



Some factors affecting the design of bolted assemblies

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SOME FACTORS AFFECTING THE DESIGN OF BOLTED ASSEMBLIES

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SOME FACTORS AFFECTING THE DESIGN OF BOLTED ASSEMBLIES

1. INTRODUCTION

In this Data Item, a bolted assembly is defined as a joint that consists of two or more members that are secured by a bolt or stud. There are countless examples of bolted assemblies in engineering and many factors must be taken into account in their design, some of which are considered here.

In a bolted assembly, the loads and stiffnesses in the bolt can be calculated using one of the methods presented in ESDU 13008* or in Section 3 of ESDU 13015†.

The loads and stiffnesses in the members during typical pre-loading of the assembly (or during hydraulic or pneumatic tensioning of the bolt or stud) are, however, less straightforward to calculate. They can be estimated using either simple linear equations or a finite element analysis (FEA) that accounts for bolt and member interactions; see the methods in ESDU 13015.

FEA may also be used to examine the non-linear behaviour of loads and stiffnesses in the members when applying an external load to a pre-loaded assembly, such as under service conditions. In Section 2, early non-linear FEA of loads in bolted joints are discussed.

Sections 3 and 4 proceed in the manner of a parametric design exercise to study the loads in a bolted assembly. They are based on a number of considerations from an FEA study carried out as part of a Master's Degree investigation at the University of Strathclyde (Derivation 13). Derivation 15 describes an experimental investigation that validates the non-linear FEA methods that were used in Derivation 13. Both Derivation 13 and Derivation 15 confirm that linear FEA methods which are often used to examine the loads during pre-loading should not be used to model member stiffnesses when an external load is applied.

In Section 5, a summary of the behaviour of member stiffnesses in bolted assemblies during external loading is given. In Section 7, a worked example calculates member stiffnesses during pre-loading and when external loads are applied, using the data given in Section 4.

1.1 Notation

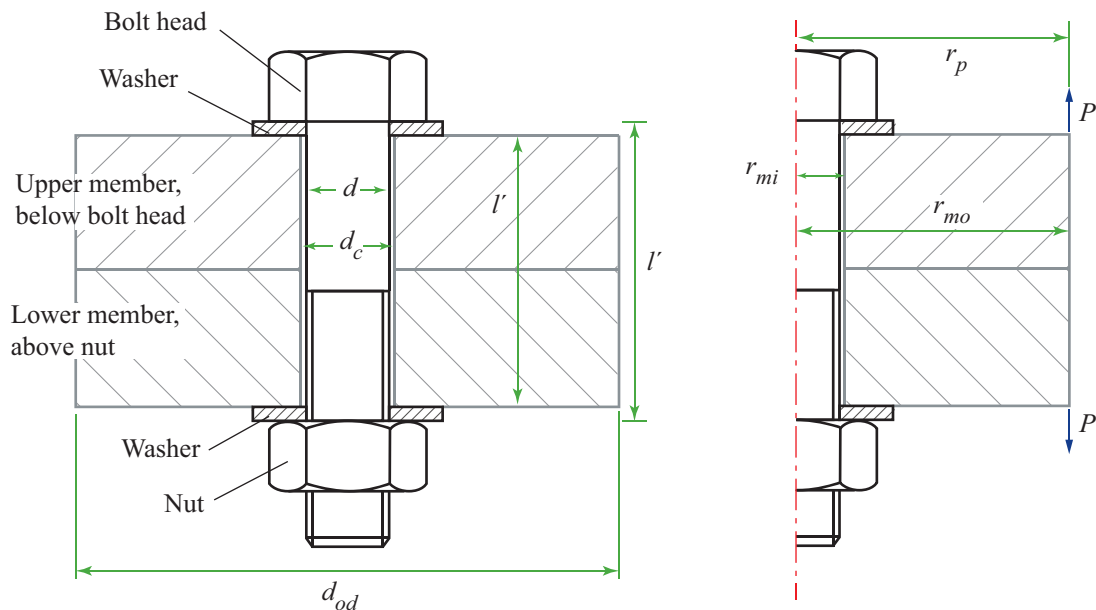
Both SI and British units are quoted but any coherent system of units may be used.

