



Damping of structural vibrations by acoustic radiation

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DAMPING OF STRUCTURAL VIBRATIONS BY ACOUSTIC RADIATION

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DAMPING OF STRUCTURAL VIBRATIONS BY ACOUSTIC RADIATION

1. NOTATION

		Units	
		<i>SI</i>	<i>British</i>
a	length of plate in x -direction	m	in
b	length of plate in y -direction	m	in
c	velocity of sound in panel material	m/s	in/s
c_0	velocity of sound in fluid surrounding panel	m/s	in/s
D	bending stiffness of isotropic plate	N m	lbf in
D_x	bending stiffness in x -direction	N m	lbf in
D_y	bending stiffness in y -direction	N m	lbf in
D_{xy}	torsional stiffness about x - and y -axes	N m	lbf in
E	Young's modulus for material of plate	Pa	lbf/in ²
f	frequency	Hz	Hz
f_c	critical frequency of structure	Hz	Hz
f_{mn}	resonance frequency of plate at mode (m, n)	Hz	Hz
H	scaling factor defined by Equation (3.2)		
h	thickness of plate	m	in
h_e	equivalent thickness of orthotropic plate	m	in
k	acoustic wavenumber	m ⁻¹	in ⁻¹
k_b	bending wavenumber for infinite plate	m ⁻¹	in ⁻¹
k_{mn}	magnitude of wavenumber vector	m ⁻¹	in ⁻¹
m_p	mass of plate	kg	*
m	modal number in x -direction		
n	modal number in y -direction		

Issued November 2007 – 48 pages

P_{mn}	time-averaged sound power radiated by plate in mode (m, n)	W	lbf in/s
R	radius of cylinder	m	in
R_r	radiation resistance	kg/s	*
S	surface area of plate	m ²	in ²
X	total reactance (sum of mechanical and radiation reactance)	kg/s	*
X_r	radiation reactance	kg/s	*
Z_m	mechanical impedance	kg/s	*
Z_r	radiation impedance	kg/s	*
$\overline{v_{mn}^2}$	mean squared velocity of surface of plate	m ² /s ²	in ² /s ²
γ	dimensionless parameter, k/k_b		
Δ_{mn}	dimensionless parameter defined by Equation (3.1)		
ζ_{mn}	equivalent viscous damping ratio of (m, n) mode		
ν	Poisson's ratio		
ρ	density of plate	kg/m ³	*
ρ_0	density of air fluid surrounding panel	kg/m ³	*
σ_{mn}	radiation efficiency of (m, n) mode		
ψ	specific damping capacity		
ψ_{mn}	specific damping capacity of mode (m, n)		

* In British units, for the system to be coherent the mass in lb must be divided by the constant 386.4.

2. INTRODUCTION

Radiation of sound from a structural component, such as a panel or plate, is accompanied by a loss of vibrational energy and hence contributes to the reduction of the amplitude of vibration displacement, particularly at resonance. Damping of the vibration as a consequence of the radiation of sound is one of many forms of damping (see ESDU 73011). The stress in the component is reduced at the resonance frequencies by damping and hence the fatigue life is increased. This Item quantifies the damping of panel vibration, as the panel radiates sound, in terms of the various modes of vibration.

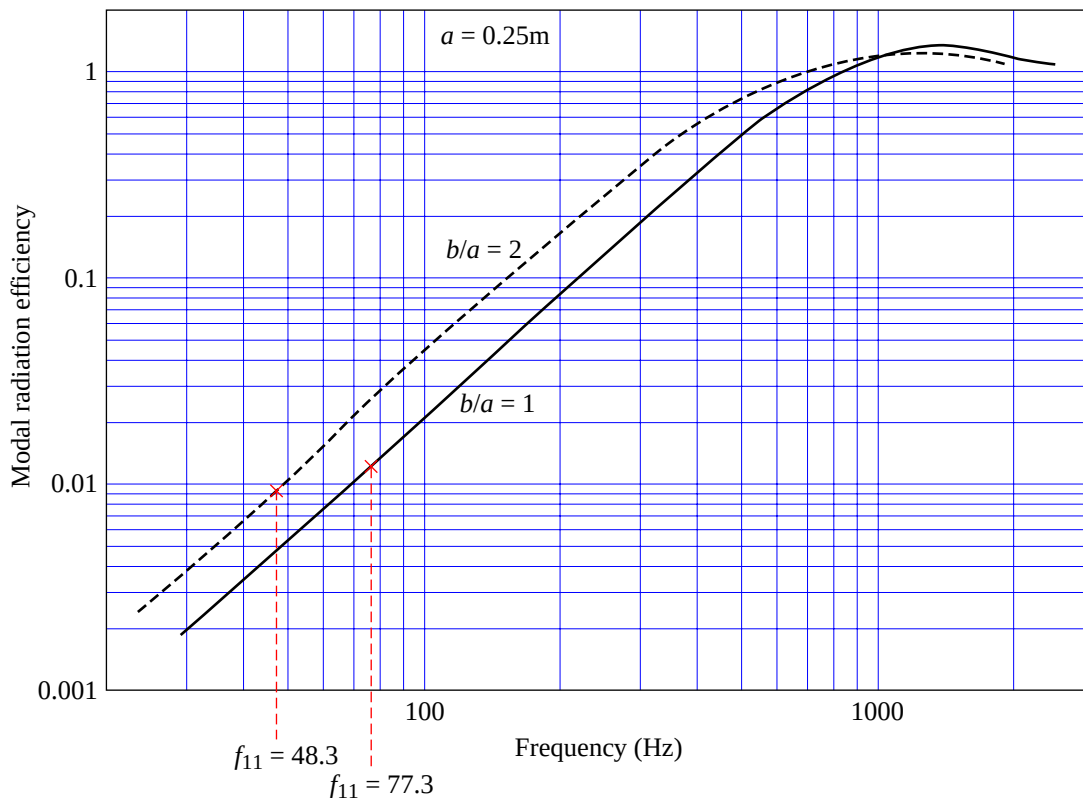
The sound generating capacity of a panel is most conveniently expressed in terms of its radiation efficiency (ESDU Items 00002 and 02009, References 18 and 20). The radiation efficiency may be calculated for a panel for which the following assumptions apply:

