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Decelerating forces on multiple-wheel undercarriages rolling or braking on precipitate contaminated runways



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DECELERATING FORCES ON MULTIPLE-WHEEL UNDERCARRIAGES ROLLING OR BRAKING ON PRECIPITATE CONTAMINATED RUNWAYS

CONTENTS

	Page
1. NOTATION AND UNITS	1
2. INTRODUCTION	4
2.1 General	4
2.2 Line-astern	4
2.3 Line-abreast	5
2.4 Dual Line-astern	5
3. ANALYSIS OF DATA FROM HARRIN'S EXPERIMENT	6
3.1 General	6
3.2 Phase Angle	6
3.3 Depth of Macro-texture	7
3.4 Aft Wheel with No Forward Wheel	9
3.5 Coefficient of Decelerating Force on Aft Wheel	9
3.6 Decelerating Force on Aft Wheel	10
3.7 Statistics	10
4. ANALYSIS OF DATA FROM BARRETT'S EXPERIMENTS	12
4.1 General	12
4.2 Evaluation of Single Wheel Data	12
4.3 Two Wheels in Line-abreast	13
5. ANALYSIS OF DATA FROM DREHER'S EXPERIMENTS	15
5.1 General	15
5.2 Single Wheel	15
5.3 Two Wheels in Line-abreast	16
5.4 Two Wheels in Line-astern	16
5.5 Dual Line-astern	17
5.6 Statistics	19
6. DISCUSSION	20
6.1 General	20
6.2 Line-astern	20
6.3 Line-abreast	21
7. SUMMARY OF MODEL ROLLING ON WATER-COVERED SURFACES	23
8. BRAKING ON WATER-COVERED SURFACES	25
8.1 Undercarriage Braking Force	26

9.	EFFECT OF SURFACES COVERED WITH SLUSH, SNOW OR ICE	28
9.1	Rolling	28
9.1.1	Slush and loose snow	28
9.1.2	Compressed snow and ice	28
9.2	Braking	28
9.2.1	Slush	28
9.2.2	Loose snow, compressed snow and ice	28
10.	REFERENCES	29
	FIGURES 1 to 20	30-39

DECELERATING FORCES ON MULTIPLE-WHEEL UNDERCARRIAGES ROLLING OR BRAKING ON PRECIPITATE CONTAMINATED RUNWAYS

1. NOTATION AND UNITS

Consistent, British, units are used throughout this Data Item.

		Units
A^*	Anderson-Darling test statistic for normal distributions	—
C_D	decelerating force coefficient for single tyres	—
c_0, c_1	constants for a given size of tyre in drag equation (see Equation (3.4))	—
c_2, c_3	constants in Equation (4.2) (see also Table 7.1)	—
c_4, c_5	constants in Equation (5.5) (see also Table 7.1)	—
c_{01}, c_{11}, c_{12}	constants in $(C_D)_{DISP}$ (see Table 7.1 and Table 5.2 of ESDU 10015, Reference 1)	—
D	inflated diameter of tyre	ft
d	depth	ft
F_C	parameter associated with effective contaminant depth (see Equations (8.5) and (8.7))	ft
Fr	Froude number based on flow conditions at forward tyre	—
$f[m]$	general function for influence of successive wheels in line-astern (see Equation (6.3))	—
G	decelerating force on single tyre	lbf
G	effective shear modulus or modulus of rigidity	lbf/ft ²
g	acceleration due to gravity	ft/s ²
i	wheel number in a line-astern array numbered from the front, or first, wheel in the array	—
j	wheel number in a line-abreast array numbered from the front, or first, wheel in array	—
L	length defining wheels in a line-astern arrangement (see Equation (5.4) and Table 7.1)	ft
M_i	parameter associated with effective contaminant depth, $i = 1$ to m (see Equations (8.5) and (8.6))	—
m	number of wheels in a line-astern arrangement	—
N^2	speed number $(= \rho_c V^2 / (\pi p_{ABS}))$	—
n	number of test points in a correlation	—