c	clearance	m	in
d	nominal diameter of unthreaded portion of bolt, see Sketch 1.1	m	in
d_c	clearance hole diameter, see Sketch 1.1	m	in
d_{od}	member contact outer diameter, see Sketch 1.1	m	in
E	Young's modulus	N/m ²	lbf/in ²
F_i	pre-load	N	lbf
k_b	bolt stiffness	N/m	lbf/in

* ESDU 13008, 'Fatigue strength of low-carbon steel stud threads under axial and combined axial and bending loading'.

† ESDU 13005, 'An introduction to some methods to calculate loads and stiffnesses in bolted assemblies'.

k_m	member stiffness	N/m	lbf/in
$k_{m(P/F_i)}$	member stiffness for a particular value of P/F_i	N/m	lbf/in
l	through thickness length of combined members, including washers, see Sketch 1.1	m	in
l'	through thickness length of combined members, see Sketch 1.1	m	in
P	applied tensile (separating) external load	N	lbf
P_b	bolt load	N	lbf
P_m	member load	N	lbf
p_c	pressure at a given radius	N/m ²	lbf/in ²
r	radius	m	in
r_b	bolt radius	m	in
r_{mi}	inner radius of member, see Sketch 1.1	m	in
r_{mo}	outer radius of member, see Sketch 1.1	m	in
r_p	radial position of external load, see Sketch 1.1	m	in
t_{m1}	upper member thickness, below bolt head	m	in
t_{m2}	lower member thickness, above nut	m	in
y	deflection at point of load application	m	in
α	non-dimensional factor, see Section 4.3.2 of ESDU 13015		
β	non-dimensional factor, see Section 4.3.2 of ESDU 13015		
δ	extension	m	in
θ	half-apex angle of pressure cone	degrees	degrees
$\mu_{kinetic}$	kinetic coefficient of friction		
μ_{static}	static coefficient of friction		
ν	Poisson's ratio		
σ_i	stress in bolt due to pre-load	N/m ²	lbf/in ²
σ_y	yield strength	N/m ²	lbf/in ²



Sketch 1.1 Notation

2. BACKGROUND

In ESDU 13015, the loads in bolted assemblies are calculated based upon the assumption of a static pre-load, F_i , in the bolt after tightening of the nut. (Note that a bolt pre-load of about 80-85 per cent of the bolt material yield strength would normally be used. However, if thermal loading might occur, then a bolt pre-load of 70 per cent of the material yield strength is more typical.) The member stiffnesses are then calculated by assuming a particular form and size for the compressed region which is assumed to remain unaffected by the later application of an external load. It is also supposed that the axial stress distribution remains uniform within a given horizontal plane and that a uni-axial stress state exists inside this compressed region. Interfacial friction and nut bolt-thread interaction effects are neglected.

Derivation 4 presents an FEA study of member stiffnesses in a bolted assembly that consists of two 16 mm thick plates clamped by an M14 bolt; note that gap elements and frictional effects were included in the model. A tensile external load, used to simulate a 'remote' separating load, was applied to the outer diameter of the members in various combinations of shear force and radial bending moments.

Derivation 4 showed that the axial member stiffnesses were very high when the external load was small in relation to the level of the pre load (less than approximately $0.4F_i$) but when the applied external load approached the pre-load level, member stiffnesses decreased and there was a greater separation of the member plates. Derivation 4 concluded that 'the results point to many deficiencies in the **linear** bolted joint theory', e.g. those theories presented in ESDU 13015.

Figure 1 (from Derivation 4) shows the effect of the post pre-load application of an external load, P , on the bolt strain (δ/l). Initially, as the external load is applied, the strains in the bolt relax slightly as the contact between the bolt-head and the members adjusts to new loading boundary conditions. The strain in the bolt only becomes appreciable when P reaches about 80 per cent of the pre-load.