- (a) the panel is surrounded by an infinite baffle which is flush with the surface of the panel,
- (b) the panel is simply-supported at all four edges,
- (c) there is negligible fluid loading on the panel and
- (d) acoustic radiation is from one side of the panel only.

The radiation efficiency, σ_{mn} , is defined for a particular mode of vibration of a plate, characterised by the modal numbers m and n . In terms of the time-averaged sound power, P_{mn} , radiated from one side of a plate the modal radiation efficiency, σ_{mn} , is given by (Reference 5, 9, and 22)

$$\sigma_{mn} = \frac{P_{mn}}{\rho_0 c_0 S \overline{v_{mn}^2}}, \quad (2.1)$$

where $\overline{v_{mn}^2}$ is the squared vibrational velocity average in time and space for the simply supported plate of surface area, S . The density of the fluid which loads the panel is ρ_0 and the velocity of sound in the fluid is c_0 . The modal radiation efficiency depends upon the dimensions of the surface of the plate and the deflected shape of the plate at the particular mode of vibration. The modal radiation efficiencies are shown as functions of frequency in Sketch 2.1 for the (1, 1) mode of a rectangular panel, loaded by air, with side lengths a and b . (See ESDU Items 00002 and 02009 and also Reference 5 for more graphs showing modal radiation efficiency as a function of frequency.)



Sketch 2.1 Modal radiation efficiency for (1, 1) mode of a baffled, simply-supported rectangular plate

In the case of a vibrating structure it is the behaviour at resonance which is of importance. The resonance frequency, f_{mn} , related to the mode (m, n) , of an isotropic, simply-supported panel of thickness h constructed from a homogeneous material of density ρ , is given by

$$f_{mn} = \frac{1}{2\pi} \sqrt{\frac{D}{\rho h} \left(\frac{\pi^2 m^2}{a^2} + \frac{\pi^2 n^2}{b^2} \right)}, \quad (2.2)$$

where D , the bending stiffness of the panel, is written as

$$D = \frac{Eh^3}{12(1-\nu^2)}, \quad (2.3)$$

and where E and ν are the Young's modulus and Poisson's ratio for the material of the panel. Hence, Equation (2.2) may be written as

$$f_{mn} = \frac{\pi}{2} \sqrt{\frac{Eh^2}{12\rho(1-\nu^2)} \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)}. \quad (2.4)$$

The modal radiation efficiency of the panel at the resonance frequency of the mode $(1, 1)$ is indicated in Sketch 2.1 for two aspect ratios.

The sound energy radiated per cycle into the farfield by the panel at the resonance frequency, f_{mn} , is P_{mn}/f_{mn} and the total energy of vibration of the plate is on average $\rho Sh \langle v_{mn}^2 \rangle$. The specific damping capacity, ψ , is defined as the ratio of the energy dissipated per cycle (in this case the energy radiated as sound) to the total (kinetic and potential) energy of vibration of the plate. Hence, for a particular mode the specific damping capacity at resonance, ψ_{mn} , is given by (Reference 5)

$$\psi_{mn} = \frac{\rho_0 c_0}{\rho h f_{mn}} \sigma_{mn}. \quad (2.5)$$

Substituting f_{mn} from Equation (2.4) into Equation (2.5) leads to

$$\psi_{mn} = \frac{2\rho_0 c_0 \sigma_{mn}}{\pi \rho h \sqrt{\frac{Eh^2}{12\rho(1-\nu^2)} \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)}}. \quad (2.6)$$

