



ESDU 10003

Issued May 2013
With Amendments A and B
September 2015

Worked example for tyre in linear motion on a contaminated runway surface. An application of ESDU 05011



**ROYAL
AERONAUTICAL
SOCIETY**

Endorsed by
The Royal Aeronautical Society

AVA Documents
Document Sharing Hub

ESDU DATA ITEMS

Data Items provide validated information in engineering design and analysis for use by, or under the supervision of, professionally qualified engineers. The data are founded on an evaluation of all the relevant information, both published and unpublished, and are invariably supported by original work of ESDU staff engineers or consultants. The whole process is subject to independent review for which crucial support is provided by industrial companies, government research laboratories, universities and others from around the world through the participation of some of their leading experts on ESDU Technical Committees. This process ensures that the results of much valuable work (theoretical, experimental and operational), which may not be widely available or in a readily usable form, can be communicated concisely and accurately to the engineering community.

We are constantly striving to develop new work and review data already issued. Any comments arising out of your use of our data, or any suggestions for new topics or information that might lead to improvements, will help us to provide a better service.

THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Performance Committee. This Committee first met in 1946 and now has the following membership:

Chairman	
Mr G.J.R. Skillen	— Independent, UK
Vice-Chairman	
Mr K.J. Balkwill	— Independent, UK
Members	
Mr P.V. Aidala*	— Northrop Grumman ESID, Bethpage, New York, USA
Dr J.D. Avery	— Independent, UK
Mr M. Baxter	— BAE SYSTEMS, Brough, UK
Mr M. Broad	— Independent, UK
Mr A. Dalton	— MOD, Boscombe Down, UK
Mr G.W.H. van Es	— National Aerospace Laboratory NLR, The Netherlands
Mr N.J. Herniman*	— Independent, France
Mr L. Kornstaedt	— Airbus Industrie, Toulouse, France
Mr D.A. Law	— Civil Aviation Authority, Gatwick, UK
Mr P.E.C.S. Magalhaes*	— Embraer, Brazil
Mr R.J. Orlowski*	— Boeing Commercial Airplane Company, Seattle, Washington, USA
Dr P. Render	— Loughborough University, UK
Mr G.E. Smith	— Independent, UK
Mr R. Storey	— Independent, UK
Captain D. Thomas	— Independent, UK
Mr S. Tsach*	— Israel Aircraft Industries, Jerusalem, Israel
Mr C.J. Turner*	— Boeing Commercial Airplane Company, Seattle, Washington, USA.

* Corresponding Member

The construction and subsequent development of the Data Item were undertaken by

Mr G.J.R. Skillen	— Consultant to ESDU
Mr R.S.D. Sayers	— Senior Engineer.

The person with overall responsibility for the work in this subject area is C.J. Loughton, Head of Performance, Mechanical Motion and System Dynamics Groups.

WORKED EXAMPLE FOR TYRE IN LINEAR MOTION ON A CONTAMINATED RUNWAY SURFACE. AN APPLICATION OF ESDU 05011

CONTENTS

	Page
1. INTRODUCTION	1
2. ASSUMPTIONS	2
2.1 Aircraft and Tyre Information	2
2.2 Runway Conditions	3
2.3 Ambient Conditions	4
2.4 Speed Range	4
2.5 Planing of tyre	5
3. DETERMINATION OF ROLLING RESISTANCE COEFFICIENT AND BRAKING COEFFICIENT OF FRICTION	5
4. CALCULATION OF BRAKING AND ROLLING FORCES ACTING ON TYRE	8
5. CALCULATION OF DRAG FORCES ACTING ON TYRE	10
6. CALCULATION OF TOTAL RESISTIVE FORCE ACTING AGAINST BRAKED TYRE DUE TO PRESENCE OF CONTAMINANT	12
7. PERFORMANCE INFORMATION	13
8. REFERENCES	13
APPENDIX A – DETAILED SOLUTION FOR WORKED EXAMPLE	14
A.1 Rolling Resistance for Tyre on Paved Surface	14
A.1.1 Rolling Resistance for Tyre on Dry Paved Surface when $T_G = 15^\circ\text{C}$	15
A.1.2 Rolling Resistance for Tyre on Water-covered Surface when $T_G = 10^\circ\text{C}$	16
A.1.3 Rolling Resistance for Tyre on Slush-covered Surface when $T_G = 5^\circ\text{C}$	16
A.1.4 Rolling Resistance for Tyre on Snow-covered Surface when $T_G = 0^\circ\text{C}$	17
A.1.5 Rolling Resistance for Tyre on Ice-covered Surface when $T_G = -5^\circ\text{C}$	17
A.2 Calculation of Drag Forces Acting on Tyre	17
A.2.1 Calculation of contaminant drag force when runway is covered by layer of water or slush	18
A.2.2 Calculation of contaminant drag force when runway is covered by layer of loose snow	23

100

WORKED EXAMPLE FOR TYRE IN LINEAR MOTION ON A CONTAMINATED RUNWAY SURFACE. AN APPLICATION OF ESDU 05011

An aircraft manufacturer wishes to estimate braking and rolling-drag forces on a single tyre of a main gear on a transport aeroplane.

