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Worked example for aircraft with multiple-wheel undercarriages braking on water-covered runway. Application of ESDU 11004



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WORKED EXAMPLE FOR AIRCRAFT WITH MULTIPLE-WHEEL UNDERCARRIAGES BRAKING ON WATER-COVERED RUNWAY. APPLICATION OF ESDU 11004

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WORKED EXAMPLE FOR AIRCRAFT WITH MULTIPLE-WHEEL UNDERCARRIAGES BRAKING ON WATER-COVERED RUNWAY. APPLICATION OF ESDU 11004

1. NOTATION AND UNITS

The notation scheme from References 1 and 2 is followed in this document. Users should be familiar with the symbols and definitions from these references. Additional notation relating to terms defined in this Data Item follow.

		<i>SI</i>	<i>British</i>
d_{TREAD}	tyre tread depth	m	ft
C_L	coefficient of lift	—	—
G	decelerating force on single tyre or all tyres from an undercarriage	N	lbf
$i, i = 1, m$	wheel number in a line-astern array numbered from the front, or first, wheel in the array	—	—
$j, j = 1, n$	wheel number in a line-abreast array numbered from the front, or first, wheel in array	—	—
L	aerodynamic lift force	N	lbf
m	number of wheels in a line-astern arrangement	—	—
n	number of wheels in a line-abreast arrangement	—	—
n_{UC}	number of undercarriages belonging to particular section of an aircraft	—	—
S_{REF}	reference area of wing	m ²	ft ²
s	slip ratio ($0 < s < 1$ for braking)	—	—
W	aircraft weight	N	lbf
Z_{FAC}	proportion of net vertical force of aircraft acting on an individual tyre from a specific type of undercarriage	—	—
ρ_A	density of air	kg/m ³	slug/ft ³

Subscripts

<i>BRAKE</i>	braking
<i>DRY</i>	dry runway surface
<i>FLUID</i>	fluid

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<i>FUS</i>	undercarriages from fuselage section of aircraft
<i>NOSE</i>	undercarriages from nose section of aircraft
<i>ROLL</i>	rolling
<i>SECT</i>	undercarriages from particular section of aircraft
<i>UCS</i>	all undercarriages belonging to an aircraft
<i>WET</i>	wet runway surface
<i>WING</i>	undercarriages from wing section of aircraft

2. INTRODUCTION

This Data Item predicts the rolling resistance and the decelerating forces due to braking and contaminant drag acting on the tyres of an aircraft that is operating on a water-covered runway surface during a landing operation. The methods described in References 1 and 2 are applied to calculate these forces. Note that the analysis only considers the decelerating forces acting at tyre-runway interfaces. Whenever the term 'decelerating force' is used in this Data Item, it refers to such forces.

The aircraft motion is analysed from a ground speed, V_G , of 150 knots until rest. The analysis begins after the aircraft has touched down. All aircraft wheels are assumed to be in contact with the runway surface for the duration of the analysis.

This analysis can be performed using software provided by ESDU (Reference 6).

When analysing the operation of an aircraft on a water-covered runway surface, the potential planing of one or more aircraft tyres must be taken into consideration. Reference 4 contains a procedure for predicting the aircraft ground speed which will result in an aircraft tyre planing during an operation. The method has not been used in this Data Item as it is applicable to cases where the contaminant depth equals or exceeds the tread depth of an aircraft tyre. In the analysis considered in this Data Item, the contaminant depth is less than the tread depth of each type of tyre used by the aircraft.

Reference 5 is a worked example which illustrates the application of the procedure from Reference 4 to determine the planing speeds of each type of tyre used by an aircraft that is similar in both specification and configuration to the one that is analysed in this Data Item.