The equivalent viscous damping ratio, ζ_{mn} , is related to the specific damping capacity, ψ_{mn} , by the relation (Reference 6)

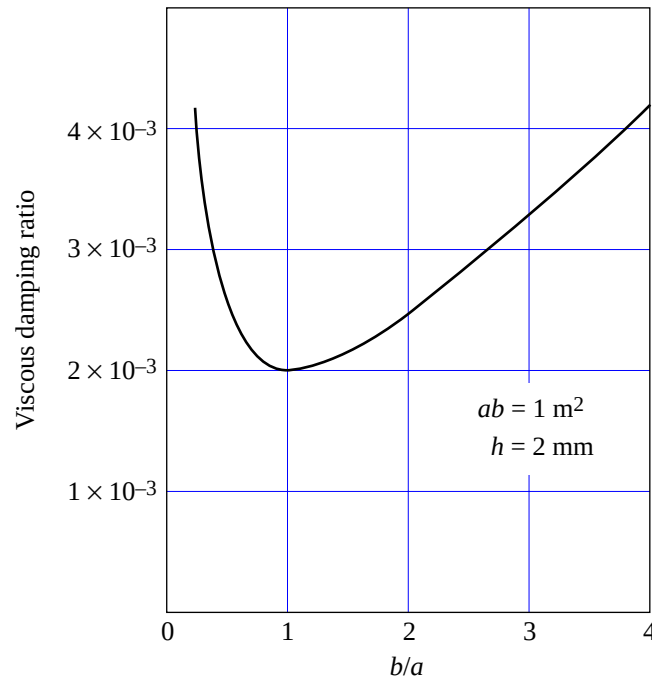
$$\zeta_{mn} = \frac{\psi_{mn}}{4\pi}. \quad (2.7)$$

Hence, the viscous damping ratio for a particular mode may be written as

$$\zeta_{mn} = \frac{\sqrt{3}\rho_0 c_0 \sigma_{mn}}{\pi^2 \sqrt{\frac{E\rho}{1-\nu^2}} \left(\frac{h^2}{ab} \right) \left(\frac{b^2}{a^2} m^2 + \frac{a^2}{b^2} n^2 \right)}. \quad (2.8)$$

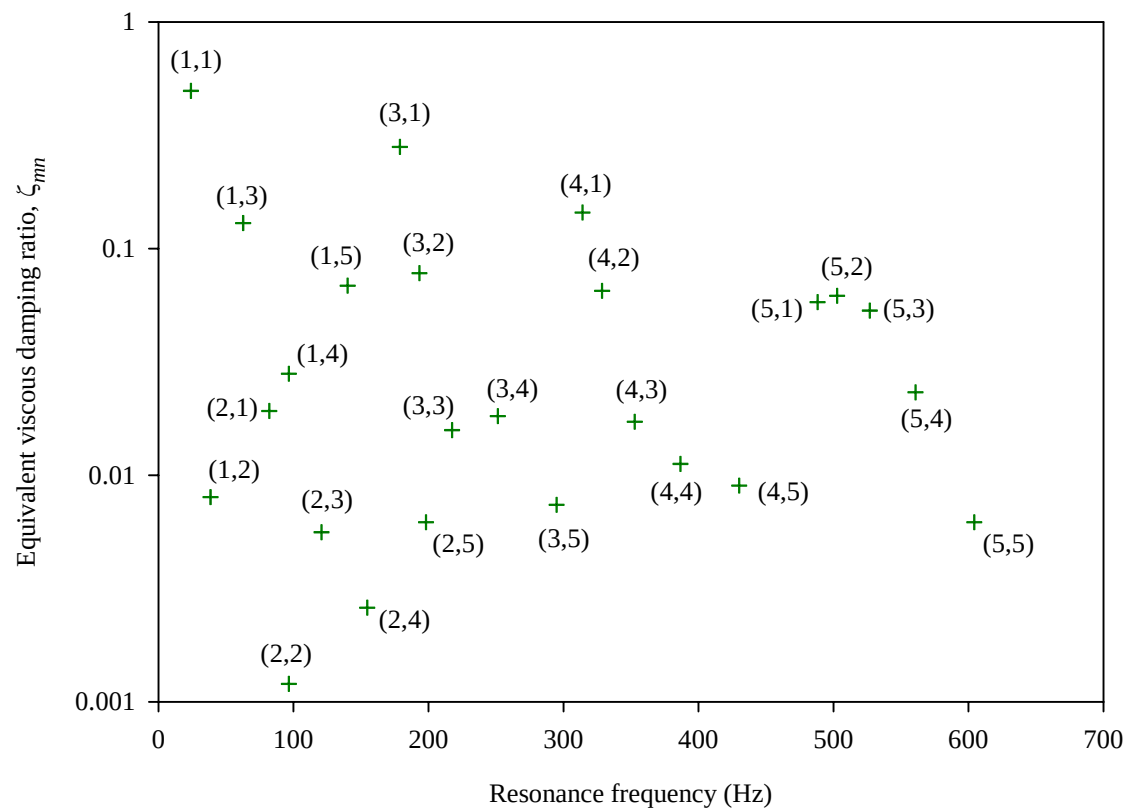
The loss factor is often used to characterise damping and is twice the value of the viscous damping ratio at the resonance frequency.