1. INTRODUCTION

This Data Item forms part of a series of worked examples. These examples illustrate application of the method for estimating resistive forces acting against a single aircraft tyre operating on a dry or contaminated runway surface described in Reference 3. A worked example, featuring continuous friction measuring equipment (CFME), is given in Reference 7, whilst Reference 10 is a worked example that illustrates the influence that the water content of snow has on the resistive forces acting on a tyre. A complete description of the mathematical model summarised in Reference 3 is given in Reference 5. Background information relating to the application of the model can be found in Reference 6.

The procedure for analysing the resistive forces acting on a multiple-wheel aircraft undercarriage operating on a contaminated runway surface is given in Reference 4. An example of its application can be found in Reference 8.

To establish aircraft performance when operating on a contaminated runway, without carrying out expensive flight testing, the forces acting on an aircraft tyre caused by the contaminant can be calculated as set out in this example.

This example illustrates a possible use of Reference 3 and considers a limited selection of cases. In practice a wider range of runway and ambient conditions would be considered and more detailed consideration given to some parameters, for example, in the following,

- the variation of the normal loads, Z , with aircraft ground speed, V_G , would be different for the acceleration and stopping cases,
- the behaviour of the anti-skid system at low speeds, where the design parameters may be different,
- the effects of brake torque and energy limits on braking forces - it should be remembered that brake energy limits usually occur at very high aircraft weights or at high ambient temperatures or at high altitude airfields, conditions not usually associated with contaminated runway operations,
- spray drag and skin friction aspects of aircraft performance are not addressed here (see Reference 1).

The following calculation process has been implemented in a computer program. This program encompasses all of the methods relating to aircraft tyres operating on dry and contaminated runway surfaces described in Reference 3. Details of the computer program are given in Reference 12.

This example involves calculating braking coefficient of friction values when the tyre specified in Section 2.1 is operating on a dry runway surface, and on the same runway surface when it is covered by the various layers of contaminant. Subsequently, associated braking forces and drag forces due to the presence of contaminant are calculated. It is assumed that the wheel is not subject to a lateral force.

Note that the values of all variables listed must be converted to a coherent set of units as specified in Reference 3.

Issued May 2013 – 35 pages
With Amendments A and B, September 2015 – 42 pages
This page Amendment B

2. ASSUMPTIONS

2.1 Aircraft and Tyre Information

The relevant tyre data are listed in Table 2.1. The tyre is assumed to have a **rib-tread**.

The Notation used throughout this Data Item is given in References 3 and 9.

TABLE 2.1 Aircraft tyre data

Tyre parameter	Value	Unit
D	41 (3.4167)	in (ft)
w	14 (1.1667)	in (ft)
p	160 (23040)	lbf/in ² (lbf/ft ²)
d_{TREAD}	0.4 (0.0333)	in (ft)
p_{ABS}	25156.2	lbf/ft ²
μ_{REF}	1.02 [†]	—

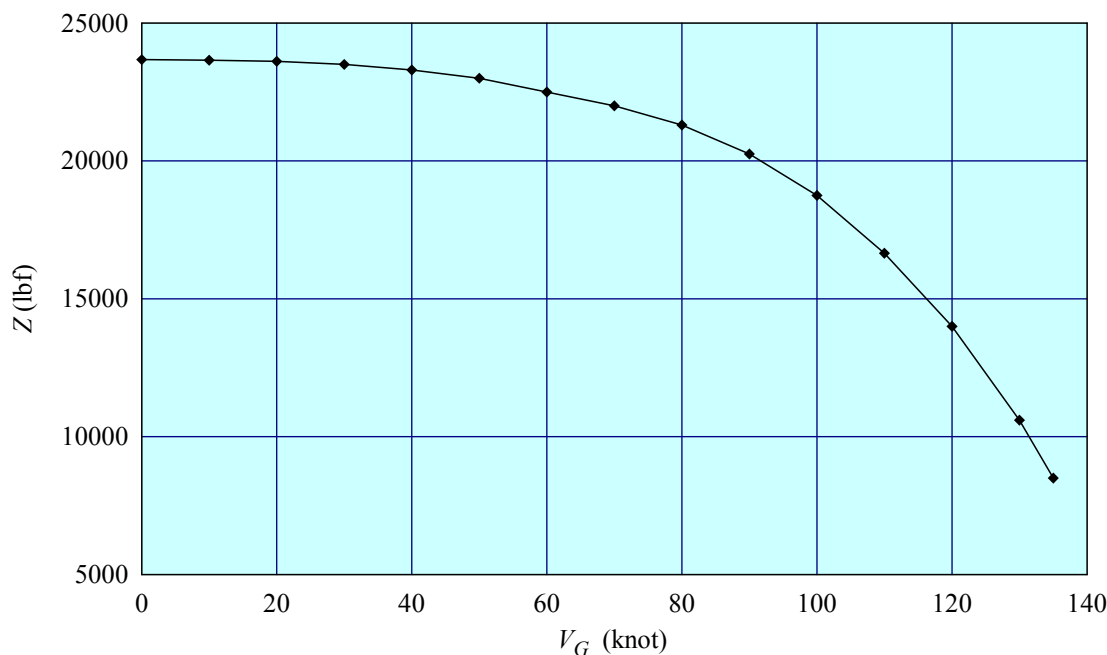
[†] This value is obtained from Table 5.1 of Reference 3.

