



An introduction to some methods to calculate loads and stiffnesses in bolted assemblies

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AN INTRODUCTION TO SOME METHODS TO CALCULATE LOADS AND STIFFNESSES IN BOLTED ASSEMBLIES

CONTENTS

	Page
1. BACKGROUND	1
1.1 Notation	2
2. GENERAL STIFFNESS EQUATIONS	4
3. BOLT STIFFNESS	5
4. MEMBER STIFFNESS	6
4.1 Cylinder Method	7
4.2 Pressure Cone Method	8
4.2.1 Derivation of stiffness	9
4.3 Finite Element Analysis Method	11
4.3.1 Linear analysis – Pre-loading stage	11
4.3.2 Non-linear analysis – Pre-loading stage	12
4.3.3 External loading stage	14
5. DERIVATION	15
6. EXAMPLE	16
6.1 Member Stiffness Calculated Using the Cylinder Method	16
6.2 Member Stiffness Calculated Using the Pressure Cone Method	16
6.3 Member Stiffness Calculated Using the FEA Method from Derivation 5	17
6.4 Calculating the Half-apex Angle, θ	17
6.5 Summary	17
7. TABLES AND FIGURES	19
7.1 Tables	19
7.2 Figures	24

AN INTRODUCTION TO SOME METHODS TO CALCULATE LOADS AND STIFFNESSES IN BOLTED ASSEMBLIES

1. BACKGROUND

A bolt is a term used to describe a threaded fastener designed to be used in conjunction with a nut to clamp together two or more members in an assembly, for example some flanges or a cover. When a bolt is tightened by turning the nut, the clamping force created in the bolt is called either pre-load or pretension. Note that pre loading may also be achieved using hydraulic tensioning. After assembly, some pre-load remains in the bolt, regardless of whether or not an external load is applied to the bolted joint (providing that the external load is not so large as to separate the member faces and that the nut has been assembled and tightened properly).

On occasion, a bolt will fail by yielding at some point during assembly or by in-service fatigue failure. The most common failures in bolts happen at known stress concentrations, such as in the fillet between the bolt head and the bolt body (shank), at the start of the threads or at the thread root fillet at the face of the nut. Improper installation increases the likelihood of bolt failure, as does the inappropriate usage of lubricants or coatings (platings), or if the bolt has been allowed to corrode, see ESDU 80614* and Derivation 3.

If an external, tensile (separating), load is applied to a bolted assembly, it is reacted partly by an increase in the bolt tension and partly by a reduction in member compression (as long as the members do not separate). When the stiffness of the members in compression is much greater than the stiffness of the bolt in tension, as is often the case, the portion of external load carried by the bolt is small and fluctuations of the load are not likely to cause a fatigue failure of the bolt. These aspects are considered in more detail in ESDU 85021†.

To examine the distribution of loads in a bolted assembly, it is essential to know the stiffnesses of the bolt and the members. Bolt stiffness can be calculated from its geometry, see Section 3 or ESDU 13008‡. However, member stiffness is less straightforward to calculate but can be determined from either experimental data or by using an analytical approach, see Section 4. Understanding the behaviour of loads and stiffnesses in the members is important for bolt tightening operations, particularly when hydraulic/pneumatic tensioning is used.

In this Data Item, the methods presented to find the loads and stiffnesses in bolted assemblies assume that a static pre load is applied to the bolt by tightening the nut. Member stiffness during pre-loading is found by examining a compressed region in the members, the form and size of which is thought to remain practically unaffected by the external load. It is also supposed that axial stress distribution stays uniform within a given horizontal plane and that a uni-axial stress state exists inside the compressed region. Interfacial friction and nut-bolt-thread interaction effects are neglected.

When an external load is applied to a bolted assembly, the methods to calculate loads and member stiffnesses are different to those used for pre loading. This is because the external load is unlikely to be applied underneath the bolt head (where the pre-load is generated). ESDU 13016** gives guidance for designing bolted assemblies when an external load is applied.

* ESDU 86014, "Applying, measuring and maintaining pretension in bolted joints".

† ESDU 85021, "Analysis of pre-tensioned bolted joints subject to tensile (separating) forces".

‡ ESDU 13008, "Fatigue strength of low-carbon steel stud threads under axial and combined axial and bending loading".