3. RUNWAY AND AMBIENT CONDITIONS

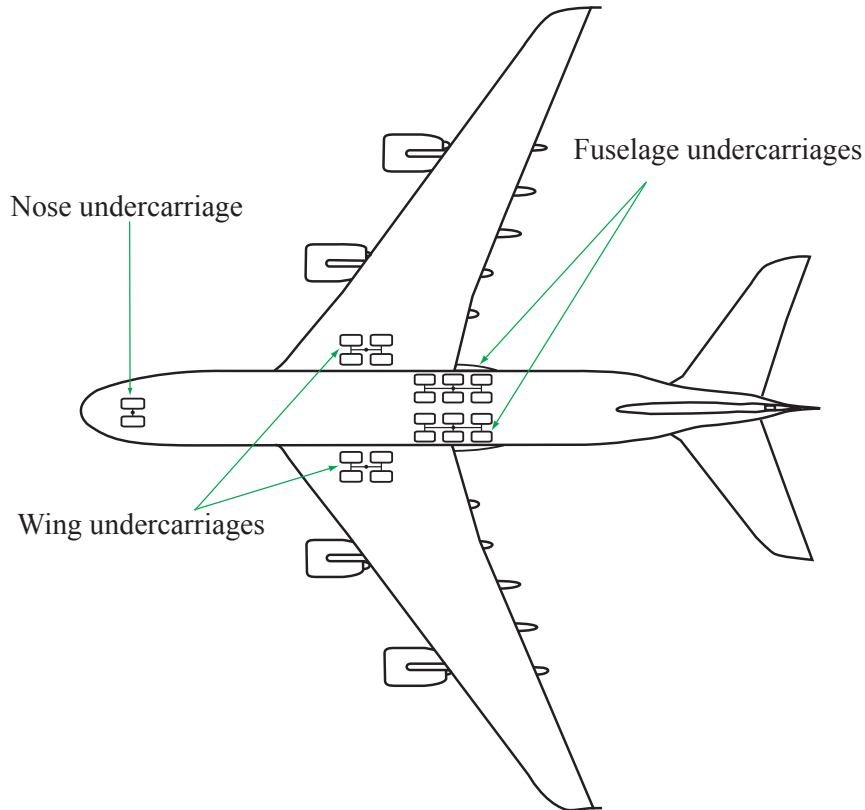
The runway surface has a macro-texture depth, d_{TEX} , of 1.27×10^{-3} m, and zero gradient. It is covered by a uniformly-distributed layer of water with a depth, d_C , of 3×10^{-3} m. For water, ρ_C is 1000 kg/m³. The runway surface temperature, T_G , is 10°C.

The micro-sharpness parameter F_1 , is assumed to have a value of 0.2. (See Reference 1.)

ISA, Sea Level pressure (14.7 psi, 1013.2 mb) and ISA, Sea Level temperature (288.15 K) are assumed. An air density, ρ_A , of 1.225 kg/m³ is assumed. There is no headwind or tailwind component.

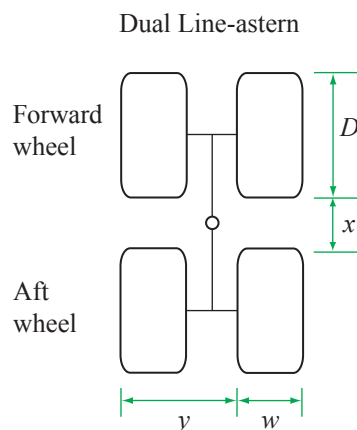
4. DESCRIPTION OF AIRCRAFT AND UNDERCARRIAGES

The aircraft under consideration comprises one nose undercarriage, two wing undercarriages and two fuselage undercarriages, as shown in Sketch 4.1.



Sketch 4.1 General layout of aircraft undercarriages

A schematic diagram illustrating the notation associated with an undercarriage, taken from Reference 2, is given in Sketch 4.2. The diagram illustrates a dual line-astern undercarriage arrangement, however the parameters are applicable to all multiple-wheel undercarriages.



Sketch 4.2 Undercarriage terminology (see Reference 2)

Relevant aircraft and undercarriage data are summarised in Tables 4.1 and 4.2, respectively.

TABLE 4.1 General aircraft data

	<i>Parameter</i>	<i>Value</i>	<i>Unit</i>
Weight [†]	W	3.4×10^6	N
	W_{NOSE}/W	0.08	—
	W_{FUS}/W	0.56	—
	W_{WING}/W	0.36	—
Aerodynamic	C_L	0.25	—
	S_{REF}	845	m ²

[†] Aircraft weight, and the proportion of aircraft weight acting on each undercarriage, is assumed to remain constant throughout the operation.