If the bolt stiffness is relatively linear, it may be used in conjunction with the bolt force to calculate the portion of load carried by the bolt. The remainder of the load reduces the interface pressure between the members. Figures 2 and 3 show the variation of bolt and member loads, respectively, with increasing

applied external load. Note that on Figures 2 and 3, and also on subsequent figures, the bolt load, P_b , the load in the members, P_m , and the applied external load, P , have been non dimensionalised with respect to the pre-load, F_i . For most of the usable range of applied external loading, the majority of the external load is reacted by a reduction in the member compression at the interface of the bolt-head and the members.

3. BASIS OF STRATHCLYDE UNIVERSITY MASTERS REPORT

Derivation 13, a report from a Master's Degree at the University of Strathclyde, presents the results of a non-linear FEA study of loads and stiffnesses in several axisymmetric, single-bolted assemblies and also an experimental investigation of a bolted assembly consisting of a 6.33 mm diameter steel aircraft standard bi-hex bolt lamping 15 mm thick members. Note that the non linear FEA of the loads and stiffnesses is needed to consider the effects of contact and separation in the joint, but the analysis assumes that the material responses are in the linear range.

Tables 8.1 to 8.3 give the combinations of the parameters used in Derivation 13.

In all cases, the pre-load level was set at 85 per cent of the respective bolt material yield strength. This gives a 24.5:1 variation in the pre-load over the range of bolt sizes reported in Table 8.1.

For most of the data, the bolt shank is modelled as a uniform, smooth cylinder and the bolt and the nut heads are modelled as a stack of integral concentric discs. Different strategies for realistic modelling of the bolt/nut thread interaction have been investigated.

The geometry of the bolted assembly examined in Derivation 4 was also evaluated in Derivation 13 and the results of the non-linear FEA were found to confirm the data in Figure 1.

4. RESULTS

4.1 Member Stiffness

Figure 4, that has been plotted using data from Derivation 4 (see Section 2), shows a typical relationship between the member load, P_m , and the load at the bolt-head interaction point (P_m/F_i) and the corresponding dimensionless displacement (δ/l) (equal to the bolt strain and the member strain).

Figure 4 can be used to find the dimensionless *effective* linear stiffness, k_m/Ed , for a given applied external load, see Equation (4.1) and further explanation is given in Section 4.7.

$$\frac{k_m}{Ed} = \frac{\Delta P_m}{F_i} F_i \frac{1}{\left(\frac{\delta}{l} \times l\right)} \frac{1}{Ed}. \quad (4.1)$$

From Figure 4 when $\frac{\Delta P_m}{F_i} = 1$ then $\delta/l = 0.000\ 26$.

From Derivation 4, $F_i = 39\ 300\ \text{N}$, $l = 32\ \text{mm}$, $E = 207\ 000\ \text{N/mm}^2$ and $d = 14\ \text{mm}$.

Therefore:

$$\begin{aligned}\frac{k_m}{Ed} &= 1 \times 39300 \times \frac{1}{0.00026 \times 32} \times \frac{1}{207000 \times 14} \\ &= 1.63.\end{aligned}$$

From Figure 4 it can be seen that the dimensionless member stiffness (and hence the member stiffness, k_m), varies from negative values during the initial stage of loading up to very high values for most of its usable range. It finally drops to a dimensionless member stiffness value of 1.63 during the last stage of loading when the members are about to separate. Note that even the minimum value of member stiffness during external loading is actually greater than the pre-load member stiffnesses indicated in Figure 3 in ESDU 13015.

