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Flow in rotating components - discs, cylinders and cavities

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Internal Flow Panel. This Panel first met in 1979 and now has the following membership:

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FLOW IN ROTATING COMPONENTS – DISCS, CYLINDERS AND CAVITIES

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FLOW IN ROTATING COMPONENTS – DISCS, CYLINDERS AND CAVITIES

1. PURPOSE AND SCOPE OF THIS DATA ITEM

This Data Item introduces the subject of flow in applications where rotating machinery components induce flow rotation. There are many examples of rotating and swirling flow in engineering such as the flow between a stationary disc and a rotating disc, in a gas turbine engine or turbocharger, and the flow in an annulus with a rotating inner cylinder, between the armature and stator of an electric motor.

The aim of this Data Item is to introduce the phenomena involved in rotating flows and to provide the reader with guidance and techniques for modelling specific rotating flow applications. The flow applications considered in this Data Item relate to disc and cylindrical geometries, *i.e.* rotating discs, cylinders and cavities, predominantly in steady-state, stable flow conditions. The correlations provided are based on analytical, experimental and computational fluid dynamics (CFD) modelling of the flow and can be used in parametric design studies and optimisation.

An Introduction to this Data Item is given in Section 3.

2. NOTATION AND UNITS

		<i>SI Unit</i>
A	coefficient, used in Equation (4.112), defined in Equation (4.113) or cross sectional area	– m^2
A_{gap}	annular rim-seal gap area	m^2
a	disc inner radius	m
B	coefficient used in Equation (4.112), defined in Equation (4.114)	–
b	disc outer radius	m
C	empirical constant in Equation (4.123) or constant of integration in Equation (5.18)	– –
C_D	drag coefficient, in Equation (4.133)	–
C_d	discharge coefficient, in Section 4.6.6.3	–
C_m	moment coefficient, Equations (4.19), (4.20), (4.28), (4.31), (4.32), (4.37), (4.39), (4.47) to (4.51), (4.63), (4.104), (4.134), (4.136) and (4.137)	–
C_{mc}	moment coefficient for a cylinder, Equations (5.4) to (5.8), (5.9), (5.22), (5.24) to (5.26)	–
$C_{m,form}$	form drag moment coefficient, defined in Equation (4.133)	–
C_n	parameter in Equation (4.42)	–

C_p	pressure coefficient, defined in Equation (4.106) or non-dimensional cavity pressure-ratio, defined in Equation (4.128)	—
$C_{p \max}$	pressure coefficient at maximum circumferential pressure difference, in Equation (4.124)	—
C_w	non-dimensional flow rate, Equations (4.16), (4.17), (4.27), (4.30), (4.87), (4.93), (4.101), (4.103), (4.115) and (4.121)	—
$C_{w_{free \text{ disc}}}$	non-dimensional flow rate entrained by a free disc, defined in Equation (4.17)	—
$C_{w_{local}}$	local non-dimensional flow rate, in Equation (4.137)	—
$C_{w_{min}}$	minimum non-dimensional flow rate to prevent inflow, Equations (4.116), (4.117), (4.118), (4.120), (4.122)	—
C_1	constant used in Equation (5.14), defined in Equation (5.15)	—
C_2	constant used in Equation (5.14), defined in Equation (5.16)	—
c	swirl ratio, Section 6.3	—
c_{eff}	effective swirl ratio, Section 6.3	—
c_p	specific heat capacity at constant pressure	J/kg K
c_v	specific heat capacity at constant volume	J/kg K
d	diameter	m
d_{bolt}	exposed bolt head diameter, in Equations (4.133), (4.136), (4.137)	m
d_h	hydraulic diameter, defined in Equations (6.5) and (6.6), used in Equation (6.4)	m
e	surface roughness, in Equation (5.9)	m
F	force or thrust, Equation (4.150)	N
F_g	geometrical factor, defined in Equation (5.37), used in Equation (5.36)	—
F_r	body force per unit volume of fluid in radial direction, Equation (A.2)	N/m ³
F_z	body force per unit volume of fluid in axial direction, Equation (A.4)	N/m ³
F_Φ	body force per unit volume of fluid in circumferential direction, Equation (A.3)	N/m ³
G	gap ratio, defined in Equation (4.45)	—
G_c	shroud clearance ratio, defined in Equation (4.105)	—
Gr_b	Grashof number, defined in Equation (6.2)	—

G_1	coefficient in Equations (4.39) and (4.41)	—
G_2	coefficient in Equations (4.39) and (4.41)	—
G_3	coefficient in Equation (4.40)	—
G_4	coefficient in Equation (4.40)	—
G_5	coefficient in Equation (4.40)	—
H	height of bolt head	m
K	flow coefficient in Equation (4.119) or constant in Equation (4.123)	—
K_f	form drag correction factor due to wake interference, in Equation (4.133)	—
K_n	parameter, used in Equation (4.41), defined in Equation (4.42)	—
K_w	stress flow rate parameter, defined in Equation (4.66)	—
k'_s	sand grain size, in Equations (4.37) to (4.39)	m
k_s	sand grain roughness, in Equation (4.37)	m
k_1	constant in Equation (4.108)	—
k_2	constant in Equation (4.110)	—
k_3	constant in Equation (4.142)	—
k_4	constant in Equation (4.144)	—
k_5	constant in Equation (4.146)	N/m ⁴
L	cylinder length	m
m	exponent in Equation (4.120)	—
$\dot{m}$	mass flow rate	kg/s
$\dot{m}_{in}$	inflow mass flow rate in a rotor-stator system, defined in Equation (4.132)	kg/s
$\dot{m}_o$	mass flow rate entrained in rotating disc boundary layer, Equation (4.14) or mass flow rate in the rotor boundary layer, in Equation (4.54)	kg/s
$\dot{m}_{out}$	outflow mass flow rate in a rotor-stator system, defined in Equation (4.131)	kg/s
$\dot{m}_s$	mass flow rate in the stator boundary layer, used in Equation (4.54)	kg/s
$\dot{m}_{superposed}$	superposed mass flow rate, defined in Equation (4.54)	kg/s

N	number of blades (integer) or number of bolts or protrusions (integer)	—
n	exponent in Equation (4.120) or power-law velocity profile exponent in Section 4.5	—
$Power$	power required to overcome frictional drag, Equations (4.135), (5.3) and (5.5)	W
P_{max}	non-dimensional maximum annulus static pressure, used in Equation (4.122), defined in Equation (4.124)	—
p	static pressure or pitch of bolts or protrusions, used in Equations (4.136) and (4.137)	Pa m
p_a	external static pressure, in Equation (4.106)	Pa
p_{cav}	rotor stator cavity static pressure, in Equation (4.125)	Pa
p_{max}	maximum annulus static pressure, in Equations (4.126), (4.128) to (4.132)	Pa
p_{min}	minimum annulus static pressure, in Equations (4.126), (4.128) to (4.132)	Pa
p^*	non-dimensional static pressure, defined in Equation (4.8)	—
R	specific gas constant	J/kg K
R_a	centre-line-average roughness, in Equation (4.37)	m
Re_{d_h}	annulus Reynolds number based on hydraulic diameter, in Section 5.4	—
Re_r	radial Reynolds number, in Equation (6.1)	—
Re_ϕ	rotational Reynolds number, defined in Equation (4.2)	—
Re'_ϕ	modified rotational Reynolds number, defined in Equation (4.76)	—
$Re_{\phi m}$	rotational Reynolds number based on annulus gap, defined in Equation (5.23)	—
$Re_{\phi, critical}$	critical rotational Reynolds number, defined in Equation (4.21)	—
$Re_{\phi, local}$	local rotational Reynolds number, defined in Equation (4.1)	—
Re_z	axial Reynolds number, defined in Equation (6.4)	—
Ro	Rossby number, defined in Equation (6.7)	—
r	radial coordinate	m
r_i	inner radius, in Equations (4.134) and (4.135)	m

r_m	mean radius, in Equations (5.10), (5.35) to (5.38)	m
r_o	outer radius, in Equations (4.134) and (4.135)	m
r_s	shaft radius, in Equations (6.5) and (6.8)	m
S	speed to radius ratio factor, Equation (5.31)	–
s	gap between rotor and stator	m
s_c	clearance between shroud and rotor	m
T	temperature	K
Ta	Taylor number	–
Ta_m	Taylor number based on the mean annulus radius, defined in Equation (5.10)	–
$Ta_{m,cr}$	critical Taylor number based on the mean annulus radius, in Equation (5.34)	–
T_c	temperature of cold surface, Section 6.1	K
T_h	temperature of hot surface, Section 6.1	K
T_q	moment or torque or torque per unit length, in Equations (5.19) and (5.20)	N m N
u	velocity, relative radial velocity (to a rotating frame of reference), Equation (4.70)	m/s
u_{dm}	mixed-out radial velocity, in Equation (4.119)	m/s
u_r	velocity component in the radial direction	m/s
u_r^*	non-dimensional radial velocity component, defined in Equation (4.5)	–
u_z	velocity component in the axial direction	m/s
u_z^*	non-dimensional axial velocity component, defined in Equation (4.7)	–
$u_{z,\infty}$	free stream axial velocity, in Equations (4.38), (5.1) and (5.2)	m/s
u_ϕ	velocity component in the tangential direction	m/s
$u_{\phi,c}$	tangential velocity of the core, Section 6.3	m/s
u_ϕ^*	non-dimensional tangential velocity component, defined in Equation (4.6)	–
u_τ	friction velocity, defined as $u_\tau = \sqrt{\tau_s/\rho}$, used in Equation (4.36)	m/s
$\bar{u}_z$	average velocity component in the z direction, Section 4.6.6.1 or average velocity of axial throughflow, Equation (6.4)	m/s m/s

v	relative tangential velocity (to a rotating frame of reference), Equation (4.71)	m/s
v_c	inviscid rotating core velocity, Equation (4.69)	m/s
x	distance along x axis or ratio of local to disc outer radius, defined in Equation (4.3) or ratio of inner to outer cylinder radius, in Equation (5.31)	m — —
y	distance along y axis or thickness of the viscous sub-layer, defined in Equation (4.35)	m m
y^+	universal coordinate, defined in Equation (4.34)	—
z	distance along z axis	m
z^*	non-dimensional axial location, defined in Equation (4.4)	—
α	coefficient defined in Equation (4.59) or ratio of radial to tangential component of stress, defined in Equations (4.40) and (4.43)	— —
α_o	coefficient for the rotor, in Equation (4.83)	—
α_s	coefficient for the stator, in Equation (4.84)	—
β	swirl ratio, defined in Equation (4.44), used in Equation (4.46)	—
β^*	swirl ratio when the superposed flow rate is zero, in Equations (4.95), (4.96) and (4.104)	—
ΔC_p	non-dimensional static pressure drop across the shroud, Equations (4.111) and (4.112)	—
Δp	static-pressure drop, used in Equation (4.125), defined in Equation (4.126)	Pa
Δp_{max}	maximum circumferential static pressure difference across the shroud, in Equation (4.124)	Pa
Δp_{shroud}	circumferentially-averaged pressure drop across the shroud, in Equation (4.121)	Pa
ΔT	temperature difference, in Equation (6.2)	K
δ	nominal boundary layer thickness	m
δ_{99}	99% velocity boundary layer thickness, in Equations (4.13) and (4.26)	m
ε_m	coefficient in Equations (4.61) and (4.62)	—
ε_M	coefficient in Equations (4.63) and (4.64)	—
ε'_m	coefficient in Equations (4.67) and (4.68)	—