** ESDU 13016, "Some factors affecting the design of bolted assemblies".

Historically the cylinder method or the pressure cone method, see Sections 4.1 and 4.2, respectively, are used to find the stiffness of the members in a bolted assembly. These methods assume that

- the force/displacement characteristics of the bolt and the members are linear and
- these characteristics are the same for pre-loading and when the external load is applied.

Neither of these assumptions are strictly correct and the member stiffnesses calculated during pre loading using the cylinder or pressure cone method tend to be greater than those found using a numerical approach. Early finite element analysis (FEA) of member stiffness is based on a linear relationship between the load and the stress distribution in the bolted joint, see Section 4.3.1. A recent, more rigorous FEA approach assumes a non linear behaviour of loads and member stiffnesses during the stages of pre loading and when the external loading is applied, see Sections 4.3.2 and 4.3.3, respectively.

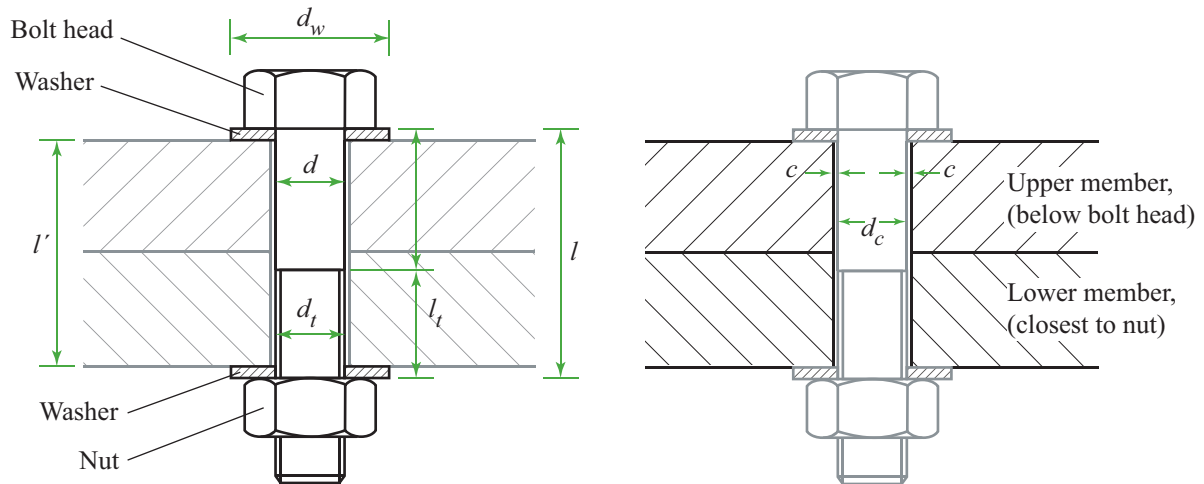
A list of derivations that have assisted in the preparation of this Item are given in Section 5. In Section 6, a worked example compares different ways that member stiffnesses can be calculated using the equations from Section 4.

1.1 Notation

Both SI and British units are quoted but any coherent system of units may be used.

A	area	m^2	in^2
A_{eq}	annular cross-sectional area of a cylinder	m^2	in^2
A	constant used in Section 4.3.1		
B	constant used in Section 4.3.1		
c	bolt-hole clearance ($d_c = d + 2c$)	m	in
D	diameter of small end of pressure cone	m	in
d	nominal diameter of unthreaded portion of bolt	m	in
d_a	outer diameter of the plate	m	in
d_c	diameter of bolt clearance hole	m	in
d_{od}	member contact outer diameter	m	in
E	Young's modulus	N/m^2	lbf/in^2
F_i	pre-load	N	lbf
k	spring constant or stiffness	N/m	lbf/in
k_{max}	maximum stiffness	N/m	lbf/in
k_0	limiting stiffness	N/m	lbf/in

l	length of the combined through thickness members, including washers	m	in
l'	length of the combined through thickness member	m	in
P	load	N	lbf
p	thread pitch	m	in
t	member thickness	m	in
x	co-ordinate	m	in
y	deflection at point of load application	m	in
α	non-dimensional factor in Section 4.3.2		
β'	l/d_w in Equation (4.5)		
β	non-dimensional factor in Section 4.3.2		
γ	D'/d_w in Equation (4.5)		
δ	change in length	m	in
ε	engineering strain		
θ	half-apex angle of pressure cone	degrees	degrees
ν	Poisson's ratio		
σ	engineering stress	N/m ²	lbf/in ²
Suffixes			
b	bolt		
d	unthreaded portion of the bolt		
m	member		
t	threaded portion of the bolt		
w	washer		
1, 2, ..., i	individual members		