Issued October 2011 – 39 pages
With Amendments A to C, September 2015 – 39 pages
This page Amendment C

n	number of wheels in a line-abreast arrangement	–
$\Pr[\]$	probability of event in $[\]$	–
p	gauge value (or value relative to atmospheric pressure) of inflation pressure	lbf/ft ²
p_{ABS}	absolute value of tyre inflation pressure	lbf/ft ²
q	kinetic pressure (see Equation (3.8))	lbf/ft ²
r	exponent used in function $f[m]$	–
S_0	reference area (see Equation (5.2))	ft ²
S_1	reference area (see Equation (3.12))	ft ²
s	slip ratio	–
$U[\]$	uncertainty associated with event in $[\]$ at 95% level of statistical significance	–
V_G	ground speed	ft/s
w	inflated width of tyre	ft
x	distance between wheels in line-astern arrangements	ft
y	lateral spacing of tyres (measured between centre lines of adjacent tyres)	ft
Z	normal (to runway) load on tyre	lbf
$\Delta[\]$	deviation of measured value of quantity in $[\]$ from model	–
$E[\]$	expected value of quantity in $[\]$ from model	–
ε	strain (see Table 4.1 of ESDU 10015, Reference 1)	–
ζ_0, ζ_1	constants in impact force equation (see Equations (3.14) and (5.3))	–
ζ_2	variable in expression for $(y/w)_{EFF}$ (see Equation (5.5))	–
ζ_{00}, ζ_{10}	constants in expression for ζ_0 and ζ_1 , respectively (see Equations (6.1a), (6.1b) and Table 7.1)	–
ζ_{20}, ζ_{21}	constants in Equation (5.6) (see also Table 7.1)	–
κ	tyre construction parameter (see Table 3.1)	–
Λ	tyre loading parameter (see Equation (3.7))	–
λ	phase angle between stress and strain (see Equation (4.1) of Reference 7)	rad
$M[\]$	measured value of quantity in $[\]$	–
μ	coefficient of friction or retarding force	–
ρ	density	slug/ft ³
σ	specific gravity of fluid contaminant	–

$\sigma[\]$	standard deviation of quantity in []	—
Φ	parameter (see for example Equation (4.1))	—
$1/\tau$	surface drainage parameter (see Equation (3.10))	—
χ^2	test statistic	—

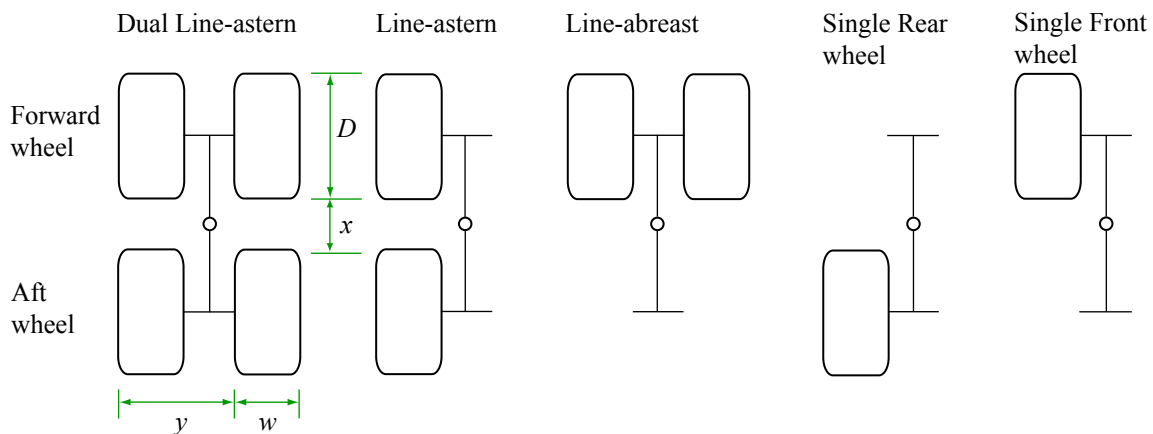
Subscripts

<i>AFT</i>	aft wheel
<i>BRAKE</i>	braking
<i>C</i>	contaminant
<i>DISP</i>	displacement of fluid
<i>EFF</i>	effective
<i>FLUID</i>	fluid
<i>FWD</i>	forward wheel
<i>IMP</i>	impact of spray on aft wheel
<i>L-Abreast</i>	wheels in line-abreast
<i>L-Astern</i>	wheels in line-astern
<i>m, n</i>	relate to number of wheels in line-astern and line-abreast arrangements, respectively
<i>ROLL</i>	rolling wheel
<i>Single</i>	single wheel
<i>SKID</i>	skidding, no wheel rotation
<i>SLIP</i>	slipping, some wheel rotation and some wheel slide
<i>TEX</i>	effective surface macro-texture
<i>TOT</i>	total for single wheel
<i>TREAD</i>	tyre tread material