The normal load acting on the tyre, Z , varies with aircraft ground speed, V_G , as listed in Table 2.2 (see also Sketch 2.1).

TABLE 2.2 Variation of normal load, Z , acting on tyre with ground speed, V_G

V_G (knot)	Z (lbf)
0	23675
10	23650
20	23610
30	23500
40	23300
50	23000
60	22500
70	22000
80	21300
90	20250
100	18750
110	16650
120	14000
130	10600
135	8500

This page Amendment B



Sketch 2.1 Variation of normal load, Z , acting on tyre with ground speed, V_G

The aircraft braking system has a relationship between braking slip ratio, s , and aircraft ground speed, V_G , governed by Equation (2.1). When V_G is equal to or greater than 30 knots (50.634 ft/s), the translation speed of the tyre footprint, V , is maintained at a constant rate of 6 knots (10.127 ft/s). When V_G is lower than 30 knots (50.634 ft/s), the anti-skid system is disengaged to enable low-speed manoeuvring and parking[†].

$$s = V/V_G \tag{2.1}$$

2.2 Runway Conditions

(a) Layers of contaminant covering runway surface

The layers of contaminant to be considered in this example are listed in Table 2.3.

(b) Runway temperature

The runway surface temperature, T_G , is 15° C, the ISA sea level temperature, when the runway surface is dry.

Values of T_G for when the runway is covered by a layer of contaminant are listed in Table 2.3.

(c) Macro-texture depth of runway surface

The macro-texture depth of the runway surface, d_{TEX} , is 0.03 in (2.5×10^{-3} ft).

This value is assumed for each instance.

[†] The relationship between V_G and V described is specific to the anti-skid system of the aircraft being considered. Other anti-skid systems may feature different characteristics.

2.3 Ambient Conditions

For all cases ISA, Sea Level pressure (2116.2 lbf/ft², 1013.2 mb) is assumed.

For wet runway cases, ISA, Sea Level temperature (288.15 K) is assumed.

For loose snow cases, $T = -1\text{ }^{\circ}\text{C}$ (272.15 K) is assumed for the contaminant temperature.

TABLE 2.3 Layers of contaminant considered for analyses

Contaminant type	ρ_C (slug/ft ³)	d_C (or d_{SNOW})	T (°C)	T_G (°C)	$\text{Pr}[z]^{\dagger}$ (%)	z
Water	1.94	3×10^{-3} m (9.8425×10^{-3} ft)	–	10	–	–
Water	1.94	15×10^{-3} m (4.9212×10^{-2} ft)	–	10	–	–
Slush	1.55	3×10^{-3} m (9.8425×10^{-3} ft)	–	5	–	–
Slush	1.55	15×10^{-3} m (4.9212×10^{-2} ft)	–	5	–	–
Slush	1.55	25×10^{-3} m (8.2021×10^{-2} ft)	–	5	–	–
Loose snow	0.194	10×10^{-3} m (3.2808×10^{-2} ft)	–1	0	5	–1.6449
Loose snow	0.194	25×10^{-3} m (8.2021×10^{-2} ft)	–1	0	5	–1.6449
Loose snow	0.194	50×10^{-3} m (0.1640 ft)	–1	0	5	–1.6449
Loose snow	0.388	10×10^{-3} m (3.2808×10^{-2} ft)	–1	0	5	–1.6449
Loose snow	0.388	25×10^{-3} m (8.2021×10^{-2} ft)	–1	0	5	–1.6449
Loose snow	0.388	50×10^{-3} m (0.1640 ft)	–1	0	5	–1.6449
Ice (solid)	–	–	–	–5	–	–

[†] $\text{Pr}[z]$ denotes the percentile from the Normal distribution that is assumed when deriving the reference coefficient of friction for the case where a tyre is braked on a runway surface that is covered by a layer of loose snow. In a practical application, a value appropriate for the situation should be selected.

2.4 Speed Range

The contaminant drag forces, rolling resistance forces and braking forces, G_{FLUID} or G_{SNOW} , G_{ROLL} and G_{BRAKE} respectively, acting against the tyre are analysed over the ground speed, V_G , range of $30 \leq V_G$ (kn) ≤ 135 . When $0 \leq V_G$ (knot) < 30 , G_{BRAKE} has a value of zero (see Section 2.1).

2.5 Planing of tyre

The partial and complete hydroplaning speeds for the tyre, V_S and V_H respectively, are listed in Table 2.4 for a water depth of 15×10^{-3} m. These values were calculated using the method described in Reference 9. (An application of the method is outlined in Reference 11.) Note that this method is only applicable when $d_C \geq d_{TREAD}$.

As the operating speed covers the range $30 \leq V_G$ (kn) ≤ 135 , partial hydroplaning of the tyre, followed by complete hydroplaning, will occur for the operation where $d_C = 15 \times 10^{-3}$ m as the V_S and V_H values for the operation are 89.9 kn and 125.7 kn respectively. When a tyre experiences partial hydroplaning spin-down will occur, there is a significant reduction in the water spray trajectory generated by it, and the fluid drag force acting on it will reach a peak value. When a tyre experiences complete hydroplaning it will lose contact with the runway surface. The braking force and rolling resistance will consequently reduce to zero under such circumstances.