The modal damping ratio depends upon the geometry of the plate. It is indicated in Sketch 2.2 how the damping ratio depends on the plate aspect ratio b/a for simply-supported plates of equal area ($a b = 1 \text{ m}^2$) and thickness 2 mm, vibrating in mode (1, 1).



Sketch 2.2 Viscous damping ratio of the mode (1, 1) as a function of panel aspect ratio b/a for simply-supported panels

The variation in damping ratio for different modes of a given panel is shown in Sketch 2.3 (aluminium alloy, $E = 72.4 \times 10^9 \text{ Pa}$, $\rho = 2800 \text{ kg/m}^3$, $\nu = 0.3$, $b/a = 2$, $a = 0.5 \text{ m}$ and $h = 2 \text{ mm}$). Pairs of numbers indicate modes (m, n). For the case illustrated in Sketch 2.3 radiation of sound is assumed from both sides of the plate, consequently ζ_{mn} values for radiation from one side of the panel are doubled.



Sketch 2.3 Variation of viscous damping ratio with resonance frequency (radiation from both sides of panel)

In Sketch 2.3 it can be observed that the largest value of the damping ratio is associated with the mode (1, 1) which has the lowest resonance frequency. It can also be seen that whilst the damping ratio tends in general to decrease with increasing resonance frequency for lower modes, there is considerable fluctuation from one mode to the next.

3. DESIGN CURVES

To obtain ψ_{mn} , the specific damping capacity of a rectangular, isotropic, simply-supported, baffled panel for a particular mode (m, n) , the design curves of Figures 1 to 16 should be used.

Introduction of a dimensionless parameter

$$\Delta_{mn} = \frac{\sqrt{3}\pi}{64} \left(\frac{c_0}{c} \right) \left(\frac{\rho}{\rho_0} \right) \psi_{mn} \quad (3.1)$$

and the scaling factor

$$H = \frac{1}{2\sqrt{3}} \left(\frac{c}{c_0} \right) \frac{h}{\sqrt{ab}} \quad (3.2)$$

lead to curves based on relationships

$$\Delta_{mn} = f(H) \text{ for } m = 1, 2, \dots \text{ and } n = 1, 2, \dots \quad (3.3)$$

which provide numerical data for the specific damping capacity of the panel for chosen modes (m, n) , whatever the material and panel geometry.

The design curves for different modes (m, n) are presented in logarithmic scales with aspect ratio b/a as a changing parameter. Each Figure refers to a particular mode. Since c , the phase speed of longitudinal waves in plates, is related to Young's modulus, E , by the formula

$$c = \sqrt{\frac{E}{\rho(1-\nu^2)}}, \quad (3.4)$$

where ν is Poisson's ratio and ρ is the density of the panel material, the relationship (3.1) between the specific damping capacity and the parameter Δ_{mn} can be presented in the form

$$\psi_{mn} = \frac{64}{\pi\sqrt{3}} \frac{\rho_0}{\rho c_0} \sqrt{\frac{E}{\rho(1-\nu^2)}} \Delta_{mn}. \quad (3.5)$$

If the values of E , ρ and ν , characteristic for the material of the panel, as well as the values of the density of air, ρ_0 , and the velocity of sound in air, c_0 , are known, the specific damping capacity can be found from Equation (3.5), provided that the value of the parameter Δ_{mn} is deduced from the design curves presented in Figures 1 to 16.

Figures 1 to 16 show the factor Δ_{mn} as a function of the scaling factor H for modes (m, n) , where $m = 1 \dots 4$ and $n = 1 \dots 4$, for aspect ratios, b/a of 1, 2, 3, 4, 6 and 8. Results for aspect ratios less than one are not shown directly. For modes for which m and n are equal, *e.g.*, mode (2, 2), the curves for a particular aspect ratio and its reciprocal are the same, *i.e.*, curves for aspect ratios 2 and 0.5 are identical. For pairs of modes (m, n) and (n, m) , the curves referring to a particular aspect ratio and its reciprocal are the same, *e.g.*, for mode (1, 3) and an aspect ratio equal to 2 the design curve is identical with the curve for mode (3, 1) and aspect ratio 0.5. Hence, in Figures 1 to 16 aspect ratios greater than one are only considered. The figures apply to simply-supported panels which are surrounded by a rigid baffle and radiate sound only from one of its sides.