TABLE 4.2 Undercarriage and tyre data

Under-carriage	$(n_{UC})_{SECT}^{\dagger}$	m	n	m_{BRAKE}	s	x (m)	y (m)	D (m)	w (m)	d_{TREAD} (m)	P_{ABS} (N/m ²)	$(Z_{FAC})_{SECT}^{\ddagger}$
Nose	1	1	2	—	0	0.000	1.050	1.270	0.455	0.010	1.511×10^6	0.040
Fuselage	2	3	2	$2^{\dagger\dagger}$	0.1	0.300	1.530	1.400	0.530	0.010	1.601×10^6	0.0467
Wing	2	2	2	2	0.1	0.300	1.530	1.400	0.530	0.010	1.601×10^6	0.045

[†] See Section 5.4 for a description of $(n_{UC})_{SECT}$.

[‡] See Equation (5.1) for a description of $(Z_{FAC})_{SECT}$. Note that $(Z_{FAC})_{SECT}$ is a generic term. See Section 5.4.

^{††} The wheels in the final row of this undercarriage are used to steer the aircraft and hence are not braked.

All tyres are assumed to have a rib tread.

From Table 4.2, the dimensions and inflation pressures for the tyres used in the fuselage and wing undercarriages are seen to be identical. Consequently, W_{FUS}/W and W_{WING}/W were assigned to ensure that a similar load will act on the tyres in each of these undercarriages.

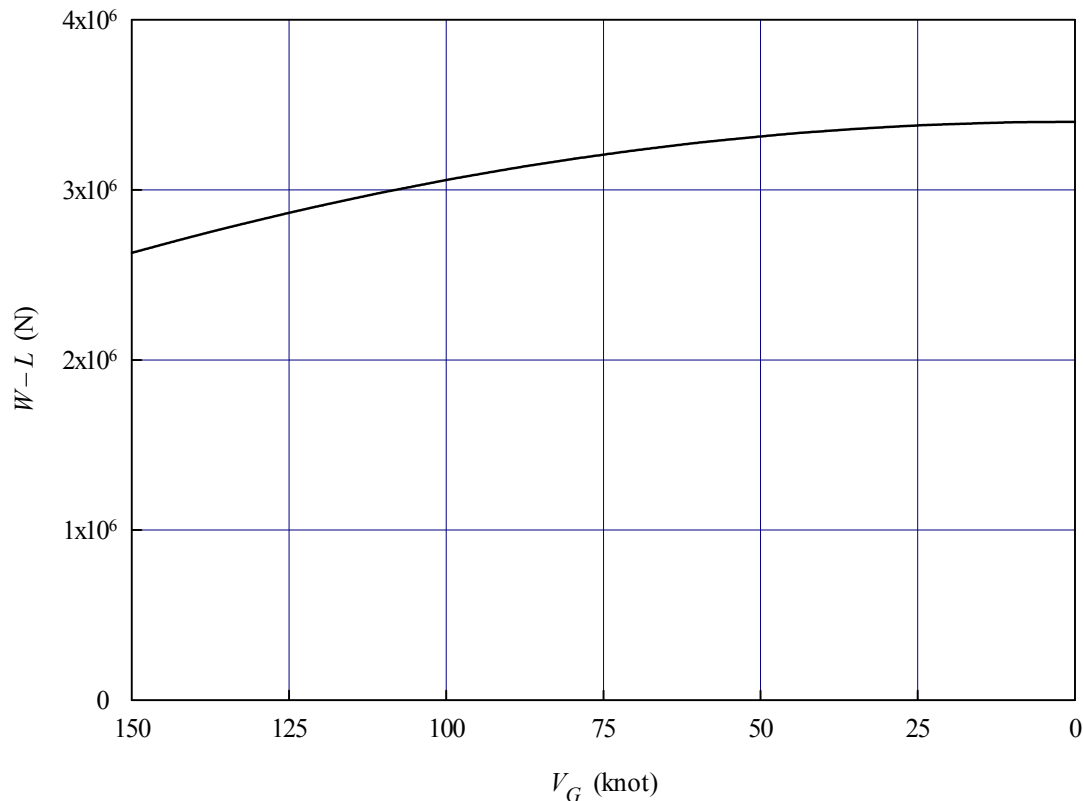
The variation of the net vertical aircraft force, $W - L$, over a ground speed, V_G , range of 0 to 150 knots, is illustrated in Sketch 4.3.