Note that information related to hydroplaning is not included in the sketches that appear in Sections 3 to 6, as these sketches illustrate coefficients and resistance forces for a water depth of 3×10^{-3} m, which is lower than the d_{TREAD} value specified in Table 2.1.

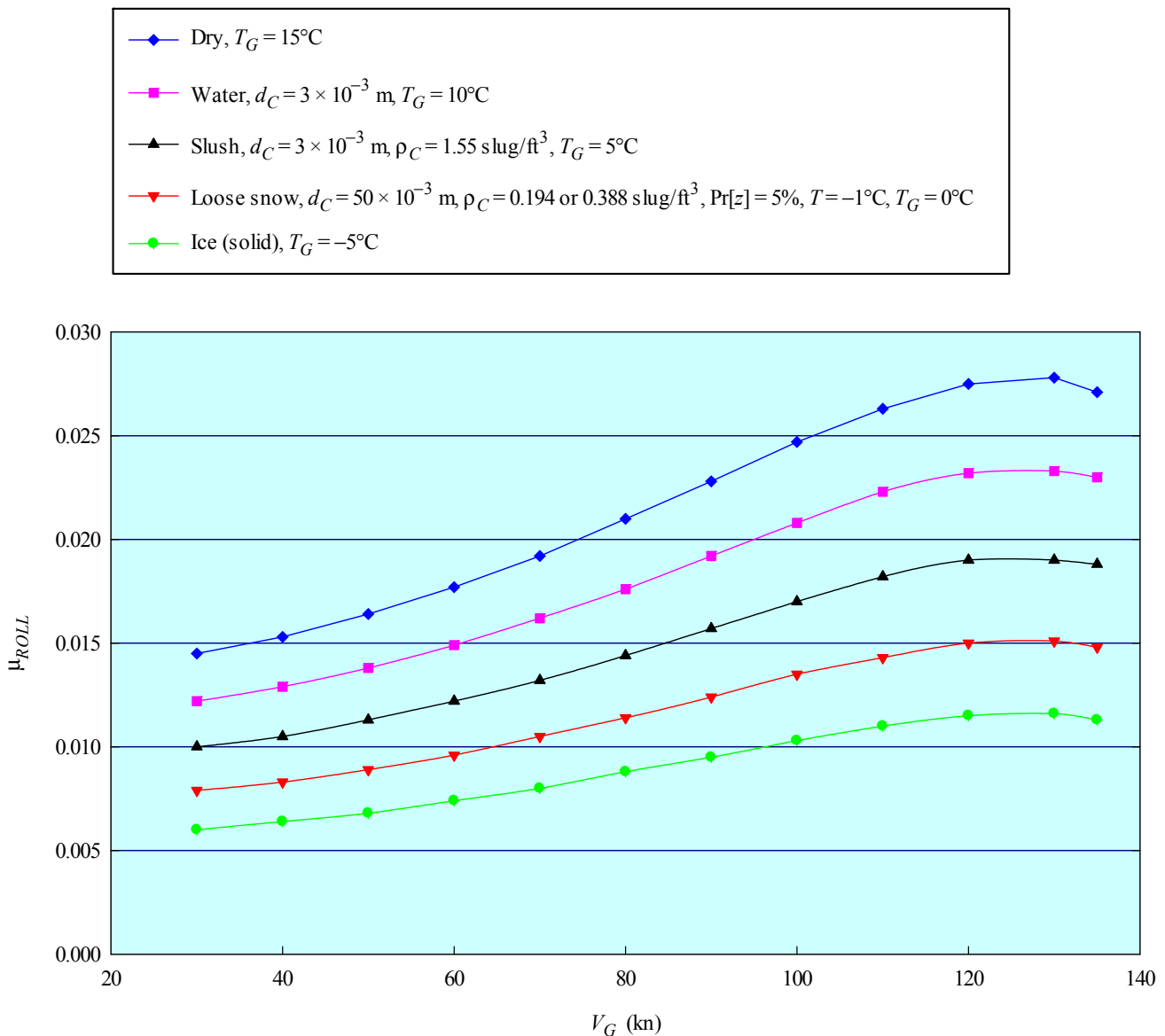
TABLE 2.4 Partial and complete hydroplaning speeds of tyre for water depth specified