Φ	non-dimensional angular position, defined in Equation (4.127), used in Equations (4.129) and (4.130)	–
ϕ	tangential coordinate	rad
γ	isentropic index or coefficient, used in Equation (4.39) and Section 4.6.3, defined in Equation (4.41)	–
γ_o	coefficient for the rotor, in Equations (4.83), (4.85), (4.88) and (4.89)	–
γ_s	coefficient for the stator, in Equations (4.84), (4.86) and (4.88)	–
λ	wavelength, defined in Equation (5.39)	m
λ_m	rotation parameter, defined in Equation (5.1)	–
λ_t	turbulent flow parameter, defined in Equation (4.91)	–
μ	dynamic viscosity	Pa s
ν	kinematic viscosity	m ² /s
ρ	density	kg/m ³
σ	ratio of mean axial and tangential components of flow, defined in Equation (4.38), used in Equations (4.39) to (4.41)	–
τ_o	shear stress at the rotor, in Section 5.1.2	N/m ²
τ_s	surface shear stress, in Section 4.4.1	N/m ²
$\tau_{\phi,o}$	tangential shear stress at the rotor, in Equation (4.65)	N/m ²
Ω	angular velocity	rad/s
Ω_a	angular velocity of inner cylinder, in Equations (5.15), (5.16), (5.19), (5.20), (5.28) and (5.31)	rad/s
Ω_b	angular velocity of outer cylinder, in Equations (5.15), (5.16), (5.19) to (5.21), (5.28) to (5.31)	rad/s
Ω_{cr}	critical angular velocity, in Equations (5.30), (5.32), (5.35) and (5.36)	rad/s
ω	angular velocity	rad/s

3. INTRODUCTION

An interest in the fluid flow associated with machinery may stem from the need to know the mechanical losses and flow structure in an electric motor, where the armature is spinning in air, or from use of fluid as a means to transfer energy as in, for example, a centrifugal compressor or rotary pump.

In this Data Item, an insight into the flow phenomena involved in the following rotating flow components is given.

- Rotating discs in Section 4 for the configurations in Table 3.1.
- Rotating cylinders in Section 5 the configurations in Table 3.2.
- Rotating cavities in Section 6 the configurations in Table 3.3.

Tables 3.1 to 3.3 provide an overview of the principal geometric configurations and relationships covered in this Data Item. These Tables can be used to enable the user to rapidly identify the principal relationships relevant to a particular geometric configuration. Further details of the development and limitations of validity for the relationships presented are given in the relevant sections.

Emphasis within this Data Item has been placed on presenting correlations based on physical modelling and experimental data which can be used in parametric design studies and optimisation.

For users seeking additional information, the subject of rotating flow has been addressed in a number of monographs and text books. The text by Dorfman²⁹ has served as a standard reference for engineering researchers interested in rotating flow. This has been complemented by the two monographs by Owen and Rogers^{76,99} and the textbook by Vanyo⁹³. A mathematical treatment on rotating flows is given by Greenspan³⁶. Valuable sections on rotating flows are given in the books by Batchelor³⁵ and Tritton⁷¹.

TABLE 3.1 Summary of the Principal Disc Configurations Covered in this Data Item

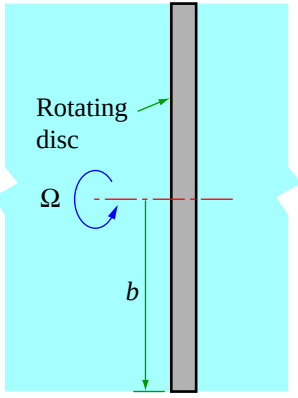
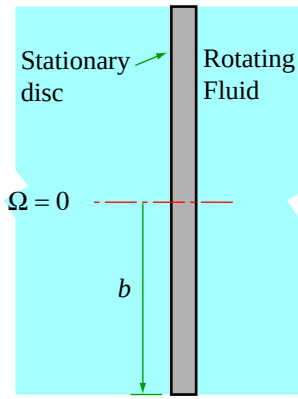
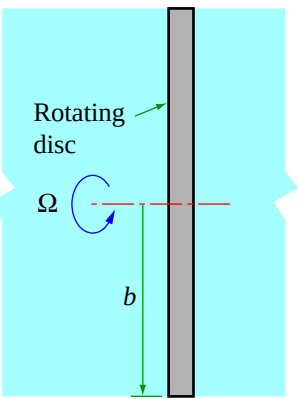
GEOMETRIC CONFIGURATION	FLOW PARAMETERS	RANGE
Free disc (Section 4.2)  (a)	Boundary layer thickness, Equation (4.13)	Laminar flow
	Non-dimensional mass flow rate entrained by a rotating disc, Equation (4.17)	Laminar flow
	Moment coefficient, Equation (4.20)	Laminar flow
	Critical Reynolds number, Equation (4.21)	Transitional flow
Rotating fluid above a stationary disc (Section 4.3)  (b)	Boundary layer thickness, Equation (4.26)	Laminar flow
	Entrained non-dimensional mass flow rate, Equation (4.27)	Laminar flow
Turbulent flow over a rotating disc (Section 4.4)  (c)	Boundary layer thickness, Equation (4.29)	Turbulent flow
	Entrained non-dimensional mass flow rate, Equation (4.30)	Turbulent flow
	Moment coefficient, Equations (4.28), (4.31), (4.32)	Turbulent flow $5 \times 10^5 < Re_\phi < 2 \times 10^6$ $10^4 < Re_\phi < 10^9$ $Re_\phi < 4.2 \times 10^6$

TABLE 3.1 Summary of the Principal Disc Configurations Covered in this Data Item (continued)

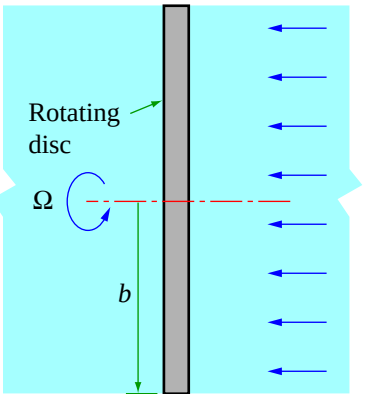
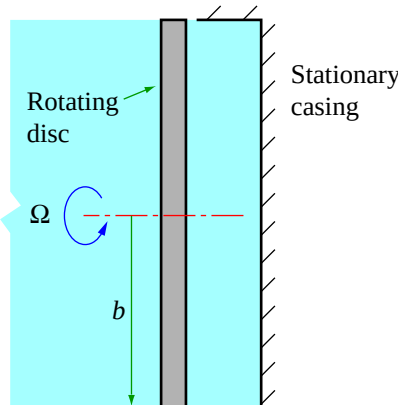
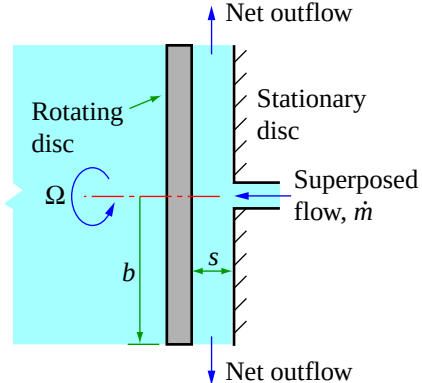
GEOMETRIC CONFIGURATION	FLOW PARAMETERS	RANGE
Turbulent flow impinging on a rotating disc (Section 4.5)  (d)	Moment coefficient, Equation (4.39)	Turbulent flow
Enclosed rotor-stator wheelspace with no superposed flow (Section 4.6.2)  (e)	Moment coefficient, Equation (4.47)	Laminar flow with no superposed flow and merged boundary layers
	Moment coefficient, Equation (4.48)	Laminar flow with no superposed flow and separate boundary layers
	Moment coefficient, Equation (4.49)	Turbulent flow with no superposed flow and merged boundary layers
	Moment coefficient, Equation (4.50)	Turbulent flow with no superposed flow and separate boundary layers
	Thrust, Equation (4.150)	Turbulent flow
Rotor-stator wheelspace with a superposed axial flow (Section 4.6.3)  (f)	Moment coefficient, Equation (4.51)	$0 < Re_{\phi} \leq 4 \times 10^6$ $14000 \leq Cw \leq 98000$ $0.01 \leq G \leq 0.18$ $a/b = 0.1333$
	Core tangential velocity, Equation (4.96)	Turbulent flow

TABLE 3.1 Summary of the Principal Disc Configurations Covered in this Data Item (continued)