The design curves are obtained with the imposed restriction that $k_b h < 1$, where k_b is the bending wave number of the infinite plate. This condition is introduced because shear deformation of the panel is disregarded. There is a danger that this inequality could be violated for thick panels.

As mentioned in Section 2, fluid loading of the panel is neglected. Fluid loading is negligible when the condition $\rho_0 c_0 \ll \omega \rho h$ is fulfilled. This inequality could be violated, if h were small, but fluid loading is unlikely to be significant for a panel in air.

A particular feature of Figure 1 for mode (1, 1) is that the parameter Δ_{mn} , as well as the specific damping capacity and the viscous damping ratio maintain constant values for a wide range of the scaling factor, H . The other modes do not exhibit this characteristic and Δ_{mn} as a function of H may have one, two or three maxima.

Since aluminium alloys are widely used in aircraft structures, additional design curves are introduced for the direct determination of the equivalent viscous damping ratio at room temperature and are presented in Figures 17 to 32. The curves can only be used for a specific aluminium alloy for which $E = 72.4$ GPa, $\rho = 2800$ kg/m³ and $\nu = 0.3$. The specific acoustic resistance of air, $\rho_0 c_0$, is taken to be 415 kg m⁻² s⁻¹, which is the value at 293 K. The curves in Figures 17 to 32 are based on relationships between viscous damping ratios, ζ , and the dimensionless parameter, $h/\sqrt{ab}$. Hence, for an aluminium alloy panel, the viscous damping ratio can be determined from Figures 17 to 32 when the geometry is known, i.e., when the thickness, h , and the lengths of the sides, a and b , are known. As with Figures 1 to 16 the panels are assumed to be simply-supported, surrounded by a rigid baffle and radiating sound from one side. Figures 1 and 17, 2 and 18, 3 and 19, etc. are related by scaling factors applied to the x - and y -coordinates, see Equations (2.7), (3.2) and (3.5). The viscous damping ratio is independent of $h/\sqrt{ab}$ for low values of $h/\sqrt{ab}$ and for all the aspect ratios represented in Figure 17, although the independence is more evident for the lower values of the aspect ratio. For a thin panel with an aspect ratio of 8, radiating sound from both sides, the viscous damping ratio may be as much as 1.6%. For the higher modes damping can be significant, but only for a limited range of values of $h/\sqrt{ab}$.

3.1 Examples

3.1.1 Example 1

An isotropic panel is made from steel. ($E = 210$ GPa, $\rho = 7970$ kg/m³ and $\nu = 0.29$). It is assumed that the panel can be treated as simply-supported, surrounded by a rigid baffle and radiating sound from one side. The panel's dimensions are: $a = 0.8$ m, $b = 1.2$ m and $h = 4.0$ mm. The atmospheric pressure is 1.013×10^5 Pa and the temperature is 20° C. The equivalent viscous damping ratios for the modes (1, 1), (1, 2), (2, 1) and (2, 2) are required.

The relationship between the specific damping capacity and Δ_{mn} is

$$\psi_{mn} = \frac{64}{\pi\sqrt{3}} \left(\frac{c}{c_0} \right) \left(\frac{\rho_0}{\rho} \right) \Delta_{mn}.$$

The velocity of sound in the panel is given by Equation (3.4) and for steel is:

$$c = \sqrt{\frac{210 \times 10^9}{7970(1 - 0.29^2)}}$$

$$c = 5364 \text{ m/s}$$

For air $c_0 = 343.5$ m/s and $\rho_0 = 1.208$ kg/m³. Hence, ψ_{mn} is given by

$$\begin{aligned}\psi_{mn} &= \frac{64}{\pi\sqrt{3}} \left(\frac{5364}{343.5} \right) \left(\frac{1.208}{7970} \right) \Delta_{mn} \\ &= 2.784 \times 10^{-2} \Delta_{mn}.\end{aligned}$$

From Equation (2.7) the viscous damping ratio in terms of Δ_{mn} is

$$\zeta_{mn} = 2.215 \times 10^{-3} \Delta_{mn}.$$

The next step is to obtain the value of the scaling factor, H .

$$\begin{aligned}H &= \frac{1}{2\sqrt{3}} \left(\frac{c}{c_0} \right) \frac{h}{\sqrt{a} \sqrt{b}} \\ &= \frac{1}{2\sqrt{3}} \left(\frac{5364}{343.5} \right) \left(\frac{4 \times 10^{-3}}{\sqrt{0.8 \times 1.2}} \right) \\ &= 0.0184.\end{aligned}$$