d_C ($\times 10^{-3}$ m)	V_S (kn)	V_H (kn)
15	89.9	125.7

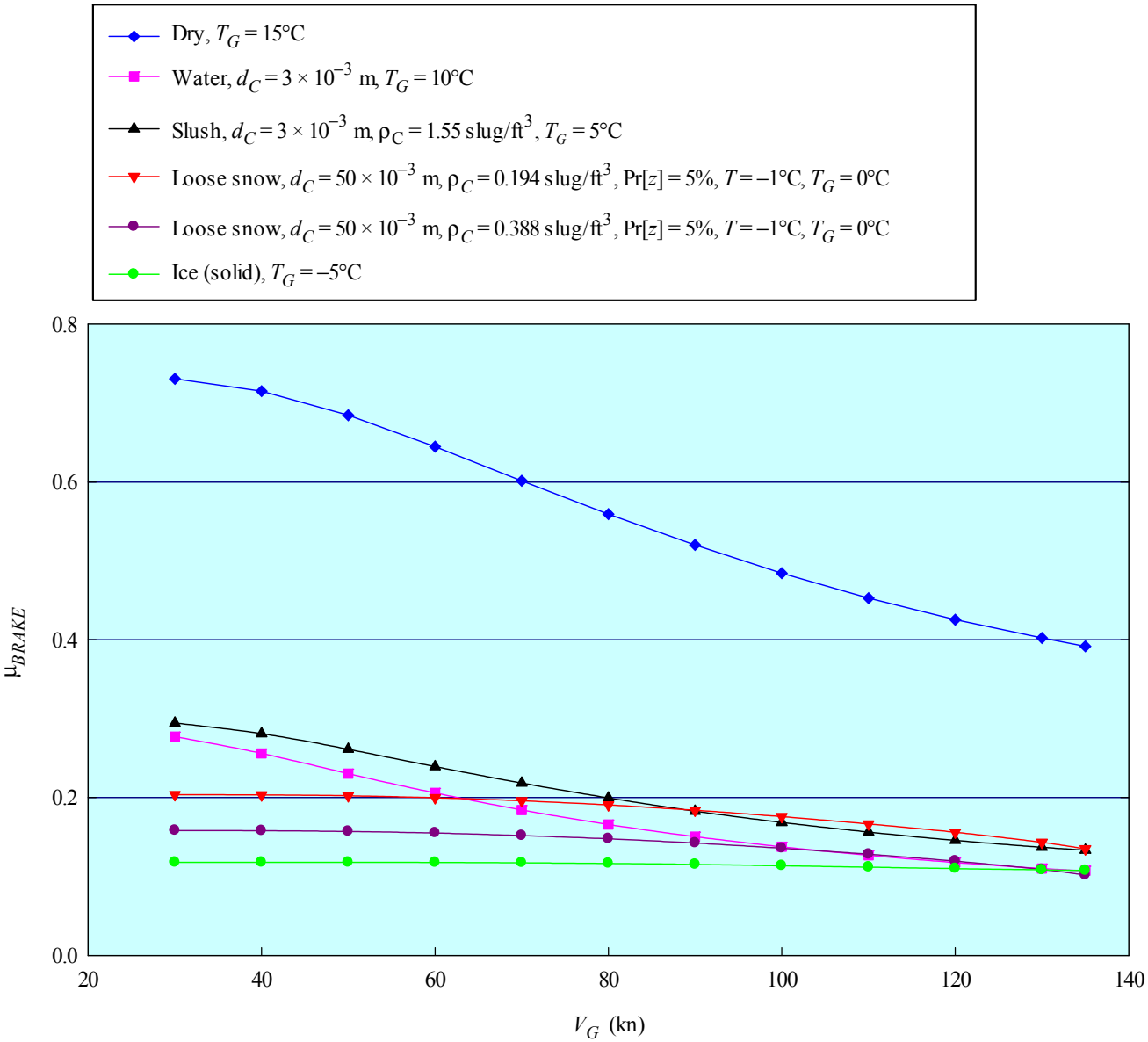
3. DETERMINATION OF ROLLING RESISTANCE COEFFICIENT AND BRAKING COEFFICIENT OF FRICTION

The rolling resistance coefficient, μ_{ROLL} , and the braking coefficient of friction, μ_{BRAKE} for the specified conditions can be calculated over the desired range of speeds, using the analysis process detailed in Appendix A, as shown in Sketches 3.1 and 3.2. (Note that a number of cases listed in Table 2.3 have been omitted for the purpose of clarity.)

Refer to Section 2.5 for the definitions of V_S and V_H .



