



Crack growth criteria for the estimation of fatigue life

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CRACK GROWTH CRITERIA FOR THE ESTIMATION OF FATIGUE LIFE

CONTENTS

	Page
1. NOTATION	1
2. INTRODUCTION	5
3. STRESS INTENSITY FACTOR	6
3.1 Crack Tip Plastic Zone	10
3.2 Plasticity-induced Crack Growth Phenomena	13
4. FATIGUE CRACK GROWTH	14
4.1 Crack Growth in the Paris Region	17
4.2 Influence of Mean Stress on Crack Growth	20
5. MODELS OF FATIGUE CRACK GROWTH	22
6. FATIGUE CRACK GROWTH FOR VARIABLE AMPLITUDE LOADING	25
6.1 Non-interaction Models	25
6.2 Effective Stress Intensity Factor Range	25
6.3 Equivalent K – Concept	26
6.4 Characteristic K – Concept	28
7. FATIGUE LIFE	29
7.1 Examples	29
7.1.1 Example 1	29
7.1.2 Example 2	31
8. REFERENCES	32
APPENDIX A STRESS COMPONENTS RELATED TO STRESS INTENSITY FACTOR	36
A1.	36
A2. References	37
APPENDIX B CRACK GROWTH MODELS	38
B1. Forman Model	38
B2. Pearson Model	38
B3. Walker Model	38
B4. Davies and Feddersen model	39

B5.	Nicholson Model	40
B6.	Elber Model	40
B7.	Wheeler Model	41
B8.	Willenborg Model	43
B9.	NASGRO Model	44
B10.	Paris Model (for stationary random loading)	45
B11.	REFERENCES	46

CRACK GROWTH CRITERIA FOR THE ESTIMATION OF FATIGUE LIFE

1. NOTATION

		UNITS	
		SI	British
A_n	coefficients of crack opening function f ; $n = 0, 1, 2$ and 3		
a	crack length or crack half-length	m	in
a_c	critical crack length	m	in
a_{eff}	effective crack size	m	in
a_i	crack length at cycle i	m	in
a_f	final crack length	m	in
a_n	crack length after N cycles	m	in
a_{OL}	crack length at tensile overload application	m	in
a_0	initial crack length	m	in
b	width or half-width of sheet	m	in
C	material constant	m/cycle	m/cycle
C_r	retardation parameter		
C_{th}	material constant		
F_p	peak rate of time history	Hz	Hz
f_{ij}	angular components of stress tensor		
f	crack opening function (K_{op}/K_{max})		
h	thickness	m	in
i	numeration of cycles		
K	stress intensity factor	N/m ^{3/2}	lbf/in ^{3/2}
K_c	fracture toughness	N/m ^{3/2}	lbf/in ^{3/2}
K_{clr}	stress intensity factor at crack growth closure	N/m ^{3/2}	lbf/in ^{3/2}

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K_{eff}	effective stress intensity factor	N/m ^{3/2}	lbf/in ^{3/2}
K_I	stress intensity factor for mode I, see Sketch 3.1	N/m ^{3/2}	lbf/in ^{3/2}
K_{Ic}	plain strain fracture toughness (mode I)	N/m ^{3/2}	lbf/in ^{3/2}
K_m	mean stress intensity factor	N/m ^{3/2}	lbf/in ^{3/2}
K_{max}	maximum value of stress intensity factor in cycle	N/m ^{3/2}	lbf/in ^{3/2}
$K_{max, req}$	required stress intensity factor to terminate retardation	N/m ^{3/2}	lbf/in ^{3/2}
K_{min}	minimum value of stress intensity factor in cycle	N/m ^{3/2}	lbf/in ^{3/2}
K_{OL}	stress intensity factor at overload	N/m ^{3/2}	lbf/in ^{3/2}
K_{op}	stress intensity factor corresponding to stress S_{op}	N/m ^{3/2}	lbf/in ^{3/2}
K_r	= ΔK , stress intensity factor range in cycle	N/m ^{3/2}	lbf/in ^{3/2}
K_{rms}	root mean-squared stress intensity factor	N/m ^{3/2}	lbf/in ^{3/2}
K_0	stress intensity factor for crack of length $2a$ in infinite sheet	N/m ^{3/2}	lbf/in ^{3/2}
l	half length of sheet or beam	m	in
M	uniform moment per unit thickness	N	lbf
m	material constant, characteristic of slope of crack growth curve in log-log plot		
N	number of loading cycles		
N_f	number of loading cycles to failure		
p	Wheeler shaping component, material constant (NASGRO)		
p_{rfr}	probability density function for stress ranges of rainflow cycles	Pa ⁻¹	(lbf/in ²) ⁻¹
q	material constant (NASGRO)		
R	stress ratio		
R_{eff}	effective stress ratio		
R_{OL}	plastic zone extent caused by tensile overload	m	in
R_p	extent of plastic zone region	m	in
r_d	outer radius of K -dominance zone	m	in