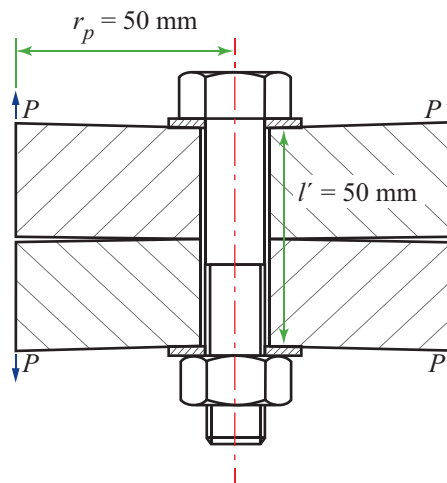
4.2 Effect of Bolt Size

4.2.1 Applying pre-load and external load

Figures 5 and 6 show through-thickness stress contours in bolted assemblies with an M5 and an M24 bolt, respectively.

In Figures 5(a) and 6(a), the bolted assembly is subject to pre-load only. In Figures 5(b) and 6(b), the external load, P is $0.475F_i$, and in Figures 5(c) and 6(c), P is $0.95F_i$.

The external loads are applied at the maximum radial distance from the bolt centre line, r_p , of 50 mm and the gripped length of the bolt (not including the washers), l' , is 50 mm for all cases, see Sketch 4.1 and Tables 8.1 and 8.2.



Sketch 4.1 Bolted assembly geometry and loading

The bigger joint separation and larger member deformation associated with the M24 bolt are due to the larger diameter of the bolt head and the greater pre-load (compared to the M5 bolt). During pre-loading of the M24 bolt, the additional diffusion of load into the members and the loss of inter-member contact pressure implies that the stresses present during pre-loading are **non-linear**. Derivation 4 shows that at the pre-loading stage, there is a partial separation of the members and an associated decrease of inter-member contact pressure to zero at a distance from the bolt centre line of about $2.5d$.

When the external load is applied to the M5 bolted assembly, see Figures 5(b) and 5(c), the inter-member contact between the plates reduces. For the M24 bolted assembly, the faces of the plates almost entirely separate when the maximum external load is applied; see Figures 6(b) and 6(c). (Note that in Figures 5 and 6, the displacements have been exaggerated in the FEA presentations.)

4.2.2 Bolt size

Figure 7 shows the variation of bolt load with external load for M5, M8, M10, M16 and M24 bolts. For each case, the external load is applied at a radial position of $r_p = 50$ mm, see Table 8.1.

From Figure 7, it can be seen that the non-linear behaviour of the bolts is similar to that described by Derivation 4. For the larger diameter bolts, the portion of the external load carried by the bolt is greater because of the higher bolt stiffness. However, even for the case of the M24 bolt, the bolt only carries 8 per cent of the external load when the bolted assembly is loaded to 95 per cent of its pre-load level. Most of the load is reacted by a reduction in the member compression and there are two reasons for this.

- The bending component of the applied external load redistributes the forces at the bolt/member interface leading to a substantial reduction in the compressive stress in the member local to the hole without a corresponding shift in the overall load carried by the bolt.
- The decrease in the inter-member contact pressure reduces the compressive stress in the members.

Given that larger diameter bolts carry more of the applied external load (which reduces the inter-member contact pressure, see Figure 6), if the aim of the design is to reduce the portion of service (externally applied) load carried by the bolt then the smallest bolt that is able to withstand the desired pre-load should be selected.

4.2.3 Effect of pre-load relative to bolt size

It is important to note that the **dimensionless** bolt load, P_b/F_i , does not depend on the magnitude of the pre-load.

Figure 8 shows that for M8 and M24 bolts (see Table 8.1), if the pre-load is reduced to a level appropriate for a smaller diameter bolt (e.g. $F_i = F_{i(\text{M5bolt})}$ or $F_i = F_{i(\text{M10bolt})}$), then the same portion of the dimensionless external load would be transmitted to the bolt in question each time. A discussion about the non-linear behaviour of loads during the pre-loading stage can be found in Section 4.3.2 of ESDU 13015 and Derivation 14.