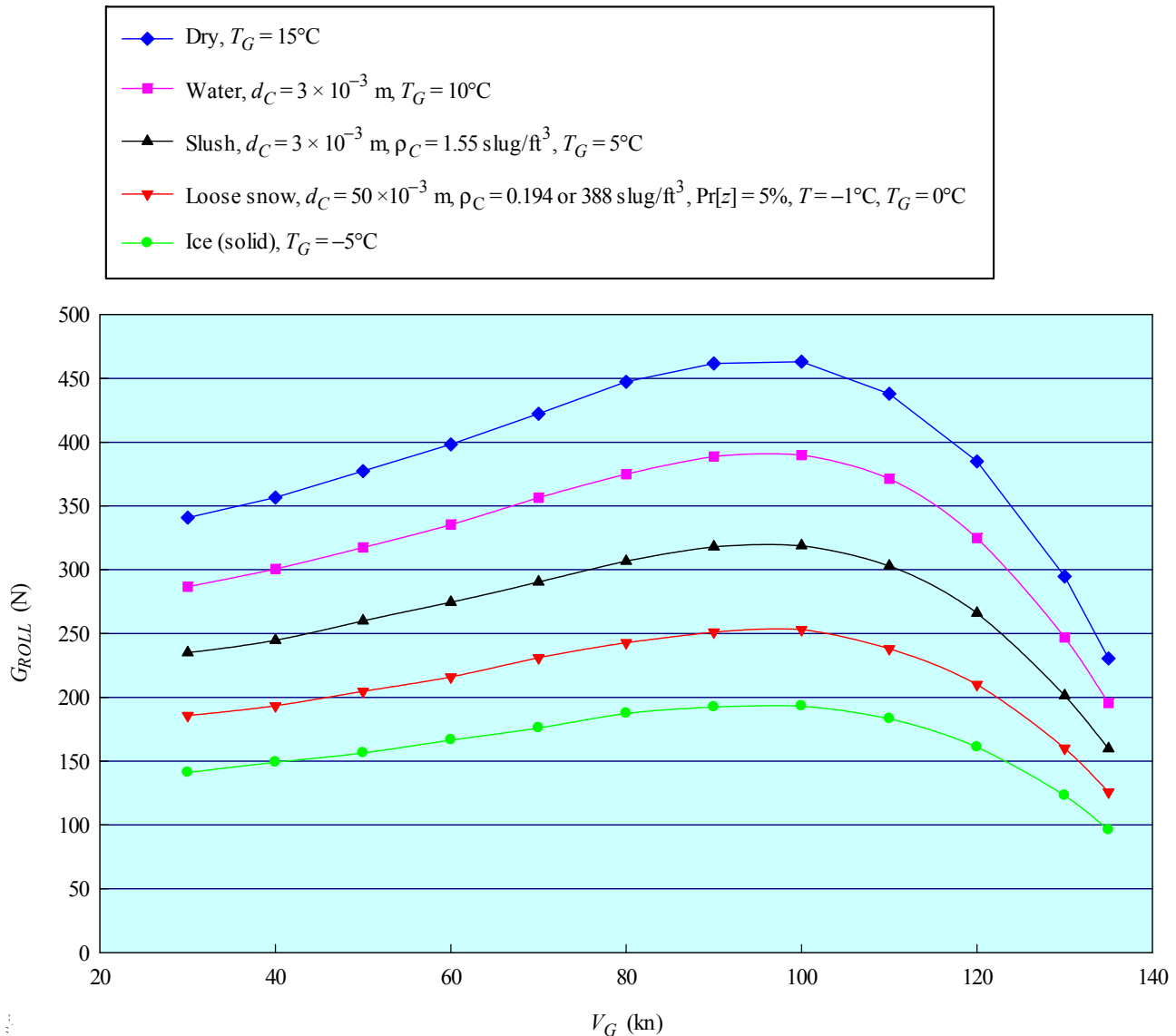
Sketch 3.1 Rolling resistance coefficient, μ_{ROLL} , acting on tyre operating on dry runway surface and runway surface covered by indicated layer of contaminant



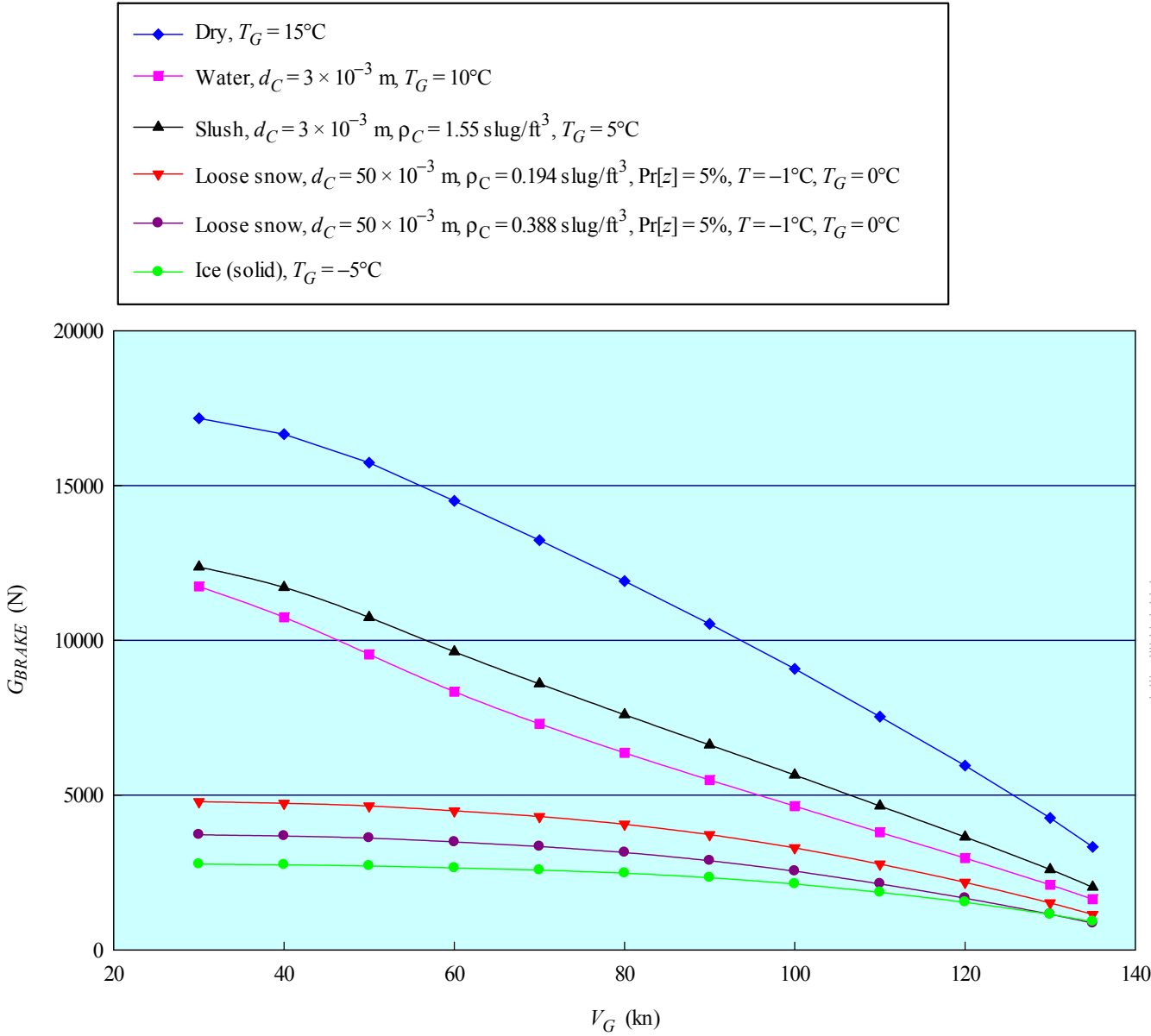
Sketch 3.2 Braking force coefficient, μ_{BRAKE} , acting on tyre operating on dry runway surface and runway surface covered by indicated layer of contaminant