r_p	radius of plastic zone	m	in
R_{pc}	extent of cyclic plastic zone region	m	in
r_{pc}	radius of cyclic plastic zone	m	in
R_{yi}	current plastic zone size at i^{th} cycle after tensile overload	m	in
$R_{y, req}$	extent of hypothetical current plastic zone	m	in
r	distance from crack tip	m	in
S	stress	Pa	lbf/in ²
S_c	applied nominal stress at crack instability	Pa	lbf/in ²
S_f	fatigue limit	Pa	lbf/in ²
S_m	mean stress	Pa	lbf/in ²
S_{max}	maximum stress in cycle	Pa	lbf/in ²
S_{min}	minimum stress in cycle	Pa	lbf/in ²
S_{op}	crack opening stress	Pa	lbf/in ²
S_r	stress range	Pa	lbf/in ²
S_{res}	compressive residual stress (Willenborg model)	Pa	lbf/in ²
S_{ri}	stress range for cycle i	Pa	lbf/in ²
S_{rms}	root mean-squared stress	Pa	lbf/in ²
S_y	yield stress	Pa	lbf/in ²
T_{fl}	fatigue life	s	s
U	effective stress range ratio (closure ratio)		
w	width	m	in
α	geometrical factor in stress intensity factor		
ΔK	stress intensity factor range	N/m ^{3/2}	lbf/in ^{3/2}
ΔK_{rms}	root mean-squared stress intensity factor range	N/m ^{3/2}	lbf/in ^{3/2}
ΔK_{th}	threshold stress intensity factor range	N/m ^{3/2}	lbf/in ^{3/2}

ΔK_0	material constant; threshold stress intensity factor range at $R = 0$	$\text{N/m}^{3/2}$	$\text{lbf/in}^{3/2}$
ΔS_{eff}	effective stress range	Pa	lbf/in^2
ΔS_{eq}	equivalent stress range	Pa	lbf/in^2
ΔS_f	fatigue limit range	Pa	lbf/in^2
ΔS_{th}	threshold stress range	Pa	lbf/in^2
δ	Irwin's plastic zone correction	m	in
κ	constraint factor		
θ	orientation angle		
σ_{ij}	components of stress tensor	Pa	lbf/in^2
σ_0	flow stress	Pa	lbf/in^2

Abbreviations used in this Item:

ASTM	American Society for Testing and Materials
CA	constant amplitude (loading)
FE	finite elements
LEFM	linear elastic fracture mechanics
$S - N$	stress versus number of cycles to failure
VA	variable amplitude (loading)

2. INTRODUCTION

This ESDU Item is an introduction to fracture mechanics as applied to the prediction of the life of components subjected to different cyclic deterministic and random excitations (for example, acoustic and thermal). Special attention is paid to acoustic excitation in the examples in Section 7 (References 14 and 50). In ESDU 06009 a stress-life analysis, associated with safe-life design, was discussed. In the present Item crack propagation models are analysed and their importance considered as essential elements in the damage-tolerant approach to fatigue damage. In designing against fatigue damage, safe-life and damage-tolerance are the two techniques most often used (Reference 56).

A safe-life structure possesses high resistance to the damaging effects of repeated loading. Its components are required to remain crack-free in service. The safe-life approach is normally used in situations where inspection is difficult or impossible to carry out, as in the case of an aircraft undercarriage. In this example the only time that the undercarriage can be inspected is when the aircraft is on the ground, *i.e.*, when the undercarriage is compressed and cracks are likely to be concealed. The design life is a fraction of the estimated life of the structural component. The estimated life is usually derived from the appropriate $S-N$ curve. It can also be obtained directly from practical tests on the actual component under the specific loading. Finally, a factor of safety must be introduced to account for the scatter in the test results and other unknown factors, as, for example, an unexpected change in the load spectrum. Usually, the design life is assumed to be one fifth (or less) of the estimated life of the component. At the end of its design life the component in service is replaced, even if no fatigue damage is observed.