2. INTRODUCTION

2.1 General

Multiple-wheel undercarriages present a specific problem when moving over wet or contaminated runways. In a line-astern arrangement (see Sketch 2.1), the forward wheel is subjected fully to the decelerating forces due to fluid displacement: the aft wheel is sheltered from impact with the free-stream but is impacted by spray from the forward wheel. In line-abreast, both wheels are subjected fully to the decelerating forces due to displacement of the free-stream. In addition, there may be mutual interference so that, depending on the spacing, the total decelerating force may be greater than the sum of that for two single wheels. The task becomes even more complex when dual line-astern and other more exotic arrangements are considered.



Sketch 2.1 Tyre arrangement and geometry

Earlier studies applied factors to single-wheel calculations without proper concern for physical and statistical probity. Therefore, the canon of data, which is quite limited, needs to be revisited and analysed in a manner consistent with that adopted in References 1 and 2 for single wheels. This means that the concepts of *speed number*, N , *drainage parameter*, $1/\tau$, *loading parameter*, Λ , and *phase angle*, λ , are to be used as the basis for correlation and modelling. In this way it is possible to avoid arbitrary factors that are not necessarily transportable from one case to another.

References 3, 4 and 5 are reports of experiments with undercarriages that are not attached to aircraft: the data are therefore not affected by interferences with an airframe. These sets of data are ideal starting points for a generalised analysis providing that sufficient ranges of all variables are covered.

2.2 Line-astern

The information in Reference 3 was obtained from experiments on a treadmill with small tyres fixed in a rig which was specifically designed for economy in investment. Despite this, the data are uniquely suitable for deducing the effect of the spray that impinges on the aft wheel in a line-astern installation.

Analysis of these data is completed in three stages:

1. deduction of effective *texture* depth of the surface and phase angle between stress and strain for the specific tyre construction using force measurements from rolling tests on a single wheel,
2. deduction and correlation of a force coefficient at the aft wheel due to spray from the forward wheel[†]

[†] Note that there are no data to determine the effects of varying fore-and-aft spacing.

- and 3. modelling decelerating forces on the aft wheel and analysing the statistics of deviations between measurements and model.

It is shown in Reference 2, that the effect of tyre-size on such coefficients as those of decelerating force is generally dependent only on the fineness ratio of the tyre. The model deduced from the tests on a small tyre (reported in Reference 3) is therefore presumed to be applicable to full-scale tyres. However, the decelerating forces on the individual tyres of the configuration were not measured in the tests reported in Reference 4. Because the sum of the *estimated* forces on each wheel is different from the total measured force, it is assumed that the forward and aft wheel interact. The effect is most easily described by deducing an increment that varies with speed number and drainage parameter (N^2/τ^2), wheel geometry (D and w) and loading (Z).

2.3 Line-abreast

Data from Reference 5 were collected in the laboratory at Bristol University using small tyres mounted on a rig that provided extensive data for single tyres from which the modelling of Reference 2 was derived. Some modifications were made to the rig in order to accommodate the varying spacing between the wheels in the configuration. However, loading and inflation pressure for the wheels were not varied in this test series. Also, comments in Reference 5 imply that there were flaws in the experiments: in the current investigation, evidence has been found that invalidates some of the data.

In addition to the set of data on small tyres as reported in Reference 5, part of the experiment conducted by NASA and reported in Reference 4 was devoted to line-abreast configurations in which inflation pressure and loading on (full-scale) aircraft tyres were changed. Studying the two sets of data in combination enables the development of a plausible model. An unfortunate feature of the reporting of the line-abreast configurations in both references is that the measured data are not presented. Only faired values are given: there is no indication of the scatter at any particular condition. It is therefore not possible to conduct a study of the statistics of the modelling in this case.

Analysis of these data is completed in stages as follows:

1. for the small tyres tested in the laboratory, compare the values of measured force with the data analysed in Reference 2
- and 2. use the data, appropriately adjusted for balance tare, to deduce the effect of lateral spacing and speed number on the components of decelerating force.

2.4 Dual Line-astern

Data, in the public domain, for configurations of this kind are limited to the testing reported in Reference 4. There is sufficient information in the reference to deduce the make-up of interference for the specific longitudinal and lateral spacings of the four wheels in the configuration. Although the loads on the wheels were kept constant for the test series, three values of inflation pressure were tested over an acceptable range of ground speed in one inch of water. Because measured values are presented in the reference, the statistics associated with this part of the reported experiments are studied.