Sketch 1.1 General notation of a bolted joint

2. GENERAL STIFFNESS EQUATIONS

The general relationship between a load, P , and a deflection in the direction of the applied load and at the point of load application, y , is given by:

$$P = f(y). \quad (2.1)$$

The spring constant, or stiffness, k , is:

$$k(y) = \lim_{\Delta y \rightarrow 0} \frac{\Delta P}{\Delta y} = \frac{dP}{dy}. \quad (2.2)$$

Simple load-deflection problems are linear so k can be written as:

$$k = \frac{P}{y}. \quad (2.3)$$

Assuming elastic behaviour, the definitions of engineering stress, σ , and engineering strain, ε , are, respectively:

$$\sigma = \frac{P}{A} \quad \text{and} \quad \varepsilon = \frac{\delta}{l}, \quad (2.4)$$

where A is the area over which the load acts and δ is the total (uniform) elongation experienced along the length, l .

Hooke's Law states that stress is proportional to strain,

$$\sigma = Ee, \quad (2.5)$$

where E is the proportionality constant, or the Young's modulus of the material. When the relationships in Equation (2.4) are substituted into Hooke's Law, an equation for the total elongation of a bar axially loaded in tension is:

$$\delta = \frac{P l}{A E}, \quad (2.6)$$

It should be noted that Equation (2.6) can also apply to a contraction due to compression when P is negative but cannot apply to a long bar loaded in compression where there is a possibility of buckling, see ESDU SS3*.

Combining Equations (2.3) and (2.6) gives an expression for stiffness:

$$k = \frac{A E}{l}. \quad (2.7)$$

3. BOLT STIFFNESS

The bolt stiffness, k_b , can be found by separating the gripped length of the bolt (the length of the combined through thickness of the members, including the washers) into a threaded portion and an unthreaded portion.

The stiffness of the threaded portion of the bolt, k_t , and the unthreaded portion of the bolt, k_d , are, respectively:

$$k_t = \frac{A_t E}{l_t} \quad \text{and} \quad k_d = \frac{A_d E}{l_d}. \quad (3.1)$$

l_t is the length of the threaded portion and l_d is the length of the unthreaded portion, see Sketch 1.1. The cross-sectional area of the unthreaded portion of the bolt, A_d , is:

$$A_d = \frac{\pi d^2}{4}, \quad (3.2)$$

where d is the diameter of the unthreaded portion of the bolt.

The area of the threaded portion of the bolt, A_t , also known as the **tensile stress area**, is given in Table 7.1 and Table 7.2 for metric and Unified screw-thread fasteners, respectively. Alternatively, A_t can be calculated:

$$A_t = \frac{\pi}{4} (d_t - 0.9382p)^2 \quad (\text{SI}) \quad \text{or} \quad A_t = \frac{\pi}{4} (d_t - 0.9743p)^2 \quad (\text{British}), \quad (3.3)$$

where d_t is the nominal major thread diameter and p is the thread pitch.

* ESDU SS3, "Guide to Items on struts, beams and shafts".

The total stiffness of the bolt is found by combining the stiffness of the threaded and the unthreaded portions of the bolt.