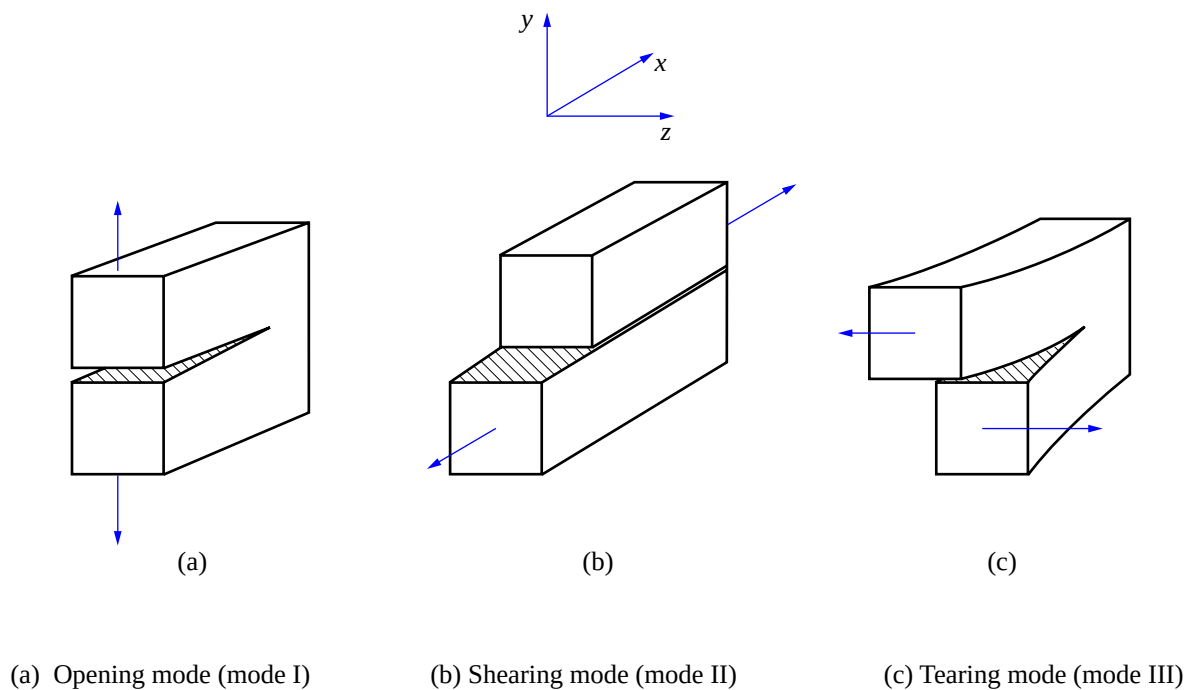
The second approach, damage-tolerance, incorporates the assumption that a flaw is present in a structural component from the outset of the damage process. Damage-tolerance methods employ fracture mechanics to analyse crack growth. The fatigue life in this case is defined in terms of the number of fatigue cycles to propagate the crack from the initial size to the critical size. The initial length of the crack refers usually to the largest flaw undetected by applied detection techniques with a defined resolution. For sufficiently low fracture toughness materials the critical size of the crack is derived from the fracture toughness and applied stress. In the case of materials of very high fracture toughness, the critical size of the crack can be estimated from the limit load for a particular component of the structure. It should be also noted that when the maximum stress intensity approaches the critical value, the crack extends in an unstable manner and fast fracture occurs. However, failure may occur earlier during the crack growth process, if the net section tensile stress achieves the value of the yield stress of the material. The damage-tolerance philosophy requires periodic inspections to detect cracks before they reach the size close to the critical.

The fatigue life of a structure or component consists of two stages. During the first stage there is a process of nucleation in which microcracks are initiated and grow. In the second stage of the process macrocracks are formed and propagate. In general, stress history, stress level, residual stresses and, finally, the geometry of the structure and the material properties decide which stage, either nucleation or crack growth, dominates the whole fatigue life of the structure. Crack initiation is essentially a surface phenomenon, contrary to the macrocrack growth process, which takes place throughout the depth of the material.