4. CALCULATION OF BRAKING AND ROLLING FORCES ACTING ON TYRE

The braking forces acting on the tyre, G_{BRAKE} , and the tyre rolling resistance, G_{ROLL} , acting on the tyre for the specified conditions can be calculated over the desired range of speeds using the analysis process detailed in Appendix A, as shown in Sketches 4.1 and 4.2.



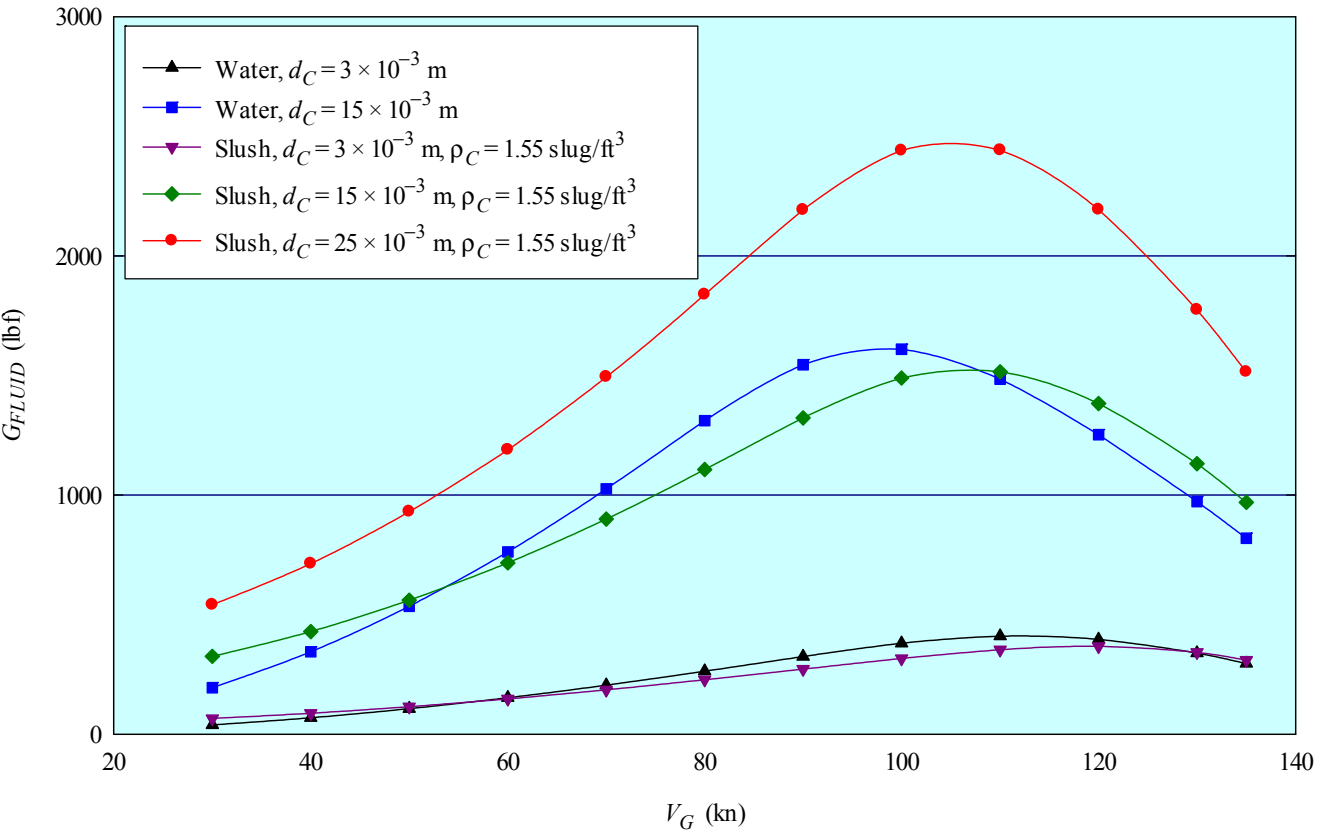
Sketch 4.1 Rolling resistance, G_{ROLL} acting on tyre operating on dry runway surface and runway surface covered by indicated layer of contaminant



