



Method to determine surface finish required to minimise local skin friction in the presence of a turbulent boundary layer

Associated software: SURFIN



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METHOD TO DETERMINE SURFACE FINISH REQUIRED TO MINIMISE LOCAL SKIN FRICTION IN THE PRESENCE OF A TURBULENT BOUNDARY LAYER

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1. INTRODUCTION

The skin friction associated with a turbulent boundary layer increases if the surface finish falls below a certain standard in terms of surface roughness. This has clear implications in the manufacture of both full-scale aircraft and wind-tunnel models where, for example, it is desired to establish a reliable datum drag measurement.

Nikuradse's measurements¹ of skin friction for water flowing through a pipe of circular cross-section with fully-developed turbulent flow confirm that, below a limiting roughness height, there is no influence on skin friction. Under these conditions the pipe is said to be hydraulically smooth or, where the working fluid is air, aerodynamically smooth.

A criterion, derived from Nikuradse's results¹, defines a limiting roughness height, non-dimensionalised with respect to a function of density, kinematic viscosity and local skin friction. Young² argues that this criterion is applicable to the boundary layer on a flat plate in the absence of a pressure gradient and also suggests suitable corrections to permit application to compressible flow.

Graphs in References 3 and 4 give the variation, with Reynolds number (based on chord), of permissible roughness on a wind-tunnel model. It is stated that this variation is based on Nikuradse's criterion¹ but its derivation is not clear. Calculations presented in Section 3 of this Item suggest that the variation corresponds to the permissible roughness immediately downstream of natural transition on a flat plate, when Nikuradse's criterion is applied to a power-law turbulent boundary-layer velocity profile, in incompressible flow, using the empirical relationships of Reference 2. An extension to this method, in which local-flow conditions and compressibility effects are taken into account, is described here, although the influence of the local pressure gradient is still ignored. For a wing of reasonably high aspect ratio the appropriate local-flow conditions may be estimated using the two-dimensional aerofoil program, VGK⁵ or the wing/body program VFP⁶.

A program (see Section 6) is provided with this Item to facilitate the calculation of the limiting roughness height either on a flat plate or with the input of appropriate freestream and local-flow conditions.

2. NOTATION

C	Sutherland constant (=120 K for air)
C_L	lift coefficient
c	chord length of flat plate or aerofoil
$\bar{c}$	mean chord
c_f	local friction coefficient $\left(= \tau_w / \frac{1}{2} \rho_\delta u_\delta^2 \right)$
c_{fi}	local friction coefficient estimate assuming incompressible flow
e	limiting roughness height above which drag is influenced by surface finish
M_δ	local Mach number at edge of boundary layer

M_∞	freestream Mach number
n	index in turbulent boundary layer velocity profile ($u/u_\delta = (y/\delta)^{1/n}$)
p_w	local pressure at wall
p_δ	local pressure at edge of boundary layer
Re_c	Reynolds number based on freestream conditions and c ($= U_\infty c/\nu_\infty$)
Re_{tr}	Reynolds number at transition ($= u_\delta x_{tr}/\nu_\delta$)
Re_x	Reynolds number based on x ($= u_\delta x/\nu_\delta$)
Re_{xturb}	Reynolds number based on x_{turb} ($= u_\delta x_{turb}/\nu_\delta$)
r	temperature recovery factor
T	local temperature in boundary layer
T_s	stagnation temperature
T_w	local adiabatic wall recovery temperature
T_δ	local temperature at edge of boundary layer
T_0	Sutherland reference temperature ($= 291.15$ K for air)
T_∞	freestream temperature
U_∞	freestream velocity
u	local velocity in boundary layer
u_δ	local velocity at edge of boundary layer
x	distance measured from leading edge (Sketch 3.1)
x_{off}	offset from effective origin of turbulent boundary layer to transition location (Sketch 3.1)
x_{tr}	distance from leading edge to transition location (Sketch 3.1)
x_{turb}	distance measured from virtual origin of turbulent boundary layer (Sketch 3.1)
y	distance measured normal to surface
γ	ratio of specific heats at constant pressure and constant volume
δ	boundary-layer thickness
θ	boundary-layer momentum thickness
μ	dynamic viscosity
μ_w	local dynamic viscosity at wall
μ_δ	local dynamic viscosity at edge of boundary layer