$$\frac{1}{k_b} = \frac{1}{k_t} + \frac{1}{k_d}, \quad (3.4)$$

or

$$k_b = \frac{k_t k_d}{k_t + k_d}.$$

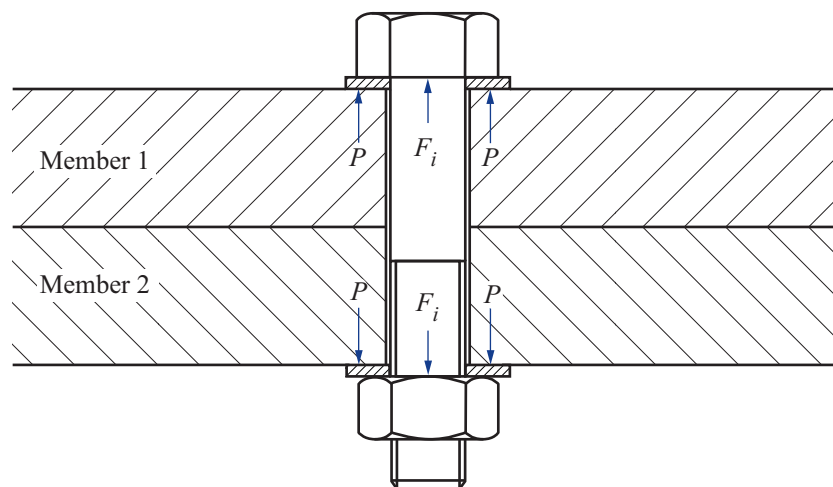
Therefore,

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}. \quad (3.5)$$

Bolt stiffness is a function of the gripped length, the cross-sectional areas of the threaded and unthreaded portions of the bolt, and the Young's modulus of the bolt material. For short bolts, the length of the unthreaded portion is likely to be small so it is expected that $k_b \approx k_t$. Conversely for long bolts, the length of the threaded portion is probably small and $k_b \approx k_d$. Some more complex calculations of fastener stiffnesses are given in ESDU 85021 and ESDU 13008.

4. MEMBER STIFFNESS

The majority of methods to estimate member stiffnesses are based on an assumed distribution of stress within the members caused by the bolt force applied by the bolt head, the nut or the washers (if they are used). During pre-loading, these forces, F_i , are reacted at the interface between the clamped members. If an external load, P , is applied, this load is assumed to be a traction force upon the same contact surfaces used in pre-loading (see Sketch 4.1). For this reason, the member stiffness does not change when the external load is applied and the members unload according to the same linear spring stiffness assumed in pre-loading. The distribution of stress can be converted to an equivalent area which may then be used to find the member stiffness, k_m .



Sketch 4.1 Loads in a bolted joint

If the clamped assembly consists of several members, then each member can be thought of as compressive spring which would act in series:

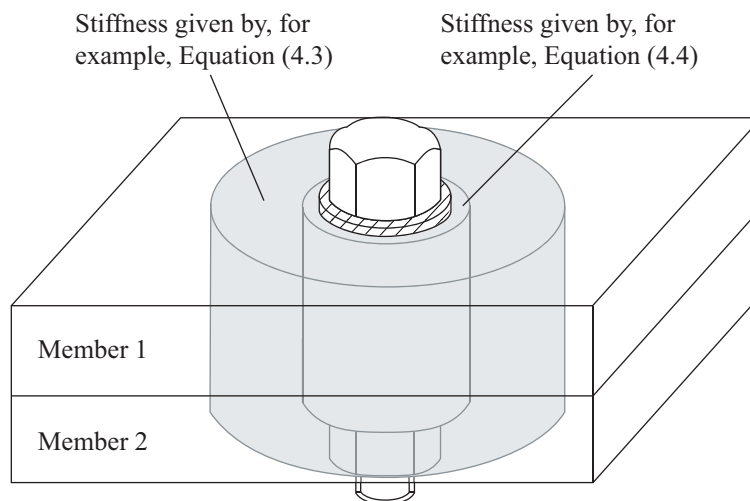
$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \dots + \frac{1}{k_i}, \quad (4.1)$$

where $k_1, k_2, \dots, k_i$ are the stiffnesses of the individual members.

If one of the members is a soft gasket then the gasket stiffness is usually so small that, for practical purposes, the stiffnesses of all of the other members may be neglected. For this particular case, the member stiffness would be approximately equal to the gasket stiffness. If a gasket is not present, the member stiffness is harder to calculate as the distribution of loads in the members is no longer uniform.

4.1 Cylinder Method

A method to find the stiffness of the members is to suppose an equivalent cylindrical region surrounding the bolt that experiences a uniform through-thickness induced stress and that outside this region, the stress is zero, Sketch 4.2. Note that this implies a discontinuity in the stress field at this boundary



Sketch 4.2 Cylinder method for calculating member stiffnesses

Rötscher (reported by Fritsche, 1962, cited by Derivation 5) assumed that the annular cross-section area of the cylinder, A_{eq} , is a function of the washer diameter, d_w , the bolt-hole diameter hole, d_c , and the length of the combined through thickness of the members, including the washers, l , Sketch 1.1.

$$A_{eq} = \frac{\pi}{4} \left[\left(d_w + \frac{l}{2} \right)^2 - d_c^2 \right]. \quad (4.2)$$