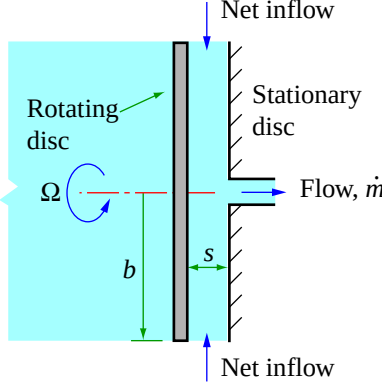
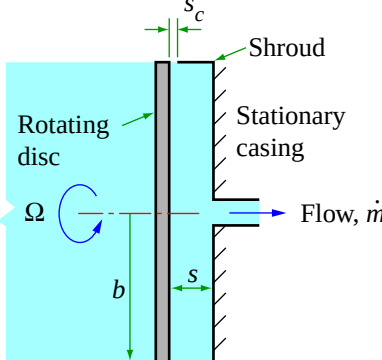
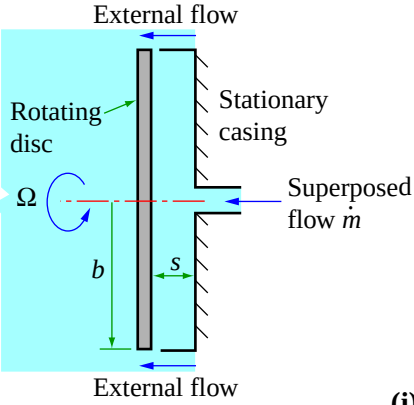
GEOMETRIC CONFIGURATION	FLOW PARAMETERS	RANGE
Rotor-stator wheelspace with radial inflow (Section 4.6.4)  (g)	Non-dimensional mass flow rate to prevent radial outflow, Equation (4.103) Moment coefficient, Equation (4.104)	$0.004 \leq G \leq 0.6$ $0 \leq C_w \leq 10^4$ $0 \leq Re_\phi \leq 4 \times 10^6$ $2 \times 10^6 < Re_\phi < 10^7$ $0 < \lambda_t < 0.06$
Rotor-stator wheelspace with stationary shroud and radial outflow (Section 4.6.5)  (h)	Minimum non-dimensional superposed flow rate to prevent ingestion, Equations (4.116), (4.117)	$G_c \ll G$ $G_c = 0.0033, 0.0067$ $G = 0.06, 0.12, 0.18$ $4 \times 10^5 \leq Re_\phi \leq 4.2 \times 10^6$
Rotor-stator wheelspace with external flow (Section 4.6.6)  (i)	Outflow mass flow rate, Equation (4.131) Inflow mass flow rate, Equation (4.132)	Turbulent flow Turbulent flow

TABLE 3.1 Summary of the Principal Disc Configurations Covered in this Data Item (continued)

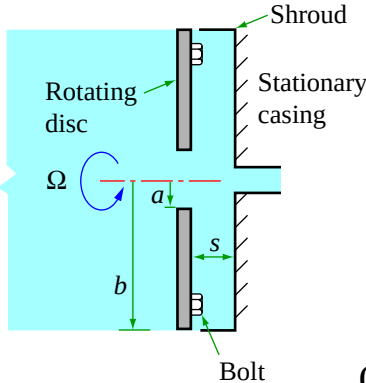
GEOMETRIC CONFIGURATION	FLOW PARAMETERS	RANGE
Rotating Disc with protrusions (Section 4.6.7)  (j)	Moment coefficient, Equation (4.136)	$1.5 < \frac{r}{a} < 2.25$ $0.001 < \frac{Cw_{local}}{Re_{\phi, local}} < 0.1$ $3 < \frac{p}{d_{bolt}} < 20$

TABLE 3.2 Summary of the Principal Cylinder Configurations Covered in this Data Item

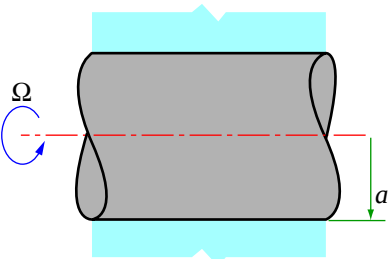
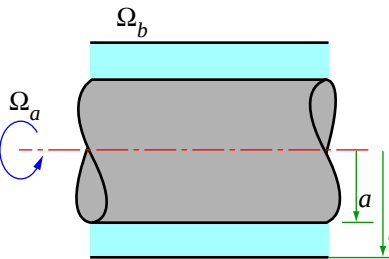
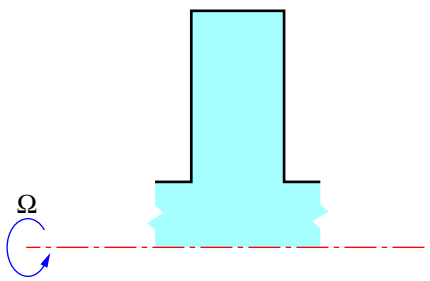
GEOMETRIC CONFIGURATION	FLOW PARAMETERS	RANGE
Rotating cylinder (Section 5.1)  (a)	Moment coefficient, Equation (5.6)	Laminar flow
	Moment coefficient, Equation (5.8)	Turbulent flow
Rotating annulus (Sections 5.2 and 5.3)  (b)	Taylor number, Equation (5.10)	Laminar to turbulent flow
	Torque per unit length for the inner cylinder, Equation (5.19)	Laminar flow
	Torque per unit length for the outer cylinder, Equation (5.20)	Laminar flow
	Critical angular velocity, Equation (5.36)	Transitional flow
	Moment coefficient, Equation (5.24)	Transitional flow $\frac{b-a}{a} > 0.07$ $64 < Re_{\phi m} < 500$
	Moment coefficient, Equation (5.25), Equation (5.26)	Turbulent flow, $Re_{\phi m} \leq 1 \times 10^4$ $Re_{\phi m} > 1 \times 10^4$

TABLE 3.3 Summary of the Principal Cavity Configurations Covered in this Data Item

GEOMETRIC CONFIGURATION	FLOW PARAMETERS	RANGE
Rotating cavity (Section 6) 	Grashof number, Equation (6.2)	Laminar to turbulent
	Rotational Reynolds number, Equation (6.3)	Laminar to turbulent
	Axial Reynolds number, Equation (6.4)	Laminar to turbulent
	Rossby number, Equation (6.7)	Laminar to turbulent

3.1 Flow Modelling Methods

There are three principal methods for modelling flow:

- analytical,
- experimental,
- computational fluid dynamics.

Use of analytical and empirical relationships, based on flow physics and experimental data, can provide insight into the physical phenomena involved and a means by which design parameters can be explored. Studies using these methods have resulted in a significant number of correlations that can readily be used for estimating important parameters in rotating flow applications such as the flow regime, mass flow rate and moment coefficients. The presentation of many of these correlations forms the principal focus of this Data Item.

Since the 1970s, with the advent of computational fluid dynamics (CFD), many fluid flow applications can be modelled. Use of CFD, as with any analytical or computational modelling method, requires appropriate attention to the model physics and boundary conditions. When modelling a flow problem, a trade-off exists between the accuracy of the CFD model to the true conditions and the length of time required to obtain solutions. The computational cost of a CFD model is highly dependent on how many dimensions the flow is modelled in, whether it is modelled as steady or unsteady, and on the physical model used for turbulent flow. Turbulence modelling options include the one- and two-equations turbulence models, direct numerical simulation and large eddy simulation models. Different standard or near-wall functions for the viscous layer can be used. Further considerations include whether it is necessary to solve the energy equation or to model fluid-solid interactions to obtain structural or thermal analysis information.

If the rotating flow concerned is unsteady with three-dimensional features, then careful consideration is required to determine whether it is necessary to model the flow as unsteady in three dimensions. If the unsteady phenomena are not significant in determining the flow parameter under investigation, it may be possible to obtain acceptable results with CFD steady-state calculations.

For any obtained solution, it is required to assess the validity of the CFD predictions. In this assessment, prior knowledge of the fluid dynamics of the problem under investigation, its modelling methods and their limitations is useful. However, appropriate validation of CFD predictions against experimental data and theoretical solutions is always required. ESDU CFD Guides provide guidance on Best Practice and Benchmarks for modelling pressure loss and flow characteristics in internal flow applications.

Prior to the advent of CFD, many of the advances in understanding rotating flows were achieved by a combination of theoretical, numerical and experimental approaches. Many of the theoretical approaches have now been augmented by CFD, often negating their detailed application. Nevertheless, the basic relationships derived from these studies for design parameters such as mass flow rates, boundary layer thickness, swirl speed, pressure distribution and moments, provide useful correlations that enable the rapid exploration of the design envelope and validation of predictions obtained using CFD.