In the case of smooth, unnotched components subjected to a high cycle fatigue process, the fatigue life refers essentially to the duration of the process of initiation of small cracks, since the time of crack propagation can be neglected in comparison with the duration of crack initiation. Crack initiation is the major part of the life of coupons in the form of small beams, subjected, for example, to narrow band excitation as part of a test to simulate acoustic loading. When most of the life of the structure is taken up with the initiation of microcracks, the stress-life approach may be applied and $S-N$ curves used to determine the useful life of the structure. It is this method which is described in ESDU 06009 (Reference 53) in the ESDU Vibration and Acoustic Fatigue Series. Data for this approach may be found, for example, in ESDU 72015 (Reference 18) and ESDU 84027 (Reference 32), as well as in ESDU 72005 (Reference 17).

In this Item, the whole attention is focused on the fatigue crack propagation stage. The estimation of the fatigue life, based on the macrocrack growth stage, requires knowledge of the crack growth curve, *i.e.*, the relation between crack growth rate and the range of stress intensity factors. The crack growth models presented are based on the concepts of linear-elastic fracture mechanics (LEFM). Fracture mechanics introduces the concept of the stress intensity factor, K , also called the stress field parameter, which characterises the elastic stress distribution around the crack tip (References 5, 23, 25, 26, 29, 33, 37, 40, 41, and 45-48).

There are three fundamental modes of crack growth which are illustrated in Sketch 2.1 (References 33, 40, 46 and 48). In Mode I – the opening mode – there is symmetry about the xy and xz planes. In Mode II – the sliding mode or in-plane shearing mode – anti-symmetry exists about the xz plane and symmetry about the xy plane. Mode III – the tearing mode – has anti-symmetry about the xy and xz planes. The opening mode, associated with the moving apart of crack surfaces, is considered in this Item. Mode I is the most common kind of crack growth since the crack growth takes place predominantly in the plane of maximum tensile stress.



Sketch 2.1 Modes of crack growth

3. STRESS INTENSITY FACTOR

The concept of the stress intensity factor was introduced by Irwin (References 2 and 3) as a result of the development of the Griffith energy balance model (References 1, 33 and 48) in which the criterion for the onset of the unstable propagation of a crack in a plate of brittle material was formulated. The theory of elasticity predicts asymptotic fields of elastic stresses in the vicinity of a crack tip (at a distance r) as being (References 33 and 48)

$$\sigma_{ij}(r, \theta) = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) + \text{higher order terms, nonsingular at the crack tip}, \quad (3.1)$$

for each of the cracking modes. The components of the stress tensor are denoted as σ_{ij} .

The polar coordinates (r, θ) , relative to the crack tip, are specified in the plane of deformation. In the close vicinity to the crack tip, the higher order terms of Equation (3.1) are negligible compared with the first term. For different cracked structural components of linear and isotropic materials, the stress field around the stationary crack, defined in Equation (3.1), has the same form. The elastic normal and shear stresses in the vicinity of the crack tip are only functions of coordinates (r, θ) and similarity factor or scaling factor K , called also “crack tip intensity factor” or simply “stress intensity factor”. The spatial part, $f_{ij}(\theta)/\sqrt{2\pi r}$, of the first dominant term in Equation (3.1) consists of the radial component, $1/\sqrt{2\pi r}$, and the known angular component, $f_{ij}(\theta)$. The angular components, f_{ij} , of the stress tensor are different functions of θ for elastic normal and shear stresses (References 5 and 46). The angular components, as well as the normal and shear stresses, are specified for mode I in Appendix A. The magnitude of the stress at the defined position (r, θ) depends only on the factor K and for this reason K is described as a stress intensity factor.