The member stiffness, k_m , can be found by combining Equation (2.7) and Equation (4.2):

$$k_m = \frac{A_{eq}E}{l} = \frac{\pi E}{4l} \left[\left(d_w + \frac{l}{2} \right)^2 - d_c^2 \right]. \quad (4.3)$$

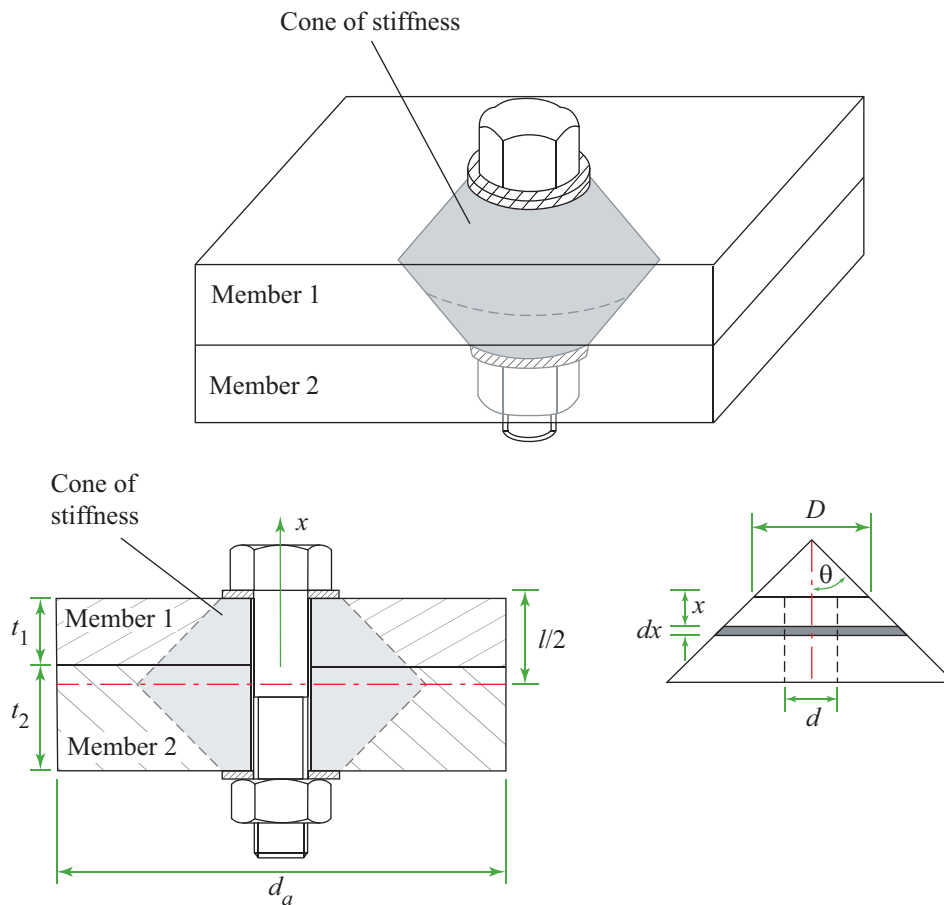
An alternative expression to Equation (4.3) is proposed by Derivation 6:

$$k_m = \frac{\pi E}{4l} \left[\left(1.5d + \frac{l}{10} \right)^2 - d_c^2 \right]. \quad (4.4)$$

Figure 1 shows a comparison of the dimensionless member stiffnesses (k_m/Ed), calculated using Equation (4.3) and Equation (4.4) when $d_w = 1.5d$ and $d_c = 1.002d$, plotted versus the aspect ratio, d/l . In Figure 1 it can be seen that $k_m/Ed \rightarrow \infty$ as d/l approaches zero. This anomaly can be avoided using the pressure cone method in Section 4.2.

4.2 Pressure Cone Method

Another method to find member stiffness is to suppose that the stress in the clamped members surrounding the bolt is contained within two conical frusta, or cones, which are symmetric around the mid plane of the assembly, Sketch 4.3. As with the cylinder method, a discontinuity exists in the stress field at the boundary of these cones. The cones are assumed to have a constant half-apex angle, θ . (Derivation 3 suggested that if θ varies with position, x , the estimate of member stiffness can be improved. However this complicates matters appreciably so only the simpler approach – with a fixed apex angle – is considered here.)



Sketch 4.3 Pressure cone method for calculating member stiffnesses