Analysis of the data from these experiments is completed in stages as follows:

1. evaluate the data for single wheels in the context of the model of Reference 2,
 2. use the approaches developed in analysing data from laboratory experiments to determine, as far as is possible, relationships for full-scale installations
- and 3. analyse statistics of correlating these full-scale data to determine probable uncertainties in using the correlation as a model.

3. ANALYSIS OF DATA FROM HARRIN'S EXPERIMENT

3.1 General

The experiments reported in Reference 3 were designed to provide insights, crucially *at low cost*, into the effect of clearing water from the path of a braked and unbraked wheel. Two methods were used: the analysis undertaken here considers only the effect of placing a free-rolling wheel in front of an unbraked wheel. Effects on a braked wheel may be considered in a separate study.

Tests considered were made with a diamond-treaded 3 x 12 inch tyre, inflated to 13 psig and rolling on a treadmill. The static, vertical load was 100 lbf. Because of the design of the force-measuring system, vertical loading on the aft wheel varies with decelerating force. A free-rolling wheel was placed in front of the test wheel: provision was made for the forward wheel to be raised to enable force measurements on the test wheel alone. Nominal water depth was close to 0.09 inch throughout the testing.

The surface of the treadmill belt was treated with sand of unspecified size "*to simulate a concrete runway*". No adjustments were made to the presented results for rolling resistance.

3.2 Phase Angle

It is assumed that the tyre used in the experiment is close in construction to a standard aircraft tyre: the modelling scheme developed in Reference 1 is therefore applicable. Rolling resistance is shown in Reference 1 to be related to the phase angle between stress and strain for the tread material *and* carcass-design, λ : it is plausible to make the assumption that the rolling resistance for the test tyre may be different from that of a standard aircraft tyre.

Numerical values for the tyre parameters, taken from Table 4.1 of ESDU 10015 (Reference 1), are repeated here in Table 3.1. A value of λ may be calculated using Equation (4.1) from Reference 7, but it should be noted that the value of λ calculated is for an aircraft type tyre and is neither appropriate for nor used in the analysis in this Data Item.

TABLE 3.1 Numerical values of tyre parameters

G_{TREAD} (MPa)	G_{TREAD} (lbf/ft ²)	κ	ϵ_{TREAD}	ρ_{TREAD} (kg/m ³)	ρ_{TREAD} (slug/ft ³)
0.48	10,000	16	3	1100	2.134

Decelerating force on a single wheel rolling through water consists of two parts: one due to the flexing of the tyre as the footprint progresses around the circumference (rolling resistance) and the second due to displacement of water by the tyre. Total decelerating force as defined in Reference 1 for a tyre rolling over a water-covered surface, is given by:

$$G_{TOT} = G_{ROLL} + G_{DISP}, \quad (3.1)$$

$$G_{ROLL} = Z \lambda \Lambda^{1/3} \left(1 + \left(\frac{\rho_{TREAD} V_G^2}{G_{TREAD}} \right)^{1/3} \left(\frac{\epsilon_{TREAD} \rho_{TREAD} V_G^2}{\kappa p_{ABS}} \right)^{2/3} \right), \quad (3.2)$$

$$G_{DISP} = q d_C \left(\frac{Z}{p_{ABS}} \right)^{1/2} (C_D)_{DISP} \quad (3.3)$$

$$\text{and} \quad (C_D)_{DISP} = c_0 \tanh \left[\left(c_1 \sqrt{\frac{N_{ROLL}^2}{\tau^2}} \right)^2 \right], \quad (3.4)$$

This page Amendment C

in which c_0 and c_1 are known functions of tyre geometry (taken from Equation (5.18) of ESDU 10015, Reference 1) and repeated here as.

$$c_0 = \left(\frac{w/D}{c_{01}} \right)^2 \quad (3.5)$$

and

$$c_1 = c_{10} + c_{11} / \left(1 + \left(\frac{w/D}{c_{12}} \right)^2 \right). \quad (3.6)$$

All component parts of Equations (3.1) to (3.4) are known for the case considered, except for the value of τ for the surface condition (see Section 3.3) and the value of phase angle, λ , for the specific construction of tyre.