Finally, values of Δ_{mn} are obtained from Figures 1, 2, 5 and 6 for $H = 0.0184$ for aspect ratio, $b/a = 1.2/0.8 = 1.5$. (Interpolation between $b/a = 1$ and 2 is required.) Thus, for modes (1, 1), (1, 2), (2, 1) and (2, 2), respectively,

$$\Delta_{11} = 0.36; \Delta_{12} = 1.9 \times 10^{-2}; \Delta_{21} = 2.1 \times 10^{-2} \text{ and } \Delta_{22} = 2.4 \times 10^{-3}.$$

Consequently, the viscous damping ratios are:

$$\zeta_{11} = 7.97 \times 10^{-4}; \zeta_{12} = 4.21 \times 10^{-5}; \zeta_{21} = 4.65 \times 10^{-5} \text{ and } \zeta_{22} = 5.32 \times 10^{-6}.$$

3.1.2 Example 2

An isotropic panel is made from an aluminium alloy ($E = 72.4$ GPa, $\rho = 2800$ kg/m³ and $\nu = 0.3$). It is simply-supported, surrounded by a rigid baffle and radiating sound from one side. The panel's dimensions are $a = 0.6$ m, $b = 1.2$ m and $h = 1.5$ mm. What are the equivalent viscous damping ratios for the modes (1, 1), (1, 2), (2, 1) and (2, 2) ?

For the aluminium alloy, $c = 5330.5 \text{ m/s}$.

$$\psi_{mn} = \frac{64}{\pi\sqrt{3}} \left(\frac{5330.5}{343.5} \right) \left(\frac{1.208}{2800} \right) \Delta_{mn}$$

$$= 0.0787 \Delta_{mn}$$

$$H = \frac{1}{2\sqrt{3}} \left(\frac{5330.5}{343.5} \right) \left(\frac{1.5 \times 10^{-3}}{\sqrt{0.6 \times 1.2}} \right)$$

$$= 0.0079.$$

From Figures 1, 2, 5 and 6 for $b/a = 2$

$$\Delta_{11} = 0.40; \Delta_{12} = 2.4 \times 10^{-3}; \Delta_{21} = 6.0 \times 10^{-3} \text{ and } \Delta_{22} = 1.5 \times 10^{-4}.$$

Hence, the specific damping capacities are

$$\psi_{11} = 0.0787 \times 0.40 = 3.15 \times 10^{-2}; \psi_{12} = 1.9 \times 10^{-4}; \psi_{21} = 4.7 \times 10^{-4} \text{ and } \psi_{22} = 1.2 \times 10^{-5}.$$

The viscous damping ratios are:

$$\zeta_{11} = 0.25 \times 10^{-2}; \zeta_{12} = 1.5 \times 10^{-5}; \zeta_{21} = 3.75 \times 10^{-5} \text{ and } \zeta_{22} = 9.5 \times 10^{-7}.$$

3.1.3 Example 3

Consider the same panel as in Example 2, but use Figures 17, 18, 21 and 22 instead.

The panel has the material properties of the aluminium alloy used to derive Figures 17 to 32.

The geometrical parameters are: $h/\sqrt{ab} = 1.5 \times 10^{-3}/\sqrt{(0.6 \times 1.2)} = 1.768 \times 10^{-3}$ and $b/a = 1.2/0.6 = 2$.

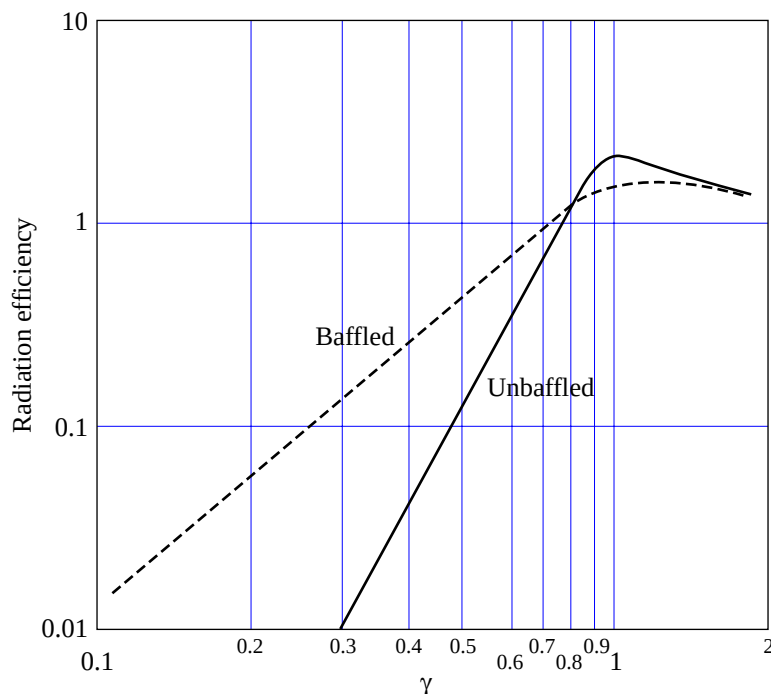
From Figures 17, 18, 21 and 22, the viscous damping ratios are found directly:

$$\zeta_{11} = 0.25 \times 10^{-2}; \zeta_{12} = 1.5 \times 10^{-5}; \zeta_{21} = 3.7 \times 10^{-5} \text{ and } \zeta_{22} = 9 \times 10^{-7}.$$