The influence of both the structural loading, and the structure’s geometrical configuration, is represented by the stress intensity factor, K . Hence, the stress intensity factor, in the case of the opening mode I, denoted as K_I , depends on the loading and the geometry of the cracked structure, such as the boundaries of the structure, and the size, location and shape of the crack. It should be noted that the stress field defined in Equation (3.1) has a singularity at a distance $r = 0$, which does not reflect the physical reality at the crack tip, since there is a small region of plastic flow. If the plastic yielding zone is small in comparison with the region of high stress in the vicinity of the crack, the stress field is adequately described by Equation (3.1) in the framework of LEFM. The annular zone of high stress, described uniquely by the stress intensity factor, is often called “the region of K – dominance”, see Sketch 3.1 in Section 3.1. The inner boundary of this region is marked by a plastic zone of radius r_p , the outer boundary is determined by the distance r_d at which higher terms cannot be neglected in comparison with the first (asymptotic) term. Usually, it is assumed (Reference 41) that it takes place at a distance where the asymptotic term of Equation (3.1) deviates more than 10% from the linear, elastic solution for the stress field around the crack represented by Equation (3.1).

The significance of the concept of the stress intensity factor is associated with the similarity principle and with the possibility to define the equivalence between the responses of two different structures of the same material subjected to different types of loading. If the resulting stress intensity factors are similar in both cases, it would be expected that the cracks would advance in a similar manner. The stress intensity factor has dimensions of stress $\times$ length^{1/2} and is conventionally expressed in $\text{MNm}^{-3/2}$ or $\text{MPa m}^{1/2}$.

For many cracked structures with different types of loading there exist analytical and numerical solutions for the stress intensity factor. An analytical solution exists for a crack in an infinite sheet under uniform, uniaxial tensile stress. For other structures, such as a crack in a finite sheet, the solutions for the stress intensity factors are approximate. The compendium of Rooke and Cartwright (Reference 23) provides information on stress intensity factors for a large number of structures with different loadings. Some stress intensity factors for basic structures under loading are given in Table 3.1.

The opening mode (mode I) stress intensity factor is usually presented in the form

$$K_I = S(\pi a)^{1/2} \alpha \quad (3.2a)$$

or,

$$K_I = K_0 \alpha \quad (3.2b)$$

where $K_0 = S(\pi a)^{1/2}$ is the stress intensity factor for a crack of length $2a$ in an infinite sheet, subjected to a uniform uniaxial tensile stress, S , and α denotes a factor of the order of unity, which depends on the geometry of the structure and the structural loading. The dimensionless factor α is called the geometry

factor (References 46 and 47) or geometry correction factor (Reference 47). Stress intensity factors for different structures with cracks and for different loadings are very often presented in the dimensionless form K/K_0 , as in the compendium of stress intensity factors (Reference 23).

The stress intensity factor of a complex, cracked structure under load can be obtained by superposition of the known stress intensity factors of simpler structures with different boundary conditions and loading (References 35 and 47). The technique of superposition is the simplest and most common approach. The compounding method is also used and is described in an ESDU Item (Reference 25). However, when more accurate values of the stress intensity factor are required, the FE technique or measurements should be applied (Reference 47).

The stress intensity factor range is the most important factor which controls fatigue crack growth. The fatigue crack growth rate is usually presented in a form related to the applied number of cycles, N , the range of stress intensity factor in a cycle, $\Delta K = K_{max} - K_{min}$ and the stress ratio, R , for the cycle, where $R = S_{min}/S_{max} = K_{min}/K_{max}$. See, for example, Sketch 6.1 for a graphic interpretation of ΔK and R . Expressions for the crack growth rate may be expressed as

$$\frac{da}{dN} = f(\Delta K, R) \quad (3.3)$$

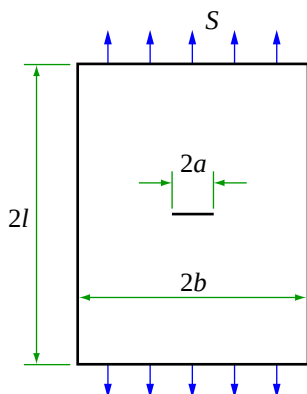
or,

$$\frac{da}{dN} = F(\Delta K) \quad (3.4)$$

where the function F depends on the stress ratio, R . Equation (3.3) expresses the similarity principle which states that for similar cyclic conditions (the same stress intensity factor range and R) the rate of crack growth will be the same in different structures of the same material.