The tyre loading parameter, Λ , the kinetic pressure, q , and the speed number, N_{ROLL} , given in Equations (3.2) to (3.4), respectively, are defined as follows

$$\Lambda = \frac{Z/(\pi D w)}{\kappa p_{ABS}}, \quad (3.7)$$

$$q = 0.5 \rho_C V_G^2 \quad (3.8)$$

and

$$N_{ROLL}^2 = \frac{\rho_C V_G^2}{\pi p_{ABS}}. \quad (3.9)$$

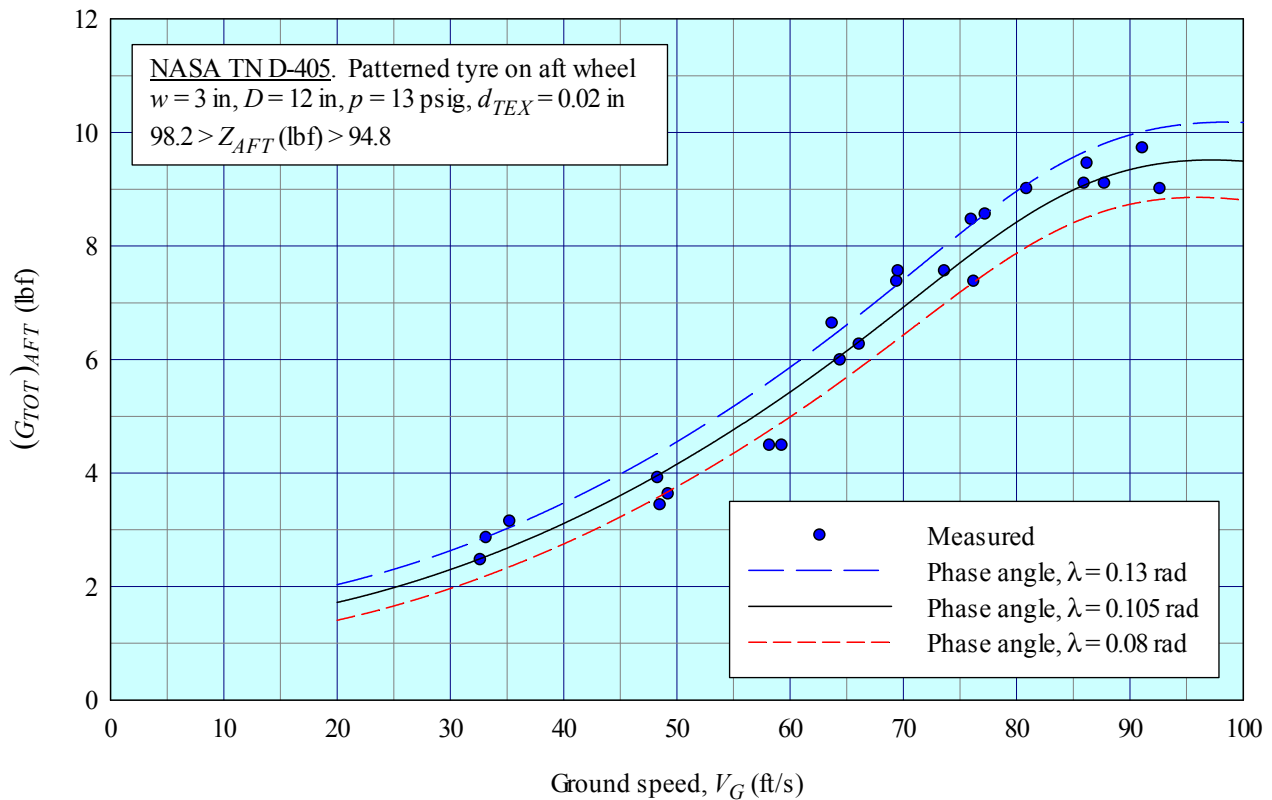
Note that for speeds less than about seventy feet *per* second the value of total force on the aft wheel is independent of depth of the macro-texture: the value of the hyperbolic tangent in Equation (3.4) is unity in these conditions. So, to match the variation of force at the lower speeds, it is sufficient to adjust the value of λ . Sketch 3.1 illustrates that, by using a value of 0.105 for λ , a plausible description of the measurements can be achieved: this value is close to the arithmetic mean of the phase angles that can be deduced from matching each individual measurement of decelerating force.