4. ROTATING DISCS

4.1 Disc Configurations and Modelling

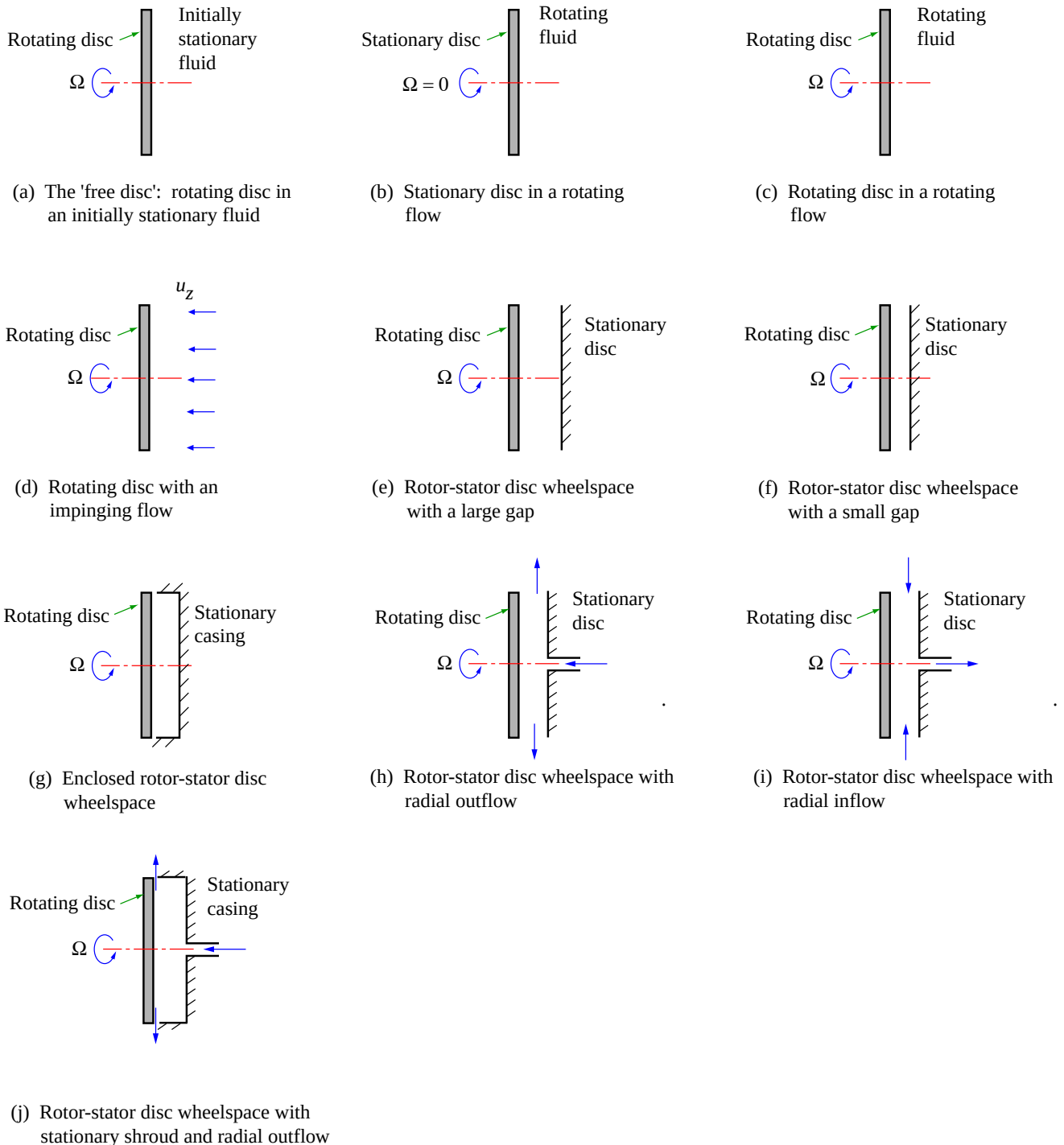
The flow associated with discs is important in a number of applications, ranging from disc drives* used for digital data storage in computers and electronic products to automotive disc brakes, flywheels, gear wheels and discs to support turbomachinery blades. Shear stresses between the disc and the fluid in which the disc is rotating dictate the power required to drive the disc to overcome frictional drag. The local flow field influences the heat transfer. Due to the combined effect of a number of factors, a universal analysis is not possible and the flow conditions and proximity of solid surfaces need to be taken into account.

A large range of geometric configurations is possible in disc flows, some of which are illustrated schematically in Sketch 4.1. In considering a simple disc, the flow characteristics are considerably different depending on whether it is rotating in an initially static fluid (Sketch 4.1(a)) or whether the disc is stationary and the surrounding fluid is rotating (Sketch 4.1(b)). A common configuration in rotating machinery is the rotor-stator wheelspace formed between a rotating disc and a stationary disc, *e.g.* Sketches 4.1(e) and (f). Wheelspace flow characteristics vary with the proximity of the stator and rotor (Sketches 4.1(e) and (f)), the presence of a stationary or rotating shroud (Sketches 4.1(g) and (j)) and with the supply of any imposed flow (Sketches 4.1(h) and (j)).

In order to develop an understanding of the flow associated with rotating discs, the case of a single disc rotating in an initially stationary fluid is considered in Section 4.2. This is further developed to model a rotating fluid adjacent to a stationary disc in Section 4.3. Turbulent flow over a single rotating disc is considered in Section 4.4. Subsequently, impinging flow on a rotating disc is presented in Section 4.5 and rotor-stator systems in Section 4.6. In particular, an introduction to the modelling of ingestion in peripheral rim seals, relevant to gas turbine engines, and of windage due to protrusions is given in Sections 4.6.6 and 4.6.7, respectively. An introduction to the determination of thrust on a rotating disc is given in Section 4.6.8.

It should be noted that the modelling methods presented here for discs do not take account of the presence of a rotating shaft. For the case of a rotating shaft driving a rotating disc, the power required to overcome frictional drag for the shaft is dependent on the relative dimensions and typically <1% of the power required to overcome frictional drag for the rotating disc. The flow associated with rotating cylinders and shafts is considered in Section 5.

* Usually, the term “disk” instead of “disc” is used in computer terminology

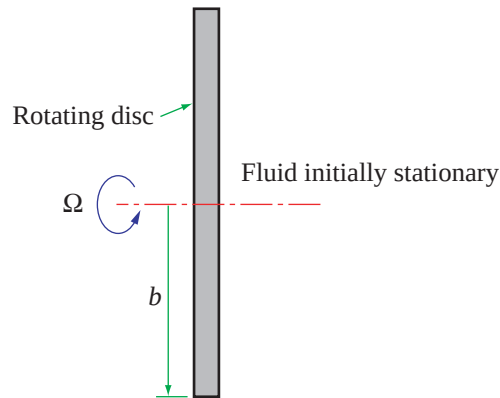


Sketch 4.1 Disc flow configurations

4.2 The 'Free Disc'

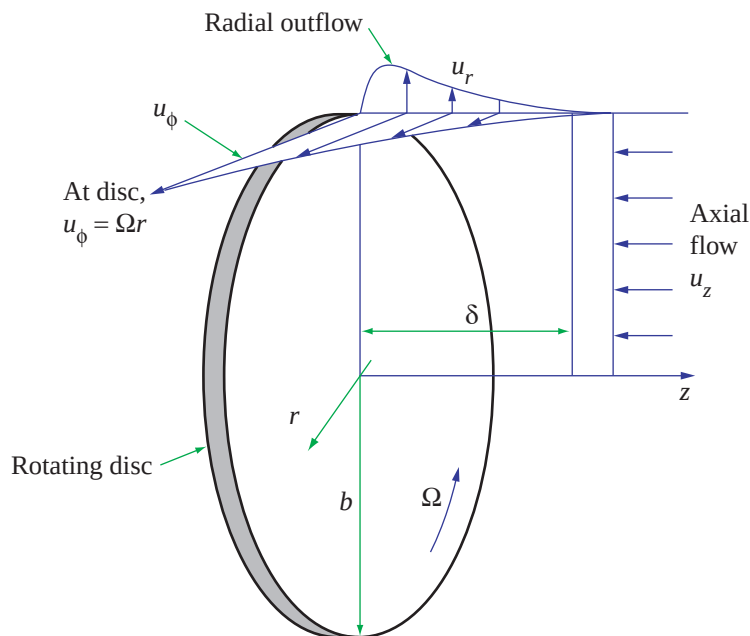
In this section, detailed consideration is given to the theoretical development for the case of laminar flow over a rotating disc (Sketch 4.1(a)). This forms the basis for understanding rotating disc flows in other applications.

The case of a single disc rotating in an initially stationary fluid is known as the 'free disc'. A disc of radius b rotating with an angular velocity, Ω , about the z axis is considered, as illustrated in Sketch 4.2. A rotating boundary layer develops on each side of the disc through which the tangential velocity of the fluid, u_ϕ , is sheared from the disc tangential velocity, Ωr , by the no-slip condition, to zero velocity in the free-stream outside the boundary layer. Due to symmetry, only one side of the disc needs to be considered in the development of the analysis.



Sketch 4.2 Rotating disc in a fluid which is initially at rest

The centrifugal forces generated by the shear between the rotating disc and the fluid cause a radial outflow of fluid in the boundary layer. As the radial component of velocity is zero at the disc surface and in the free-stream, in order to satisfy conservation of mass and provide the radial outflow, fluid must be entrained axially into the boundary layer. The radial outflow resulting from a disc rotating in a fluid is sometimes referred to as the pumped flow and, in the case of a free disc, as the 'free-disc pumping effect'. The principal flows and associated boundary layers for this case are illustrated in Sketch 4.3.



Sketch 4.3 Radial and tangential boundary layers and associated inflow and outflow induced by a rotating disc

Inside the disc boundary layer, the flow may be laminar or turbulent. The flow regime is governed by the relative magnitudes of inertia and viscous effects as defined by the local rotational Reynolds number

$$Re_{\phi, local} = x^2 Re_{\phi} \quad (4.1)$$

where the rotational Reynolds number, Re_{ϕ} , is defined by

$$Re_{\phi} = \frac{\rho \Omega b^2}{\mu} \quad (4.2)$$

and the radius ratio, x , is given by

$$x = \frac{r}{b} \quad (4.3)$$

The flow near the axis of rotation tends to be laminar. The rotational Reynolds number, Re_{ϕ} , for which flow becomes unstable (transitional) is of the order of 2×10^5 , as discussed further in Section 4.2.2. For $Re_{\phi} > 2 \times 10^5$, the flow can become turbulent even at small radii with a transition region between the laminar and turbulent flow zones.

4.2.1 Laminar flow over a free disc

Laminar flow over a single rotating free disc can be modelled using the steady state, axisymmetric Navier-Stokes equations given in Appendix A. These equations (Equations (A.5) to (A.8)) can be transformed to an exact set of ordinary differential equations that can be solved numerically. This flow problem was originally investigated by von Kármán⁵ and subsequently by Cochran⁸.

For the region normal to the rotating disc and for radii less than the outer disc radius,

i.e. for $z > 0$ and $r < b$,

the boundary conditions consistent with the no-slip condition at the rotating surface are

$$u_r = 0, \quad u_{\phi} = r\Omega, \quad u_z = 0 \quad \text{at } z = 0$$

$$\text{and} \quad u_r = 0, \quad u_{\phi} = 0 \quad \text{as } z \rightarrow \infty$$

The value of the axial velocity normal to the disc, u_z , as z tends to infinity is not specified. It adjusts to a non-zero negative value in order to provide the necessary flow for the free disc pumping effect.

The steady-state, axi-symmetric continuity and Navier-Stokes equations, Equations (A.5) to (A.8), given in Appendix A, can be made non-dimensional using the following transformations:

$$z^* = z \sqrt{\frac{\Omega}{\nu}} \quad (4.4)$$

$$u_r^*(z^*) = \frac{u_r}{r\Omega} \quad (4.5)$$

$$u_{\phi}^*(z^*) = \frac{u_{\phi}}{r\Omega} \quad (4.6)$$

$$u_z^*(z^*) = \frac{u_z}{\delta \Omega} \quad (4.7)$$

$$p^*(z^*) = \frac{p}{\mu \Omega} = \frac{p}{\rho \nu \Omega} \quad (4.8)$$

The superscript* in Equations (4.4) to (4.8) has been used to signify that a particular variable is non-dimensional. For example, z^* is a non-dimensional axial location, normal to the disc, and u_r^* is a non-dimensional radial velocity component. In Equation (4.5), the notation $u_r^*(z^*)$ is used to signify that the non-dimensional radial velocity u_r^* is a function of the non-dimensional axial location (z^*) normal to the disc.