TABLE 3.1 Some Stress Intensity Factors

Central crack in rectangular sheet of finite width in tension



$$\begin{aligned} \frac{K_I}{K_0} &= \frac{1 - 0.5\frac{a}{b} + 0.326\left(\frac{a}{b}\right)^2}{\sqrt{1 - \frac{a}{b}}} \\ &= \sqrt{\sec\left(\frac{\pi a}{2b}\right)} \quad (\text{approximate}) \end{aligned}$$

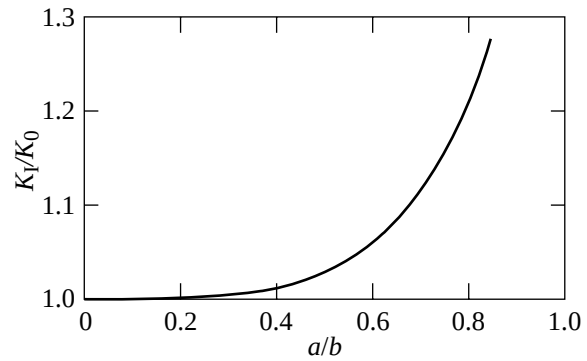
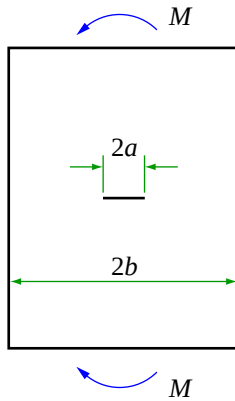
$$\text{for } \frac{a}{b} < 0.85, \text{ where } K_0 = S\sqrt{\pi a}.$$

See Compendium (Reference 23), Section 1.1.1, page 10.

TABLE 3.1 Some Stress Intensity Factors (Continued)

Central crack in rectangular sheet of finite width in bending

M is uniform moment per unit thickness

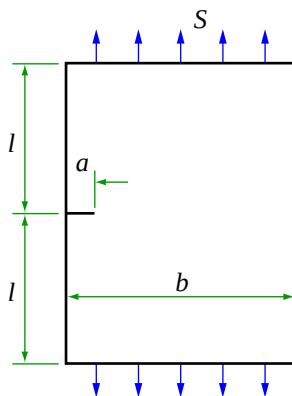


$$K_0 = \frac{3\sqrt{\pi}Ma^{3/2}}{4b^3} = \frac{3Ma\sqrt{\pi a}}{4b^3}$$

Tensile stress must keep crack open.

See Compendium (Reference 23), Section 1.1.2, page 12.

Single edge crack in a finite width sheet in tension

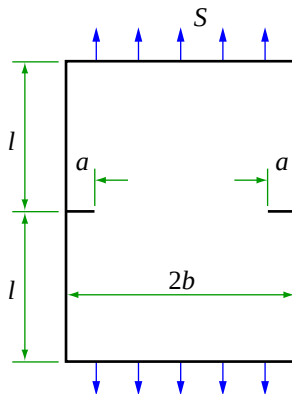


$$\frac{K_I}{K_0} = 1.12 - 0.23\left(\frac{a}{b}\right) + 10.6\left(\frac{a}{b}\right)^2 - 21.7\left(\frac{a}{b}\right)^3 + 30.4\left(\frac{a}{b}\right)^4$$

for $\frac{l}{b} \geq 1.0$ and $\frac{a}{b} \leq 0.6$, where $K_0 = S\sqrt{\pi a}$.

See Compendium (Reference 23), Section 1.1.20, page 84.

Edge cracks in a finite width sheet in tension



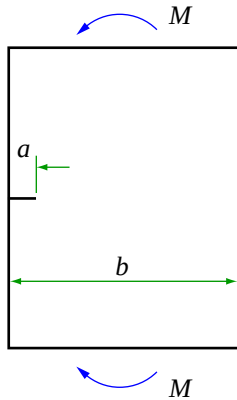
$$\frac{K_I}{K_0} = 1.12 + 0.20\left(\frac{a}{b}\right) - 1.20\left(\frac{a}{b}\right)^2 + 1.93\left(\frac{a}{b}\right)^3$$

for $\frac{l}{b} \geq 3.0$ and $0 < \frac{a}{b} \leq 0.7$, where $K_0 = S\sqrt{\pi a}$.

See Compendium (Reference 23), Section 1.2.1, page 110.