Note that W refers to the aircraft weight and L refers to the aerodynamic lift force acting on the aircraft.

The net vertical force, $W - L$, was calculated using the following relationship.

$$W - L = W - 0.5 \rho_A S_{REF} C_L V_G^2 \quad (4.1)$$

Values for parameters other than V_G can be found in Sections 3 and 4 of this Data Item.



Sketch 4.3 Net vertical aircraft load as a function of aircraft ground speed

5. DETAILED ESTIMATION OF DECELERATING FORCES ON FUSELAGE UNDERCARRIAGE

This Section demonstrates in detail the calculation of the decelerating forces acting on the tyres that belong to one of the fuselage undercarriages from the aircraft described in Section 4. In the case considered the tyres operate in a braked state at a V_G of 100 knots. A similar procedure would be followed to calculate the decelerating forces acting on the tyres belonging to each type of undercarriage during the entire operation, which covers a ground speed range, V_G , of 150 to 0 knots.

A slip ratio of 0.1 is assumed for all braked tyres (see Table 4.2).

5.1 Background Information

When a multiple-wheel undercarriage operates on a contaminated runway surface, the depth of contaminant that a row of wheels in the undercarriage passes through varies according to the location of the row in the undercarriage. As a row of wheels passes through a layer of contaminant, it displaces some contaminant, thereby reducing the depth of contaminant that the next row of wheels encounters. Additionally, spray projected by wheels in a 'forward' undercarriage row generally impinges on the wheels in the next, or 'aft', row. Hence, tyres in an aft row move through an effective contaminant depth, which is a function of both the depth of the remaining contaminant on the runway surface and the quantity of contaminant that impinges on the tyres in the aft row.

5.2 Runway Parameters

As noted in Section 3, the macro-texture depth of the runway surface, d_{TEX} , is 1.27×10^{-3} m, and the depth of the water layer covering the surface, d_C , is 3×10^{-3} m.

The surface drainage parameter, $\frac{1}{\tau}$, is calculated from the equation that appears in Table 7.1 of Reference 2, that is

$$\begin{aligned}\frac{1}{\tau} &= \frac{d_C}{d_C + d_{TEX}} \\ &= \frac{3 \times 10^{-3}}{3 \times 10^{-3} + 1.27 \times 10^{-3}} \\ &= 0.7026 .\end{aligned}$$

5.3 Aircraft Parameters

The aircraft has a weight, W , of 3.4×10^6 N (see Table 4.1). The proportion of aircraft weight acting on the fuselage undercarriages, W_{FUS}/W , taken from Table 4.1, is 0.56. The weight distribution is assumed to be constant throughout the operation.

When the aircraft operates at a ground speed, V_G , of 100 knots in the configuration listed in Table 4.1, the net vertical load that the aircraft exerts on the runway, $W - L$, will be 3.058×10^6 N. This value is found from Equation (4.1).

5.4 Fuselage Undercarriage Parameters

Fuselage undercarriage data are specified in Table 4.2.

The proportion of aircraft weight acting on an individual wheel belonging to an undercarriage from a particular part of an aircraft, $(Z_{FAC})_{SECT}$, is given by,

$$(Z_{FAC})_{SECT} = \frac{\frac{W_{SECT}}{W}}{(n_{UC})_{SECT} \times m \times n} . \quad (5.1)$$

Note that $(Z_{FAC})_{SECT}$, $(n_{UC})_{SECT}$ and W_{SECT} are generic terms used to represent the part of the aircraft under consideration. The suffix *SECT* is always replaced by the suffix that denotes the part of aircraft under consideration. For example, analyses involving undercarriages that belong to the fuselage part of the aircraft, the suffix *FUS* is used in place of *SECT*.