4. FACTORS AFFECTING DAMPING BY ACOUSTIC RADIATION

4.1 Effect of Baffles

The estimations of the modal radiation efficiencies in this Item make use of the Rayleigh integral. The sound pressure in the far field of the vibrating panel may be predicted by means of the Rayleigh integral, provided that the panel is surrounded by a rigid, infinite baffle. Such an assumption cannot be realised in practice. Numerical methods for the estimation of radiation efficiencies from unbaffled plates have been presented by Williams (Reference 12) and Laulagnet (Reference 17). For modes with a resonance frequency below the critical frequency the radiation efficiency of the baffled panel is greater than that of the unbaffled panel. For modes with a resonance frequency above the critical frequency the results for baffled and unbaffled panels are similar. The radiation efficiencies obtained by Laulagnet for a simply-supported plate (dimensions not given, but presumably square) are shown in Sketch 4.1 as a function of γ , the ratio of the frequency to the critical frequency, for the (1, 1) mode for both a baffled and unbaffled cases.



Sketch 4.1 Radiation efficiencies of a baffled and unbaffled, simply-supported square plate for the (1, 1) mode (Reference 17)

For $\gamma = 0.5$ the value of the radiation efficiency for the unbaffled panel is 0.0733 and for the baffled panel 0.355. Thus, for the case of an aluminium alloy panel ($a = 0.5$ m, $b = 0.5$ m and $h = 2$ mm) the viscous damping ratio for the unbaffled panel is approximately 0.41×10^{-4} and 0.2×10^{-2} for the baffled panel.

An unbaffled panel is completely unattached to any other structural component. In this sense the unbaffled panel is just as unrealistic as the baffled panel and the real situation would be somewhere between the two cases.

4.2 Effect of Panel Boundary Conditions

The radiation efficiencies used in the estimation of the viscous damping ratios are based on rectangular panels which are simply supported at all four edges. The extensive results provided by Wallace (Reference 5) are all for the simply-supported case. Maidanik (Reference 1) has reasoned intuitively that

the radiation efficiencies for clamped plates are twice those for the simply-supported plates. Gomperts (Reference 7) has considered radiation efficiencies for a wide range of different types of edge restraints, but did not consider the fully clamped or the fully free cases. For mixed boundary conditions for which two opposite edges were simply supported and the two other edges clamped, he found that the radiation efficiency was slightly greater than for the entirely simply-supported plate. Williams and Maynard (Reference 10) devised an approximate method for the fully clamped case and quoted results for a non-square plate in the (1, 1) and (6, 6) modes. Berry *et al* (Reference 15) have provided more extensive numerical results and have shown that although for the fully clamped plate the radiation efficiencies are generally greater than for the simply-supported plate, in the case of the modes (1, 1), (1, 2) and (2, 1) the reverse is the case. The viscous damping ratio, due to sound radiation, is influenced by the radiation efficiency and the resonance frequency of the plate, as indicated by Equation (2.5). Mead (Reference 19) has compared the specific damping capacity of the simple-supported and fully-clamped plate for the special case where the sound pressure is constant over the entire plate.

Leppington *et al* (References 9 and 11) have devised an asymptotic method for obtaining the radiation efficiencies of plates with all edges clamped. The results for the clamped case are derived from the radiation efficiencies for the simply-supported plate which are estimated also by an asymptotic method. As an asymptotic method is adopted, the modal radiation efficiencies for both the simply-supported and fully-clamped conditions are strictly accurate only for large values of ka and kb and high modal numbers. Comparison (Reference 20) of the results obtained by the asymptotic method (Reference 11) for clamped and simply-supported panels indicates that the radiation efficiency of the clamped panels increases for the higher modes, but the increase is less than the doubling previously suggested (References 1).

4.3 Radiation from one or two Sides

For the case when a panel is free to radiate sound into infinite space on either side of the baffled panel, the viscous damping ratio given by Equation (2.8) for one-sided radiation has to be multiplied by two. When one side of a panel is radiating into a wing or flap box, the situation is less clear. Some guidance may be obtained from Fahy (Reference 3) who considered the radiation of sound from an elastic panel which formed one side of a box the other sides of which are rigid. At a frequency which is below the critical frequency and above a limiting frequency when coupling between the structural and the acoustic modes is effective, the radiation efficiency is the same as that of a panel radiating into free space. At low frequencies the radiation efficiency depends upon γ^3 and so has low values for small γ . At frequencies above the critical frequency, σ is reported by Fahy to be half that of the freely radiating panel.