TABLE 3.1 Some Stress Intensity Factors (Continued)

Single edge crack in a finite width sheet in bending



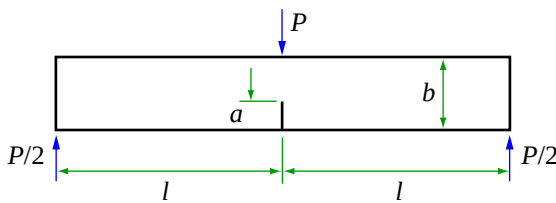
$$\frac{K_I}{K_0} = 1.12 - 1.39\left(\frac{a}{b}\right) + 7.32\left(\frac{a}{b}\right)^2 - 13.1\left(\frac{a}{b}\right)^3 + 14.0\left(\frac{a}{b}\right)^4$$

$$\text{for } \frac{a}{b} \leq 0.6, \text{ where } K_0 = \frac{6M\sqrt{\pi a}}{b^2}$$

and M is the bending moment per unit thickness.

See Compendium (Reference 23), Section 1.1.21, page 86.

Edge crack in three-point bending



$$\text{For } \frac{l}{b} = 4,$$

$$\frac{K_I}{K_0} = 1.11 - 1.55\left(\frac{a}{b}\right) + 7.71\left(\frac{a}{b}\right)^2 - 13.5\left(\frac{a}{b}\right)^3 + 14.2\left(\frac{a}{b}\right)^4$$

$$\text{and for } \frac{l}{b} = 2,$$

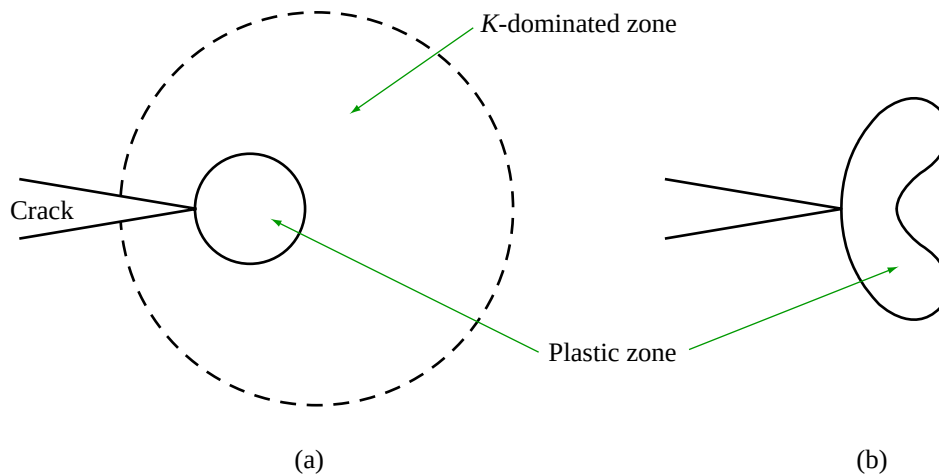
$$\frac{K_I}{K_0} = 1.09 - 1.73\left(\frac{a}{b}\right) + 8.20\left(\frac{a}{b}\right)^2 - 14.2\left(\frac{a}{b}\right)^3 + 14.6\left(\frac{a}{b}\right)^4,$$

where $K_0 = \frac{6M\sqrt{\pi a}}{b^2}$ and $M = \frac{Pl}{2}$ and P is the applied load per unit thickness.

See Compendium (Reference 23), Section 1.1.21, page 86.

3.1 Crack Tip Plastic Zone

The stress intensity factor describes not only the stress field in the K -dominated zone in the vicinity of the crack tip but also the crack tip plastic zone. The plastic zone at the crack tip is shown in idealized form in Sketch 3.1a as a circular region. In reality the contours of constant values of stress in the case of plane stress for the plastic zone resemble a “pair of wings”, rather than a circle (References 8 and 33), as indicated in Sketch 3.1b.