Substitution of Equations (4.4) to (4.8) in the Navier-Stokes equations, i.e. Equations (A.5) to (A.8), gives, with simplifying assumptions, the following set of coupled ordinary differential equations:

$$\frac{du_z^*}{dz^*} + 2u_r^* = 0 \quad (4.9)$$

$$\frac{d^2 u_r^*}{dz^{*2}} - u_z^* \frac{du_r^*}{dz^*} - u_r^{*2} + u_\phi^{*2} = 0 \quad (4.10)$$

$$\frac{d^2 u_\phi^*}{dz^{*2}} - u_z^* \frac{du_\phi^*}{dz^*} - 2u_r^* u_\phi^* = 0 \quad (4.11)$$

$$\frac{d^2 u_z^*}{dz^{*2}} - u_z^* \frac{du_z^*}{dz^*} - \frac{dp^*}{dz^*} = 0 \quad (4.12)$$

For the region where

$$z > 0 \quad \text{and} \quad r < b,$$

the boundary conditions, consistent with the no-slip condition at the rotating surface, are

$$u_r^* = 0, \quad u_\phi^* = 1, \quad u_z^* = 0, \quad p^* = 0 \quad \text{at } z^* = 0$$

$$\text{and} \quad u_r^* = 0, \quad u_\phi^* = 0 \quad \text{as } z^* \rightarrow \infty$$

Equations (4.9) to (4.12) can be solved using numerical techniques. The solution is tabulated in Table B.1 and presented graphically in Figure 1 using the numerical results from Owen and Rogers⁷⁶. Since it is non-dimensional, any particular solution to these equations is independent of the specific values of rotational speed, density and viscosity.

Examination of Figure 1 reveals the relative magnitudes of u_ϕ^* , u_z^* and u_r^* . Fluid in contact with the surface rotates at the same angular velocity as the surface, in accordance with the no-slip condition, and experiences the same centripetal acceleration. As the disc begins to rotate, a boundary layer forms in the circumferential direction. Fluid in the boundary layer just above the surface also begins to rotate but cannot maintain the same centripetal acceleration of the surface and acquires an outward radial component. As the radial component of velocity increases in magnitude, a secondary boundary layer develops in the radial direction. The stresses in this secondary boundary layer provide a central force and centripetal acceleration but smaller than that at the surface. At distances away from the boundary layer thickness, the fluid is not rotating and there is no mechanism available to sustain radial flow. Conservation of mass demands a flow to match the outward radial flow. This results in fluid with zero momentum being drawn in axially, being given momentum in the boundary layer and then being pumped radially outward with high angular momentum. The rotating boundary layer associated with a free disc is commonly referred to as an **Ekman layer**.

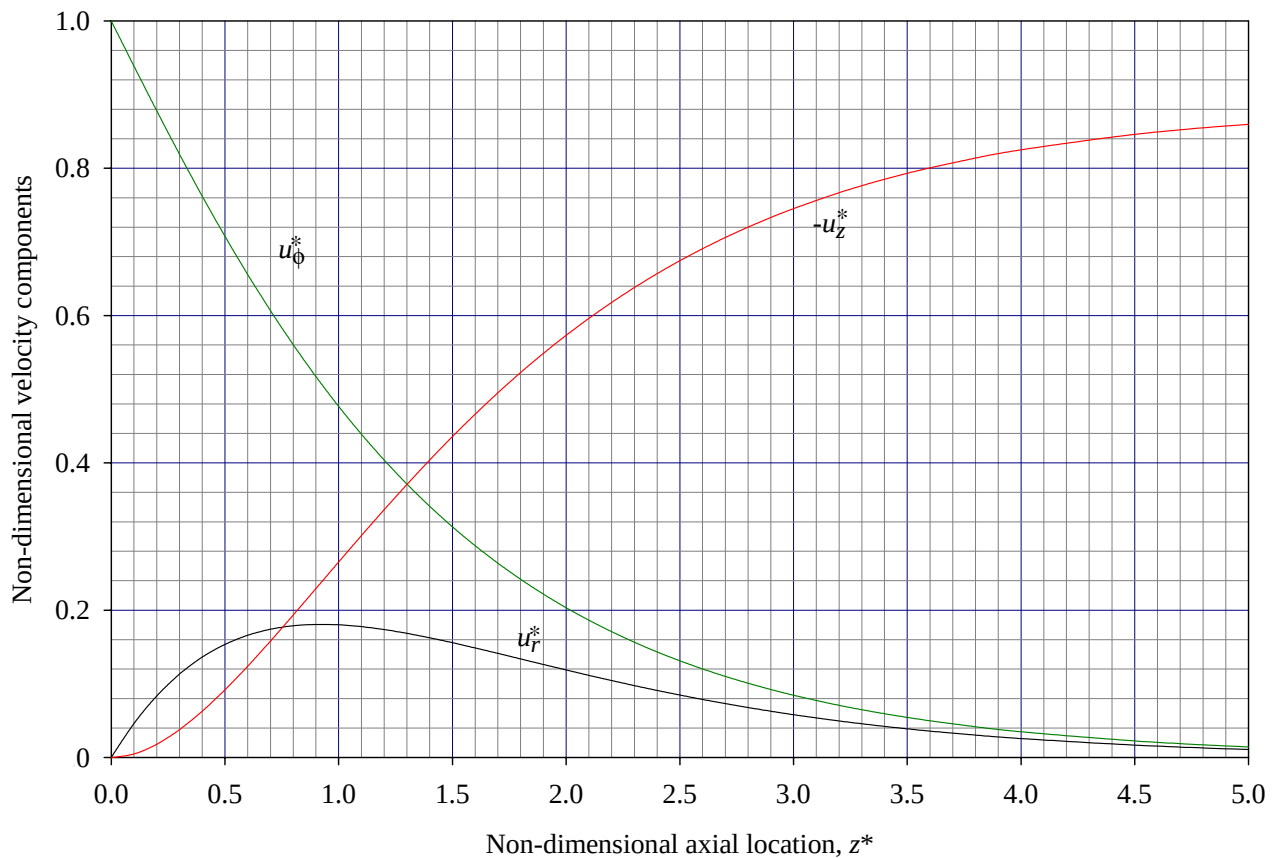


FIGURE 1 Non-dimensional laminar flow velocity components for the ‘free disc’

4.2.1.1 Boundary layer thickness

Typically, the boundary layer thickness, δ_{99} , is defined as the axial distance away from the rotating disc at which the tangential velocity is 1% of the disc speed, *i.e.* $u_\phi = 0.01\Omega r$. Values of boundary layer thickness for laminar flow over a free disc are given in Table B.1. The value of $u_\phi^* = u_\phi/\Omega r = 0.01$ corresponds to a value for z^* of about 5.5, which on substitution in Equation (4.4) gives the following relationship for the laminar velocity boundary layer on a free disc:

$$\delta_{99} \approx 5.5 \sqrt{\frac{\nu}{\Omega}} = 5.5 \sqrt{\frac{\mu/\rho}{\Omega}}. \quad (4.13)$$

It should be noted that this boundary layer thickness is not a function of the disc radius. Furthermore, Equation (4.13) has been developed based on the distribution of the tangential velocity typically used in rotating disc applications. A slightly different value for the constant would arise for a boundary layer based on the radial velocity profile.

4.2.1.2 Entrained mass flow rate

The mass flow entrained by the boundary layer on one side of the rotating disc is equal to the axial flow rate into the boundary layer. This can be found by integration:

$$\dot{m}_o = -2\pi\rho \int_0^r r[u_z]_{z \rightarrow \infty} dr = -\pi\rho r^2 \sqrt{(\Omega v)} u_{z=\infty}^* \quad (4.14)$$

From Table B.1, the corresponding value of u_z^* for $z^* = \infty$ is $u_z^* = -0.8845$. The total mass flow rate for a disc of radius b , is then

$$\dot{m}_o = 2.779\rho b^2 \sqrt{\Omega v} \quad (4.15)$$

It is often convenient to use a non-dimensional mass flow rate, C_w , defined by

$$C_w = \frac{\dot{m}_o}{\mu b} \quad (4.16)$$

Restating Equations (4.14) and (4.15) using Equation (4.16), gives the non-dimensional mass flow rate entrained by a rotating disc as

$$C_w = 2.779 Re_\phi^{0.5} \quad (4.17)$$

where Re_ϕ is defined in Equation (4.2).

4.2.1.3 Total moment and moment coefficient

A frequent requirement in rotating machinery is the evaluation of the power required to overcome viscous drag due to rotation of discs and cylinders. The frictional moment on one side of the disc can be obtained by integrating the tangential component of the fluid stress at the surface over the area concerned.

The total moment, or torque, T_q , can be evaluated from

$$T_q = -\frac{1}{2}\pi b^4 \rho v^{0.5} \Omega^{1.5} \left[\frac{\partial u_\phi^*}{\partial z^*} \right]_{z^*=0} \quad (4.18)$$

It is often convenient to use the moment coefficient. This is derived from the moment definition for drag on a rotating disc. The moment coefficient for one side of the disc of radius b is defined by

$$C_m = \frac{T_q}{0.5\rho\Omega^2 b^5} \quad (4.19)$$

From Table B.1, $[\partial u_\phi^*/\partial z^*]_{z^*=0} = -0.6159$. Substitution in Equation (4.18) and considering the rotational Reynolds number in Equation (4.19), the moment coefficient for one side of a disc is given by

$$C_m = 1.935 Re_\phi^{-0.5} \quad (4.20)$$

It should be noted that the modelling approach used in Equations (4.14) to (4.20) assumes that edge effects are negligible and does not take into account the effects of any shaft, which are typically small by comparison to that of a disc (see Section 5).