3.3 Depth of Macro-texture

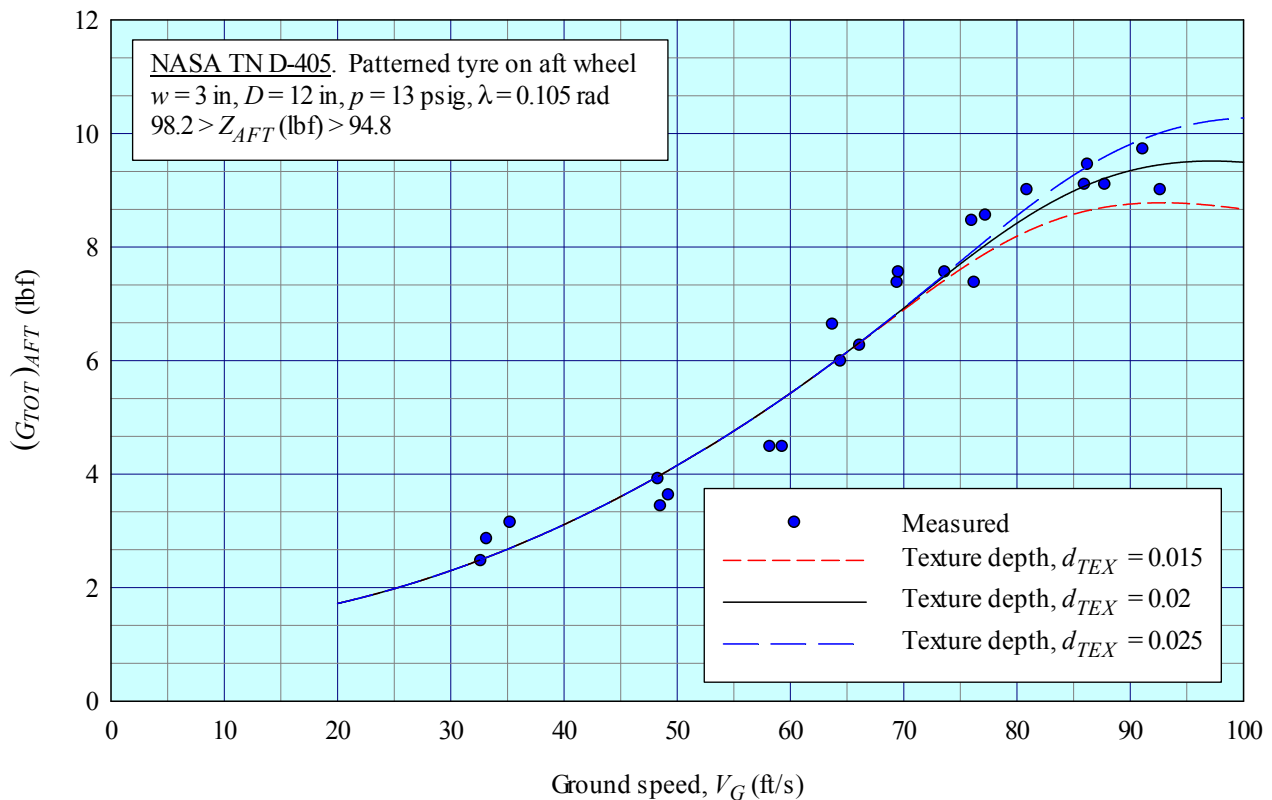
By definition, the surface drainage parameter is given by

$$\frac{1}{\tau} = \frac{d_C}{(d_C + d_{TEX})}. \quad (3.10)$$

The effect of varying the value of texture depth on the calculation of decelerating force is shown in Sketch 3.2: that sketch confirms that modelled decelerating force is independent of texture depth at the lower speeds. To achieve a close match to the mean of the measured data at speeds close to those for maximum force, an appropriate value for texture depth is 0.02 inch. This value, together with that deduced for phase angle ($\lambda = 0.105$ rad), is recommended for use when analysing data from Reference 3.



Sketch 3.1 Effect of speed and phase angle on total decelerating force on aft wheel



Sketch 3.2 Effect of speed and depth of macro-texture on total decelerating force on aft wheel

3.4 Aft Wheel with No Forward Wheel

With use of the values deduced for phase angle (Section 3.2) and effective texture depth (Section 3.3) the modelled effect of speed on total decelerating force is given in Figure 1. The figure also shows the measured data. These measured data are affected by uncertainties in the measuring process. However, the experiment was designed not only to collect data from rolling tests. A major consideration, apart from cost, which was minimised, was to collect information from braking the aft wheel. These (braking) forces are almost an order of magnitude greater than those measured in rolling. Thus, although the uncertainties appear to be substantial in comparison with these measurements, they are of the order of one *per cent* of full-scale on the load cells. Furthermore, the modelling describes the measurements with much the same level of uncertainty (see Section 3.6).