4.4 Fluid Loading of Panels

Sound radiation occurs only when a fluid is in contact with the panel. Fluid loading by an unbounded fluid on an infinite plate below the critical frequency has no effect on the damping, but results in an additional mass effectively attached to the plate (Reference 2). Above the critical frequency power is transferred from the plate to the fluid and a damping effect occurs (Reference 2).

Fluid loading of the panel can be explained in terms of a radiation impedance which has both a real and an imaginary part. The real part is directly related to the radiation of sound and the radiation efficiency. The radiation impedance has many features in common with the mechanical impedances of plates, which have been considered in ESDU 04009 (Reference 21). The impedances included in the ESDU compendium are for structures vibrating *in vacuo*. When a surface of the plate is in contact with a fluid and sound is radiated, the impedance of the plate, Z , is the sum of the radiation impedance Z_r , and the mechanical impedance, Z_m , i.e., $Z = Z_r + Z_m = R + iX$, where R and X are the real and imaginary parts of the total impedance. The radiation impedance may be written

$$Z_r = R_r + i X_r, \quad (4.1)$$

where R_r is the radiation resistance, which represents the loss of energy of the vibrating plate due to sound radiation, and X_r is the radiation reactance. The radiation resistance is given by

$$R_r = \rho_0 c_0 S \sigma, \quad (4.2)$$

where σ is the radiation efficiency of the vibrating plate. Below the critical frequency, the imaginary part of the radiation impedance is the radiation reactance

$$X_r = \rho_0 c_0 S \chi, \quad (4.3)$$

where χ is the nondimensional reactance. The imaginary part of the impedance of the plate for a particular mode (m, n) is the sum of the imaginary part of the mechanical impedance, ωm_p , and the radiation reactance, X_r . Thus,

$$X = \omega_{mn} m_p + \rho_0 c_0 S \chi_{mn} \quad (4.4)$$

or,

$$= \omega_{mn} (m_p + \rho_0 c_0 S \chi_{mn} / \omega_{mn}). \quad (4.5)$$

The second term of Equation (4.5), $\rho_0 c_0 S \chi_{mn} / \omega_{mn}$, is interpreted as an added mass to the plate due to the radiation of sound. The added mass is often called the virtual mass and is inversely proportional to the resonance frequency of the plate, ω_{mn} , for mode (m, n) . Hence, an increase in the virtual mass is associated with a decrease of the resonance frequency. There is the possibility that the effect of the virtual mass can be significant in the case of thin plates. As an example, consider an aluminium alloy plate (dimensions are $a = b = 0.5$ m and $h = 2$ mm) vibrating in air in the $(1, 1)$ mode. For this plate $\rho = 2800$ kg/m³ and hence $\omega_{11} = 243.2$ rad/s. The reactance, χ_{11} , depends upon γ , the ratio of acoustic wavenumber, k , to the bending wavenumber in an infinite plate, k_b . For the plate vibrating in mode $(1, 1)$, $\gamma = 7.95 \times 10^{-2}$. Chang and Leahey (Reference 8) indicate a value for χ_{11} of 4.53×10^{-2} for this value of γ . The added or virtual mass is 1.93×10^{-2} kg, which is small compared with the mass of the plate (1.4 kg). The effect of this virtual mass on the natural frequency, ω_{11} , is small ($\omega_{11} = 243.2 \sqrt{1.4/1.4045} = 242.8$ rad/s). Fluid loading may be neglected when the previously stated condition, $\rho_0 c_0 \ll \omega \rho h$, is satisfied. In the case of panels vibrating in air the reactive part of the radiation impedance can, generally, be neglected, but needs to be considered for panels radiating sound into water. If there is any effect of fluid loading, it is the lower order modes which are effected (Reference 22).

4.5 Effect of Shear

The viscous damping ratios estimated in this Item are limited to thin plates in which rotary inertia and shear deformation are neglected. The condition for no shear in a plate is $k_b h < 1$ (Reference 21). Hence, for the resonance modes of a simply-supported isotropic plate, $k_b = k_{mn} = (\pi^2 m^2 / a^2 + \pi^2 n^2 / b^2)^{1/2}$,

and, consequently,

$$\frac{\pi h}{\sqrt{ab}} \left(\frac{b}{a} m^2 + \frac{a}{b} n^2 \right)^{1/2} < 1$$

or,

$$\frac{h}{\sqrt{ab}} < \frac{1}{\pi \left(\frac{b}{a} m^2 + \frac{a}{b} n^2 \right)^{1/2}}.$$