope in two-dimensional inviscid flow. The lift-curve slope of the same section in three-dimensional inviscid flow is reduced to a_{1T} . Three-dimensional viscous effects reduce the section lift-curve slope further to a_1 .



Sketch 5.2 Local properties of symmetrical section at spanwise station η

The effect of the wing downwash field as well as the influence of viscosity, causes the local flow direction in the streamwise plane normal to the wing to be different to the freestream direction. The effective angle of attack is the angle between the flow direction local to the wing section and the section chord line, whereas geometric angle of attack is the angle between the freestream direction and the section chord line. The angle between the freestream direction and the local flow direction is the so-called induced incidence.

Sketch 5.3 shows a symmetrical aerofoil section at a geometric angle of attack α_s and indicates the velocity V_T of the notional two-dimensional inviscid flow at an effective angle of attack α_{eT} . The sketch also shows α_i , the inviscid component of induced incidence due to the wing downwash field,

$\alpha_T (= \alpha_{eT} + \alpha_i)$, the geometric angle of attack appropriate to three-dimensional inviscid flow, and $\Delta\alpha_{vl} (= \alpha_s - \alpha_T)$, the increment in induced incidence due to lift-dependent viscous effects.

As shown later the local lift-dependent profile drag coefficient ΔC_{dp} is related to $\Delta\alpha_{vl}$.