4.2.2 Transitional and turbulent flow structures

For low values of the local rotational Reynolds number, the flow over a rotating disc is laminar. As the rotational Reynolds number rises, the flow becomes unstable with either small patches of turbulence or a more regular oscillating vortex disturbance. This disturbed flow then breaks down into fully turbulent flow at a higher critical value of the local rotational Reynolds number.

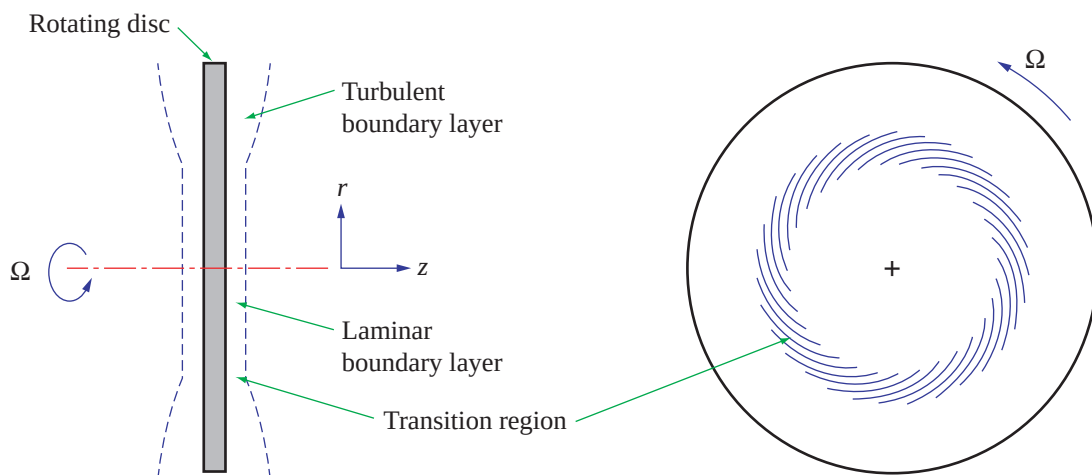
Experimental observations by Gregory *et al.*¹⁷ for a rotating perspex disc showed that the flow becomes unstable at local rotational Reynolds numbers, $Re_{\phi,local} = 1.78 \times 10^5$ to 2.12×10^5 , breaking down into a system of spiral vortices, as illustrated schematically in Sketch 4.4. The transition to complete turbulence occurred for $Re_{\phi,local} = 2.7 \times 10^5$ to 2.99×10^5 . The experiments by Theodorsen and Regier¹⁰ for a polished rotating disc found that laminar flow begins to become unstable for $Re_{\phi,local} \approx 3.1 \times 10^5$. For a roughened disc surface this value can be as low as $Re_{\phi,local} = 2.2 \times 10^5$. Fully turbulent flow establishes for $Re_{\phi,local} \approx 7 \times 10^5$.

These data combined with theoretical investigations provide evidence for the initial breakdown of laminar flow at $Re_{\phi,local} \approx 2 \times 10^5$. The critical rotational Reynolds number for the initial breakdown of laminar flow is then given by

$$Re_{\phi,critical} = x^2 Re_{\phi} = \frac{\rho \Omega r^2}{\mu} \approx 2 \times 10^5 \quad (4.21)$$

The actual value at which laminar flow breaks down depends on the specific local condition of the flow and the surface roughness.

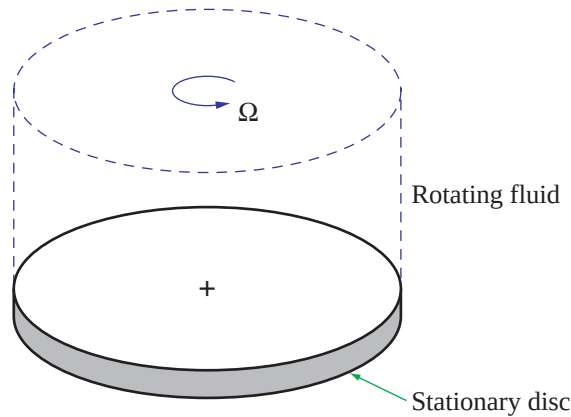
Example 1 in Section 7.1 illustrates the use of Equations (4.13), (4.17) and (4.19) to (4.21).



Sketch 4.4 Schematic diagram of the boundary layers on a rotating disc

4.3 Rotating Fluid above a Stationary Disc

Interest in the case of a rotating fluid above a stationary disc (Sketch 4.5) originally arose from modelling atmospheric flows. Analysis for this configuration provides valuable insight for the case of flow between a rotating and a stationary disc. The equations for the 'free disc', Equations (4.4) to (4.7), provide the basis for the model considered by Bödewadt⁹, Rogers and Lance²⁷, and Nydahl⁴¹ and Schlichting⁴⁷.



Sketch 4.5 Rotating fluid above a stationary disc

The boundary conditions for an infinite mass of rotating fluid at a uniform angular velocity, Ω , in contact with a non-rotating surface are:

$$\begin{aligned} u_r &= 0, & u_\phi &= 0, & u_z &= 0 & \text{at } z = 0; \\ \text{and} & & u_r &= 0, & u_\phi &= r\Omega & \text{as } z \rightarrow \infty. \end{aligned}$$

In this case, it can be assumed that the pressure is constant through the boundary layer thickness. The pressure distribution in the fluid that is rotating at a constant angular velocity, Ω , can be obtained using

$$\frac{\partial p}{\partial r} = \rho r \Omega^2 \quad (4.22)$$

On substitution, the following set of coupled ordinary differential equations result:

$$\frac{d^2 u_r^*}{dz^{*2}} - u_z^* \frac{du_r^*}{dz^*} - u_r^{*2} + u_\phi^{*2} - 1 = 0 \quad (4.23)$$

$$\frac{d^2 u_\phi^*}{dz^{*2}} - u_z^* \frac{du_\phi^*}{dz^*} - 2u_r^* u_\phi^* = 0 \quad (4.24)$$

$$\frac{du_z^*}{dz^*} + 2u_r^* = 0 \quad (4.25)$$

Pressure does not appear in Equations (4.23) to (4.25) as the pressure in the main body of rotating fluid is known. The same value is assumed in the boundary layer approximation. The non-dimensional boundary conditions are:

$$\begin{aligned} u_r^* &= 0, & u_\phi^* &= 0, & u_z^* &= 0 & \text{at } z = 0; \\ \text{and} & & u_r^* &= 0, & u_\phi^* &= 1 & \text{as } z \rightarrow \infty. \end{aligned}$$

Solutions for the non-dimensional velocity components have been performed by Nydahl⁴¹ and more recently updated by Owen and Rogers⁷⁶. These are presented in Table B.2 and plotted in Figure 2. A schematic of the boundary layer velocity profiles is given in Sketch 4.6.

Fluid in the boundary layer near the stationary surface has less centripetal pressure differential than the rotating fluid. Peripheral fluid flows radially inward through the thin boundary layer towards the axis. Conservation of angular momentum of the inward flow causes the tangential velocity to be slightly elevated above the free stream rotational speed at approximately $z^* = 1.5\delta_{99}$ which leads to the flow reversals.

The radial inward flow is the mechanism responsible for the accumulation of un-dissolved sugar particles at the centre of a stirred cup of tea.

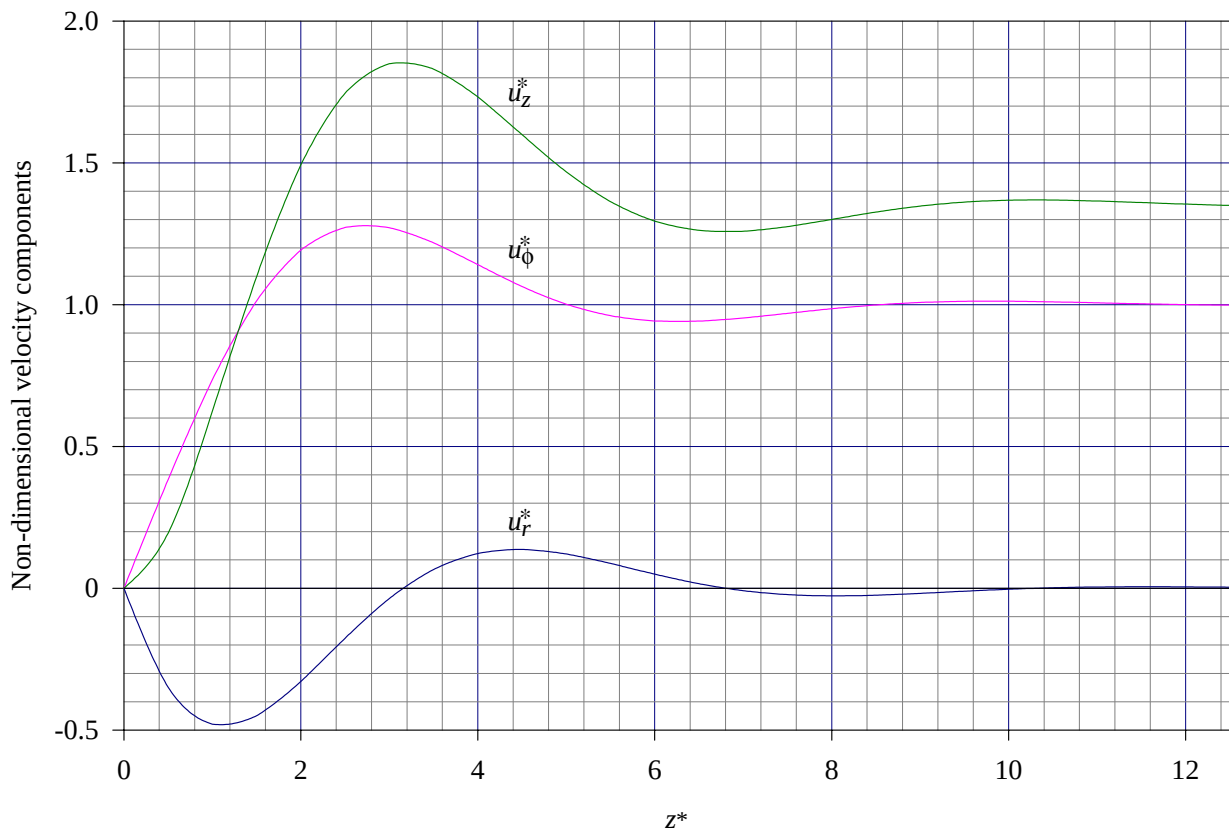
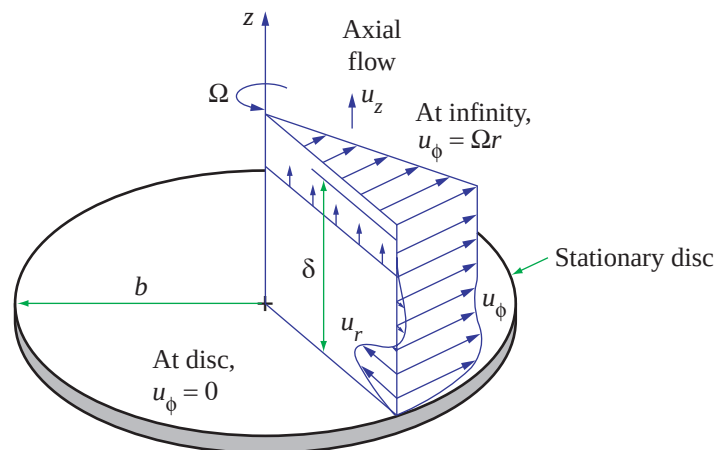