For the fuselage undercarriages from this aircraft, $m = 3$, $n = 2$, $(n_{UC})_{FUS} = 2$ and

$$\frac{W_{SECT}}{W} = \frac{W_{FUS}}{W} = 0.56 .$$

Hence, the $(Z_{FAC})_{SECT}$ value for a wheel that belongs to a fuselage undercarriage, $(Z_{FAC})_{FUS}$, is

$$(Z_{FAC})_{FUS} = \frac{0.56}{2 \times 3 \times 2} = 0.0467.$$

Subsequently, the net vertical load, Z , acting on an individual tyre from a fuselage undercarriage will be,

$$\begin{aligned} Z &= (Z_{FAC})_{FUS} \times (W - L) \\ &= 0.0467 \times (3.058 \times 10^6) \\ &= 142.8 \times 10^3 \text{ N.} \end{aligned}$$

The braking system slip ratio, s , is 0.1.

5.5 Rolling Resistance of Fuselage Undercarriage Tyres

5.5.1 Individual fuselage tyre

With use of mathematical model described in Table 5.2.1 of Reference 1, the rolling resistance of a single tyre from an undercarriage operating on a paved runway surface, $((G_{ROLL})_i)_j$, can be determined. (Note that $((G_{ROLL})_i)_j$ is equivalent to the term G_{ROLL} , the rolling resistance of a single tyre, that appears in Table 5.2.1 of Reference 1.)

A detailed application of the procedure can be found in Appendix 1 of Reference 3. For the case where V_G is 100 knots, $((G_{ROLL})_i)_j = 2.299 \times 10^3$ N.

5.5.2 All tyres from a fuselage undercarriage

The total rolling resistance of the set of tyres that belong to the fuselage undercarriage, $((G_{ROLL})_m)_n$, is,

$$\begin{aligned} ((G_{ROLL})_m)_n &= m \times n \times ((G_{ROLL})_i)_j \\ &= 3 \times 2 \times (2.299 \times 10^3) \\ &= 13.79 \times 10^3 \text{ N.} \end{aligned}$$

5.6 Contaminant Drag Force Acting on Undercarriage Tyres Due to Water on Runway Surface

The decelerating force due to contaminant drag acting on a multiple-wheel undercarriage arrangement due to the presence of water on a runway surface, $((G_{FLUID})_m)_n$, is determined using the following relationship taken from Table 7.1 of Reference 2.

$$((G_{FLUID})_m)_n = q \left(S_0(C_D)_{DISP} + S_1(m-1)^{2/3}(C_D)_{IMP} \right) \Phi, \quad (5.2)$$

where q is the kinetic pressure of water,

$(C_D)_{DISP}$ is the decelerating force coefficient for a single tyre resulting from the displacement of a fluid (see Reference 2 for further information),

$(C_D)_{IMP}$ is the decelerating force coefficient for a single tyre resulting from the impact of spray on an aft wheel (see Reference 2 for further information),

S_0 and S_1 are reference areas

and Φ is a factor that accounts for interaction between columns of tyres in line-abreast.

At a ground speed, V_G , of 100 knots (51.44 m/s), the kinetic pressure of the water contaminant, q , is

$$\begin{aligned} q &= \frac{\rho V_G^2}{2} \\ &= \frac{1000 \times 51.44^2}{2} \\ &= 1.323 \times 10^6 \text{ N/m}^2. \end{aligned}$$

The speed number, N^2 , is

$$\begin{aligned} N^2 &= \frac{\rho_C V_G^2}{\pi p_{ABS}} \\ &= \frac{1000 \times 51.44^2}{\pi \times (1.601 \times 10^6)} \\ &= 0.5261. \end{aligned}$$

The coefficient of decelerating force due to displacement of fluid by a single wheel, $(C_D)_{DISP}$, is

$$(C_D)_{DISP} = c_0 \tanh \left[\left(c_1 / \left(\frac{N^2}{\tau^2} \right) \right)^2 \right],$$

where c_0 and c_1 are drag equation constants.

The drag equation constants, c_0 and c_1 , are calculated using the following equations.

$$c_0 = \left(\frac{w/D}{c_{01}} \right)^2$$

and

$$c_1 = \frac{c_{11}}{1 + \left(\frac{w/D}{12} \right)^2}.$$

From Table 7.1 of Reference 2, $c_{01} = 0.2661$ and $c_{11} = 1.52$.