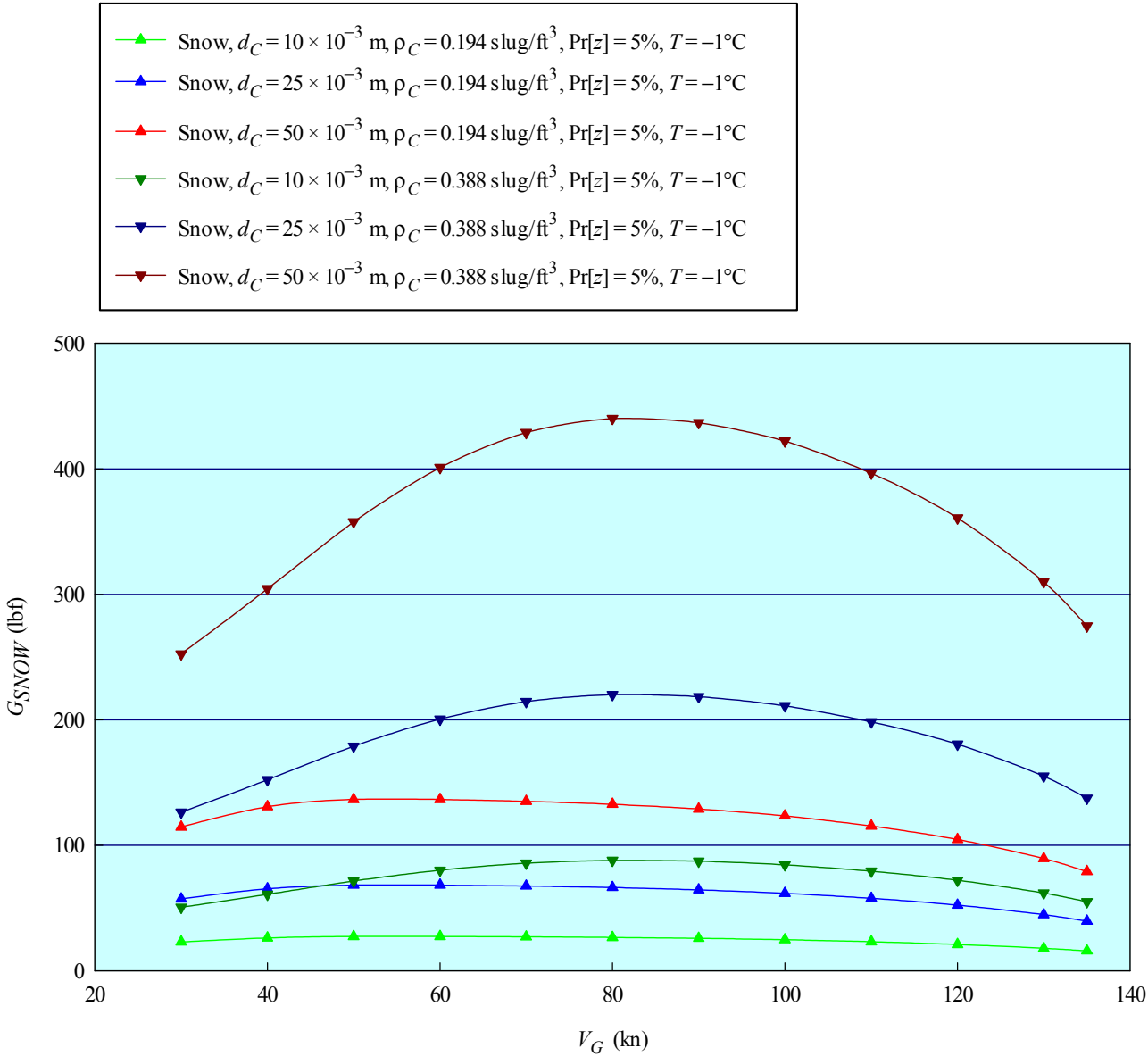
Sketch 4.2 Braking force, G_{BRAKE} , acting on tyre operating on dry runway surface and runway surface covered by indicated layer of contaminant

5. CALCULATION OF DRAG FORCES ACTING ON TYRE

The contaminant drag force acting on the tyre, which results from a layer of water/slush or snow covering a runway, G_{FLUID} or G_{SNOW} , respectively, can be calculated over the desired range of speeds using the analysis process detailed in Appendix A, as shown in Sketches 5.1 and 5.2.



Sketch 5.1 Fluid drag force, G_{FLUID} , acting on tyre operating on runway surface covered by indicated layer of water or slush



Sketch 5.2 Snow drag force, G_{SNOW} , acting on tyre operating on runway surface covered by indicated layer of snow