FIGURE 2 Non-dimensional laminar flow velocity components for a stationary disc in a rotating fluid



Sketch 4.6 Radial and tangential boundary layers and associated inflow and outflow induced by a rotating fluid adjacent to a stationary disc

4.3.1 Boundary layer thickness

Typically, for this application the boundary layer thickness, δ_{99} , is defined as the axial distance away from the disc at which the tangential velocity is 99% of the free stream rotational speed, *i.e.* $u_\phi = 0.99\Omega r$. The non-dimensional tangential velocity component, $u_{\phi*} = u_\phi/\Omega r = 0.99$ corresponds to a value for z^* of about 8 (Table B.2), which on substitution in Equation (4.4) gives the following relationship for a rotating laminar boundary layer on a stationary disc:

$$\delta_{99} \approx 8 \sqrt{\frac{\nu}{\Omega}} \quad (4.26)$$

The boundary layer thickness is thus approximately 1.5 times that produced by the free disc. In the case of the 'free disc', the axial flow towards the disc tends to suppress the boundary layer thickness while in this case the axial flow away from the surface and conservation of angular momentum tend to thicken the boundary layer.

4.3.2 Entrained mass flow rate

The net radial flow through a cylinder of radius b can be obtained by integrating the radial velocity. From Table B.2, with $u_{z=\infty}^* = 1.3794$, the non-dimensional mass flow rate for the entrained flow is

$$C_w = -4.239 Re_\phi^{0.5} \quad (4.27)$$

4.4 Turbulent Flow over a Rotating Disc

The analysis of turbulent flow over a single rotating disc provides the basis for the modelling of more complex rotating flow applications such as the flow between a rotating and a stationary disc.

The sum of the momentum of the flow across a boundary layer thickness can be integrated to give relationships for the boundary layer thickness, total mass flow rate and frictional moment on the disc. The technique involves a number of simplifications of the flow physics, however, it provides valuable correlations which are valid under limited conditions.

The momentum integral method can be applied to almost any boundary layer flow. An integral momentum technique applied to the turbulent flow on a free disc was performed by **von Kármán**⁵. From this, incorporating calculation corrections from Dorfman²⁹, a relationship for the moment on one side of the disc was determined, Equation (4.28), along with relationships for the boundary layer thickness, Equation (4.29), and non-dimensional entrained mass flow, Equation (4.30), using a 1/7th power-law assumption for the velocity profile:

$$C_m = 0.073 Re_\phi^{-0.2} \quad (4.28)$$

$$\delta = 0.525 r \left[\left(\frac{r}{b} \right)^2 Re_\phi \right]^{-0.2} \quad (4.29)$$

$$C_w = 0.219 Re_\phi^{0.8} \quad (4.30)$$

Equation (4.28) is in close agreement with measurements from Theodorsen and Regier¹⁰ for $5 \times 10^5 < Re_\phi < 2 \times 10^6$. At higher rotational Reynolds numbers, however, Equation (4.28) significantly under-predicts the measurements by Bayley and Owen³⁹.

Dorfman²⁹ obtained an alternative solution for the moment coefficient, assuming logarithmic profiles for the flow near the rotating disc:

$$C_m = 0.491 [\log_{10}(Re_\phi)]^{-2.58} \quad (4.31)$$

Equation (4.31) is valid for $10^4 < Re_\phi < 10^9$.

For a disc rotating in air at rotational Reynolds numbers, $Re_\phi < 4.2 \times 10^6$, **Bayley and Owen**³⁹ give the following correlation based on their experiments:

$$C_m = 0.0655 Re_\phi^{-0.186} \quad (4.32)$$

For rotational Reynolds numbers in the range $10^5 < Re_\phi < 10^7$, Equations (4.32) and (4.31) differ by less than 5%. Although Equation (4.28) has been relatively widely used for the prediction of the moment coefficient on a rotating disc and it is in a reasonable agreement with the measurements from Theodorsen and Regier¹⁰, at higher rotational Reynolds numbers, it significantly under-predicts the measurements obtained by Bayley and Owen³⁹. To determine the appropriate moment coefficient, it is therefore necessary to establish the rotational Reynolds number range for the application concerned and then use Equations (4.28), (4.31) or (4.32) accordingly.

4.4.1 Smooth surfaces

A surface can be regarded as hydraulically smooth if the sand-grain roughness¹¹ of the surface, k_s , is

$$k_s < y \quad (4.33)$$

where y is the thickness of the viscous sub-layer.

In general, roughness has little effect on the shear stress, provided the flow is laminar or the roughness elements do not exceed the thickness of the laminar sub-layer and protrude through the sub-layer to disrupt the flow. The thickness of the viscous sub-layer is conventionally taken as corresponding to a universal coordinate of $y^+ \approx 5$ where

$$y^+ = \frac{y \sqrt{\tau_s / \rho}}{\nu} \quad (4.34)$$

Other values are sometimes quoted for the thickness of the viscous sub-layer, but $y^+ \approx 5$ represents the approximate non-dimensional wall distance at which the viscous shear stress and the Reynolds shear stress become equal. For $y^+ \approx 5$, the thickness of the viscous sub-layer is

$$y = \frac{5\nu}{\sqrt{\tau_s / \rho}} \quad (4.35)$$

4.4.2 Rough surfaces

Turbulent flow on rough surfaces can be classified according to the roughness parameter¹¹:

$$k_s u_\tau / \nu \quad (4.36)$$

where u_τ is the friction velocity defined as

$$u_\tau = \sqrt{\tau_s / \rho}.$$

Three regimes can be defined, namely:

- (i) hydraulically smooth: $k_s u_\tau / \nu < 5$.
- (ii) transitional: $5 < k_s u_\tau / \nu < 70$.
- (iii) completely rough: $k_s u_\tau / \nu > 70$.

For the completely rough regime, Dorfman¹⁹ developed the solution given in Equation (4.37) for the momentum integral equation in terms of the size of the individual grains used, k'_s .

Owing to the proximity of grains, the sand-grain roughness, k_s , is typically 60% of the sand-grain, size *i.e.*

$$k_s \approx 0.6 k'_s$$

The centre line average roughness¹⁰¹, R_a , is typically about 50% of the equivalent sand-grain roughness¹⁰³,

$$R_a \approx 0.5 k_s \approx 0.3 k'_s,$$

allowing the following relationships:

$$C_m = 0.054 \left(\frac{k'_s}{b} \right)^{0.272} = 0.062 \left(\frac{k_s}{b} \right)^{0.272} = 0.075 \left(\frac{R_a}{b} \right)^{0.272} \quad (4.37)$$

Equation (4.37) is in reasonable agreement with data of Kanaev¹³ for a disc rotating in water with $k_s/b = 3.33 \times 10^{-3}$ but underestimates the data of Theodorsen and Regier¹⁰ for $k_s/b = 8.3 \times 10^{-4}$ by approximately 15%.

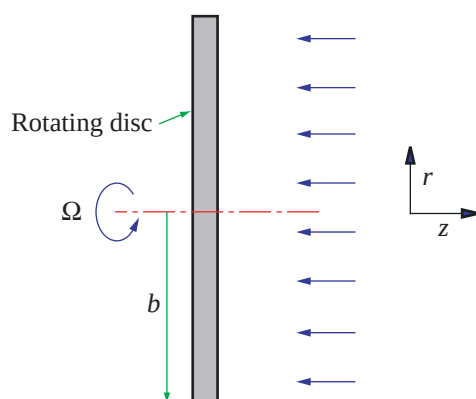
4.5 Impinging Flow on a Rotating Disc

For the case of a rotating disc with uniform superimposed flow impinging perpendicularly to the surface (Sketch 4.7), the moment coefficient has been found to be proportional to the ratio of the velocity axial component of the flow outside the boundary layer and to the tangential velocity component at the outer radius of the disc. Dorfman²⁹ defined this ratio by

$$\sigma = \frac{2}{\pi} \frac{u_{z,\infty}}{\Omega b} \quad (4.38)$$

where:

- $u_{z,\infty}$ is the axial velocity component outside the boundary layer,
- and Ωb is the tangential component of velocity evaluated at the outer radius.



Sketch 4.7 Rotating disc with uniform superimposed flow impinging on the surface

Tifford and Chu¹⁵ obtained solutions to the **laminar flow** equations for the moment coefficient for impinging flow on a rotating disc for various values of σ , and $r \ll b$. These are presented in Table B.3. Owen and Rogers⁷⁶ note that these solutions need to be used with caution as they do not hold for the whole disc.