μ_0	Sutherland reference viscosity ($= 18.27 \times 10^{-6}$ kg/ms for air)
μ_∞	freestream dynamic viscosity
ν	kinematic viscosity of fluid
ν_w	local kinematic viscosity at wall
ν_δ	local kinematic viscosity at edge of boundary layer
ν_∞	freestream kinematic viscosity
ρ	density of fluid
ρ_s	stagnation density
ρ_w	local density at wall
ρ_δ	local density at edge of boundary layer
ρ_∞	freestream density
τ_w	local shear stress at wall

3. APPLICATION OF NIKURADSE'S CRITERION TO FLAT-PLATE FLOW

3.1 Incompressible Flow

3.1.1 Free transition

Nikuradse¹ gives the criterion for hydraulic smoothness for incompressible flow in pipes of circular cross-section as:

$$\frac{e}{\nu} \sqrt{\frac{\tau_w}{\rho}} = 5.0. \quad (3.1)$$

Young² suggests that, if the effect of the weak pressure gradient in the pipe is neglected, this criterion can be applied to the boundary-layer flow on a flat plate so, for incompressible flow:

$$\frac{e}{\nu_\delta} \sqrt{\frac{\tau_w}{\rho_\delta}} = 5.0. \quad (3.2)$$

Hence:

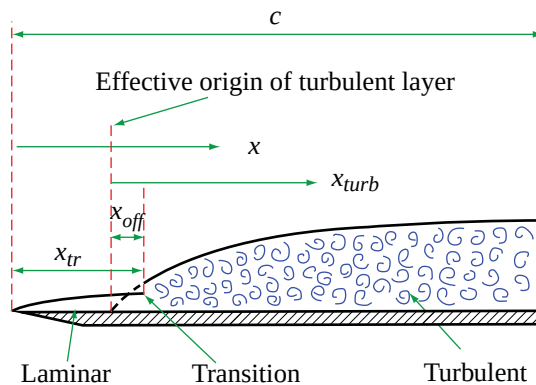
$$\frac{e}{\nu_\delta} \sqrt{\frac{\frac{1}{2} \rho_\delta u_\delta^2 c_f}{\rho_\delta}} = 5.0$$

or:

$$\frac{eu_{\delta}}{v_{\delta}} \sqrt{\frac{c_f}{2}} = 5.0. \quad (3.3)$$

In the case of a flat plate at zero incidence, $u_{\delta} = U_{\infty}$ and $v_{\delta} = v_{\infty}$ and:

$$\frac{e}{c} Re_c \sqrt{\frac{c_f}{2}} = 5.0. \quad (3.4)$$



Sketch 3.1 Boundary-layer transition and effective origin of turbulent layer

The velocity profile of a turbulent boundary layer on a flat plate can be approximated as:

$$\frac{u}{u_{\delta}} = \left(\frac{y}{\delta}\right)^{1/n}.$$

Young² suggests that appropriate choices of n are, 7 for $5 \times 10^5 > Re_{xturb} > 1 \times 10^7$ and 9 for $1 \times 10^6 > Re_{xturb} > 1 \times 10^8$.

The turbulent boundary layer has an effective origin, which lies x_{off} upstream of the transition location (Sketch 3.1). The effective Reynolds number, Re_{xturb} , for the turbulent layer is thus based on x_{turb} (Sketch 3.1)[†].

An empirical relationship for the local skin-friction coefficient on a flat plate, with a turbulent boundary layer (for $n = 7$), is given by²:

$$c_f = 0.0592 Re_{xturb}^{-1/5}. \quad (3.5)$$

Combining Equations (3.4) and (3.5) gives:

$$\frac{e}{c} = \frac{29.06 Re_{xturb}^{1/10}}{Re_c}. \quad (3.6)$$

[†] In this analysis it is assumed that x_{tr} is constant in the spanwise direction. In practice this transition line may be somewhat irregular (see e.g. Reference 7).