3.5 Coefficient of Decelerating Force on Aft Wheel

Measurements of forces on *individual* wheels in a tandem arrangement are unique to Reference 3: the deduction of decelerating force is therefore possible, unequivocally, only from these data. So, given the modelling developed in Sections 3.1 to 3.3, a force coefficient can be deduced from the forces measured on the aft wheel.

In principle, the forward wheel may be assumed to clear the *path* of the aft wheel of all fluid. It is therefore inappropriate to seek a factor on a single wheel to quantify the effect of the forward wheel on the trailing wheel. In the analysis that follows, an attempt is made to describe the effect in terms of a coefficient of decelerating force as an increment on that due to the impact of fluid on to the forward wheel.

Now, Froude number represents the ratio of dynamic forces,

$$\rho_C V_G^2 (d_C + d_{TEX}) \left(\left(\frac{Z}{p_{ABS}} \right)_{FWD} \right)^{1/2},$$

and static forces, that is the weight of water displaced

$$\rho_C g d_C \left(\frac{Z}{p_{ABS}} \right)_{FWD}.$$

So the square of Froude number for this case is

$$Fr^2 = \frac{V_G^2}{\left(\frac{g}{\tau} \right) \left(\left(\frac{Z}{p_{ABS}} \right)_{FWD} \right)^{1/2}}, \quad (3.11)$$

where τ is defined in Equation (3.10).

Three sets of data are available for different loadings on the forward wheel. The force coefficients deduced from the tests under these conditions are given in Figure 2 as a function of Froude number.

It is plausible to assume that all surface water is removed from the path of the aft wheel. Forces on that wheel due to fluid presence are therefore assumed to arise solely as a consequence of impingement of spray from the forward wheel on to the aft wheel. Spray intensity is presumed to be proportional to depth of water and it is therefore assumed that an appropriate reference for impingement force is given by S_1 defined as

$$S_1 = w d_C. \quad (3.12)$$

Furthermore, spray intensity may be assumed to be proportional to the kinetic (dynamic) pressure in the free-stream. Thus, impact force coefficient is defined as

$$(C_D)_{IMP} = \frac{G_{TOT} - G_{ROLL}}{q S_1}. \quad (3.13)$$

The correlating function shown on Figure 2 is given by

$$(C_D)_{IMP} = \zeta_0 \tanh\left[(\zeta_1/Fr)^5\right], \quad (3.14)$$

where, for this tyre $(\zeta_0, \zeta_1) \equiv (0.3, 45)$. These numbers *may* be related to the geometry of the tyres, the mechanical properties of the tyres, and to the longitudinal disposition of the arrangement of wheels.

3.6 Decelerating Force on Aft Wheel

It is assumed that Equation (3.14) is an appropriate description of the data given in Figure 2. This is based on considering the force measurements that are given in Figure 3. All forty-seven measurements of decelerating force are included on that figure. Each of these is paired with that calculated using either Equations (3.1) to (3.4) for the instrumented wheel alone or, for the dual-wheel arrangement, using

$$G_{DISP} = q S_1 (C_D)_{IMP}. \quad (3.15)$$

Uncertainties in the measuring process are again a feature in Figure 3. However, when due regard is taken of measured values in relation to the full-scale capabilities of the balance, the confidence limits shown on the chart are acceptable. These uncertainties are discussed in Section 3.7.

3.7 Statistics

In order to study the uncertainty in the correlation process, define

$$\Delta[G_{TOT}] = E[G_{TOT}] - M[G_{TOT}]. \quad (3.16)$$

There are four distinct test runs to be studied, the test-wheel alone and the three cases of the test-wheel with forward wheel loaded with 23, 63 and 103 lbf. Close inspection of Figure 3 reveals that there is an apparent difference in standard deviation of $\Delta[G_{TOT}]$ between the tests. However, using Bartlett's test for the significance of changes in variance from test-to-test, it can be shown that these apparent differences are not significant at the five *per cent* probability level. It is therefore valid to treat the 47 measurements as one population.