Impinging **turbulent flow** on a rotating disc was considered by Truckenbrodt¹⁶. Using an integral momentum technique and assuming a general $1/n$ power-law profile for the boundary layer velocity, the following relationship can be developed for the moment coefficient:

$$C_m = \gamma(G_1\alpha + G_2\sigma)Re_\phi^{-\frac{2}{n+3}} \quad (4.39)$$

where $\alpha^2 + G_3\alpha\sigma = G_4 + G_5\sigma^2 \quad (4.40)$

and $\gamma = \left[K_n \frac{(n+3)(1+\alpha^2)^{\frac{n-1}{2(n+3)}}}{(5n+11)(G_1\alpha + G_2\sigma)} \right]^{\frac{n+1}{n+3}} \quad (4.41)$

The coefficient K_n is given by

$$K_n = \frac{1}{\frac{2n}{C_n^{n+1}}} \quad (4.42)$$

The ratio of the radial to the tangential component of stress, α , is determined using

$$\alpha^2 = \frac{4(3n+2)(2n+1)(n+1)}{n^2(16n^3 + 85n^2 + 145n + 66)} \quad (4.43)$$

Values for the coefficients G_1 , G_2 , G_3 , G_4 , G_5 , C_n , K_n and α are given in Table B.4.

4.6 Rotor-Stator Systems

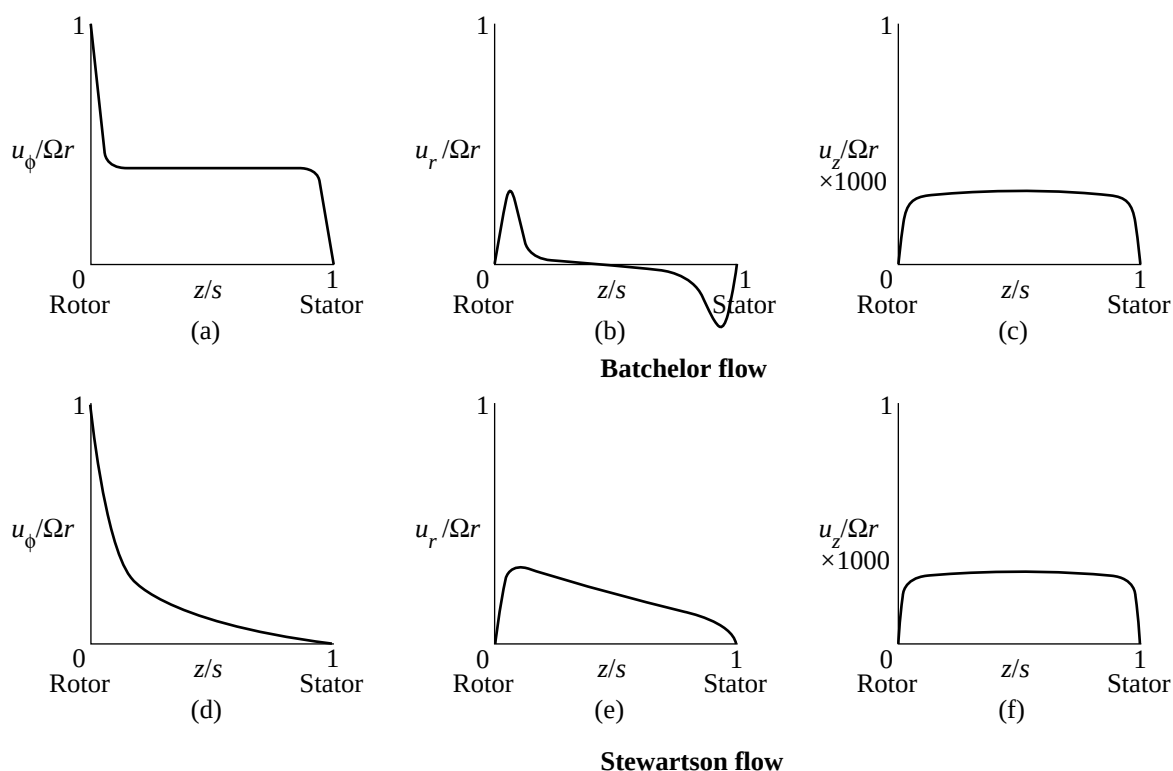
A common configuration in rotating flow applications is the flow over a stationary surface in close proximity to a rotating disc (Sketches 4.1(e) to 4.1(j)). Examples include the casing for hard-disk drives for digital memory, casings for gear boxes and flywheels, and support discs for stationary blades and components in gas turbine engines. Such configurations are here described as rotor(rotating disc)-stator (stationary disc or casing) systems and the gap between the rotating and stationary surfaces is described as the wheelspace, following gas turbine terminology.

In aircraft and electric power generation gas turbine engines, it is common practice for turbine disc cavities to be purged with coolant air taken from appropriate stages of the compression system. This air controls the disc temperature distribution and limits the inflow of hot mainstream gas into the cavity between the rotating disc and the adjacent stationary casing. However, the use of this air for turbine rim sealing is detrimental to engine cyclic performance, and turbine efficiency can be adversely affected by the seal air outflow into the main annulus. The lifetime of the rotor and the cyclic and component performance of the engine, are therefore strongly dependent on turbine rim sealing efficiency. The combined requirements for the determination of the power required to overcome frictional drag, local flow characteristics and associated heat transfer have provided a sustained impetus to investigate a number of rotor-stator wheelspace configurations.

4.6.1 Batchelor and Stewartson models

Prior to the advent of a significant quantity of experimental data and CFD predictions, various theories were developed for the flow structure in a rotor-stator wheelspace. In the 1950s, Batchelor¹² proposed a model which assumes that the core of fluid between the rotating disc and the stator rotates at an angular velocity with a value between that of the stator and the rotor. This model implied the existence of boundary layers on both the rotor and the stator. In order to supply the rotor boundary layer, fluid is entrained from the core and pumped radially outwards on the rotor. At the stator, a radial inflow of fluid and an outflow from the stator boundary layer to the core occur.

In what has become known as the 'Batchelor-Stewartson controversy', Stewartson¹⁴ proposed a model which assumes that the tangential velocity in the rotor boundary layer reduces from the rotor speed value to zero away from the boundary layer with no core rotation, as in the 'free disc' flow. Characteristic velocity profiles for Batchelor and Stewartson models are illustrated in Sketch 4.8.



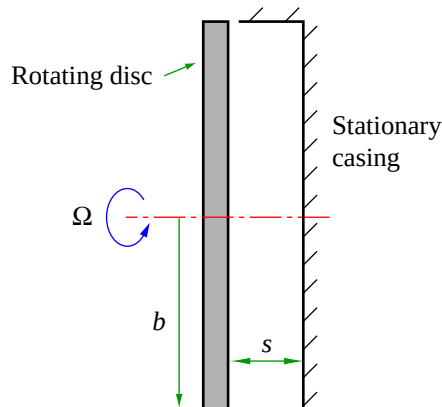
Sketch 4.8 Characteristic velocity profiles in a rotor-stator wheel-space:
(a) to (c) Batchelor flow¹²; (d) to (f) Stewartson flow¹⁴.

It is generally accepted that both the Batchelor flow model, characterised by three zones with two boundary layers separated by a central rotating core, and the Stewartson model, characterised by the tangential velocity tending to zero outside the rotor boundary layer, are valid depending on the flow conditions^{63, 128}. Velocity measurements in a rotor-stator disc system undertaken by Picha and Eckert²² showed that core rotation was minimal when the wheel-space is open to atmosphere; however, when a stationary shroud at the outer radius exists, a core of rotating fluid was measured. For a given rotor speed, the angular velocity of the core rotating fluid was found to reduce as the rotor-stator gap was increased. Subsequent CFD modelling^{38, 65, 128} has provided, for certain conditions, further supporting evidence for the existence of a rotating fluid core with a tangential velocity component that is a fraction of the rotor speed.

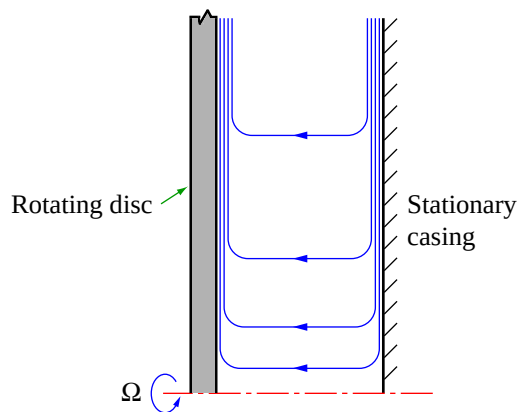
An important parameter in determining the flow structure is the supply of any superposed flow to the cavity. In general, for a rotor-stator cavity, core rotation reduces with an increase in superposed radial outflow provided by a net flow of fluid supplied at the axis to the cavity. Flow initially characterised by the Batchelor model with a rotating core tends towards that of the Stewartson model with little or no core rotation as the radial outflow increases.

4.6.2 Enclosed rotor-stator system

For an enclosed rotor-stator disc system, with no superposed flow supplied or extracted at the axis (Sketch 4.9), the Batchelor flow model normally best describes the flow structure with fluid pumped radially outwards in the rotor boundary layer moving axially across to the stator in the boundary layer over the cylindrical outer shroud. Between the rotor and stator boundary layers, there is an inviscid core of fluid that rotates at some fraction of the rotor angular velocity. As shown in Sketch 4.8(a), the tangential velocity between the rotor and stator discs takes the value of tangential shear between the disc angular velocity in the rotor boundary, to a constant value in the rotating core, and then reducing to zero in the boundary layer on the stator disc. Sketch 4.10 shows typical streamlines for a cross section of a rotor-stator wheel-space.



Sketch 4.9 Idealised enclosed rotor-stator wheelspace with perfect seal between the stationary casing and the rotor rim



Sketch 4.10 Batchelor's model for enclosed rotor-stator wheelspace flow

Typically, the speed of rotation of the core, ω , is approximately 40% that of the rotor, *i.e.* $\omega = 0.4\Omega$. This fraction can be characterised by the swirl ratio,

$$\beta = \frac{\omega}{\Omega} \quad (4.44)$$

In general, the core rotation speed reduces with net radial outflow and increases with radial inflow, with the angular velocity of inflow being an important factor.

The gap ratio, G , is defined as the ratio of the axial clearance between the discs and the outer radius of the rotating disc:

$$G = \frac{s}{b} \quad (4.45)$$

Daily *et al.*³² found that the swirl ratio was $\beta = 0.475$, 0.45 and 0.42 for gap ratios, $G = 0.0276$, 0.069 and 0.1241 , respectively. This can be correlated approximately by

$$\beta = 0.49 - 0.57G \quad (4.46)$$