Sketch 3.1 Plastic zone at the crack tip

Cracks of different lengths, occurring in different components, have the same stress field and identical sizes of the crack tip plastic zone, if they have the same stress intensity factor, K_I . If two cracks have the same extent of the crack tip plastic zone and identical stresses acting on the boundary of the plastic zone, it would be expected that the stresses and strains within the yield zone would be the same in both cases. It is important to be able to estimate the size of the plastic zone, as LEFM would be invalidated if the plastic region were too large. Estimates of the size of the crack tip plastic zone have been made by Irwin (References 33 and 41). Irwin argued that, due to the presence of the crack tip plastic zone, the fatigue crack behaves as if it were longer than it is in reality. The effective crack size in a component under plane stress* has been defined as

$$a_{eff} = a + \delta, \quad (3.5)$$

where

$$\delta = \frac{K_I^2}{2\pi S_y^2}$$

is the plastic zone correction and S_y is the yield stress. However, in the case when the plastic zone is small in comparison with the length of the crack, it is not necessary to take Irwin's correction into account.

For monotonic loading, (from minimum to maximum stress during a cycle), the extent of the plastic plane stress zone obtained using Irwin's approach is proportional to the square of the stress intensity factor,

$$R_p = 2r_p = 2\delta = \frac{1}{\pi} \left(\frac{K_I}{S_y} \right)^2, \quad (3.6)$$

where r_p is the plastic zone radius.

* Expressions for the extent of the plastic zone depends upon whether plane stress or plane strain exist. In the case of the plane stress conditions, stress components exist in, say, the xy-plane, but there is no stress in the z-direction. A state of plane stress may occur in a structure in which the dimension in the z-direction is much smaller than the dimensions in the x and y-directions. In plane strain there are strain components in the xy-plane and no strain in the z-direction, although a stress component occurs in the z-direction. Many expressions in fracture mechanics apply only to the states of plane stress and plane strain.

For the case of plane strain, the plastic zone under monotonic loading is of extent

$$R_p = \frac{1}{3\pi} \left(\frac{K_I}{S_y} \right)^2 \quad (3.7)$$

Another approach, due to Dugdale (References 4 and 46), provides an asymptotic estimation of the crack tip plastic zone size under conditions of plane stress as

$$R_p = 2r_p \approx \frac{\pi}{8} \left(\frac{K_I}{S_y} \right)^2 \quad (3.8)$$

Both the Irwin and Dugdale models provide results agreeing to within 20%.

In the case of cyclic loading, during the first stage of the cycle (from minimum to maximum loading) the monotonic plastic zone of the extent described by Equations (3.6) or (3.8) is developed due to yielding in tension. However, during the unloading stage, part of the monotonic plastic zone is in compression (References 41 and 46). This part of the plastic zone is described as a cyclic plastic zone of size, R_{pc} , given by

$$R_{pc} = 2r_{pc} = \frac{1}{4\pi} \left(\frac{\Delta K}{S_y} \right)^2 \quad (3.9)$$

for plane stress conditions and

$$R_{pc} = 2r_{pc} = \frac{1}{12\pi} \left(\frac{\Delta K}{S_y} \right)^2 \quad (3.10)$$

for plane strain conditions, where S_y is the yield stress and $\Delta K = K_{max} - K_{min}$ in constant amplitude loading. For materials which cyclically harden or soften the yield stress, S_y , is replaced by the cyclic yield stress.

The stress intensity factor has a meaning only if the plastic zone is much smaller in comparison with the length of the crack and the geometrical dimensions of the component. The methods of LEFM can then be used. In general, the plastic zone is small when the maximum loading stress is less than the yield stress: $S_{max} < 0.8 S_y$, or more specifically when the net section stress of the component does not exceed $0.8 S_y$. The restrictions in using LEFM approach in the context of the extent of the plastic zone (References 46, 48 and 49) are presented in terms of the geometrical parameters of the cracked component, such as the crack length, a , and the component width, w , as: $R_p \leq a/4$ and $(w-a) \geq 4R_p$, for the case of a specimen with an edge crack and under monotonic loading. The length of the component uncracked ligament at the crack growth plane is denoted by $(w-a)$, where w (width) is the component dimension parallel to the crack propagation direction and perpendicular to the tension which causes the cracking.

The relation between the extent of the plastic zone and the thickness, h , of the specimen indicates the specimen state, either plane strain or plane stress. When $h \ll 2.5(K_I/S_y)^2$, i.e., when the plastic zone of size, R_p , is compared with the thickness, h , of the component, a state of plane stress exists in the component (Reference 45). For the case of $h \geq 2.5(K_I/S_y)^2$ in the majority of metal specimens the state of plane strain prevails around the crack tip.