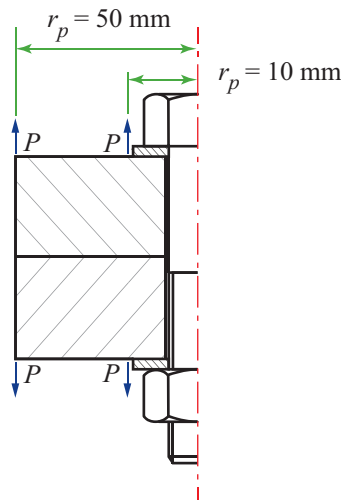
4.3 Effect of Geometry

4.3.1 Radial position of applied external load

The non-linear behaviour of loads and stiffnesses in a bolted assembly is related to the bending of the members and the radial position of the external load, r_p . Figure 9 shows the portion of the applied external load carried by an M10 bolt with r_p between 10 and 50 mm, see Sketch 4.2 and Table 8.2.

If the external load is applied closer to the bolt-head, then the bolt carries more of this load and the effect of bending in the member is reduced. (Note that at $r_p = 10$ mm, the load position is just outside the assumed-cylindrical M10 bolt-head).

- Figure 9 shows that the load behaviour becomes linear when r_p is less than approximately 15 mm.



Sketch 4.2 Radial position of external load

Figure 9 also shows a calculation using a 'classical theory' linear equation (Equation (4.12) in ESDU 13015, using the pressure cone method with a half-apex angle, $\theta = 30^\circ$). This theory is only applicable to pre-load conditions but has been assumed conventionally to apply to the subsequent external loading phase.

- The dimensionless member stiffness calculated when applying the external load at $r_p = 10$ mm remains about 1.64 times larger than the dimensionless member stiffness calculated for the same geometry under pre-loading conditions.

Figure 9 confirms that the member stiffness alters substantially between pre-loading and the application of an external load and this difference is accentuated when the load is applied further from the bolt hole centre line. However, for the pre-loading stage, reasonable agreement can be seen for an M10 bolt between the member loads calculated using FEA and the classical theory (with $\theta = 30^\circ$) for the pre-loading stage, see Figure 10.

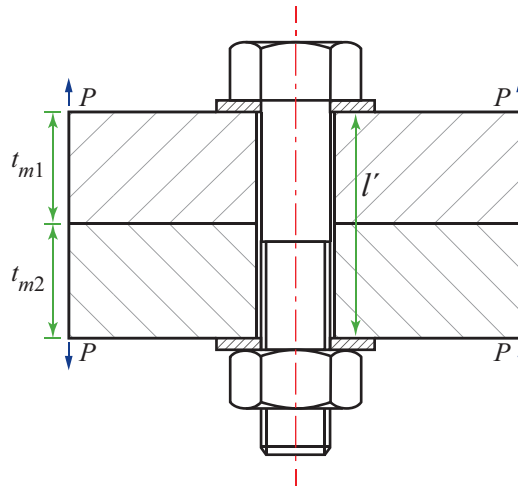
The assumption in classical theory is that the behaviour of the members in both the pre-load and external loading stages is the same so applying an external load at any location simply causes a load reversal (unloading) in the members according to the same stiffness characteristics as in pre-loading.

This treatment is based in turn on an implied application of St Venant's Principle, which is not, however, valid for the configurations treated in this study. The basis of the St Venant understanding is that an external separating load, applied at any point in a relatively stiff member, might be replaced by a statically-equivalent separating load at the point of contact between the bolt head and the member. This would then have the same effect as slackening the bolt tension induced during pre-load and thus produce a load reversal in the member, according to the classical theory.

However, St Venant's Principle includes the important caveat that the replacement of the real load with the statically-equivalent load should take place at a sufficient distance from the region where accurate stress analysis is required. As the point where the statically-equivalent load is applied is in the region of interaction between the bolt head and the member, that caveat is not normally satisfied and the approach is therefore invalid.

4.3.2 Gripped length of members

Figure 11 shows the portion of external load carried by the M10 bolt in an assembly with gripped lengths, l' , of between 20 and 90 mm (note that the members are of equal thickness), see Sketch 4.3 and Table 8.2.



Sketch 4.3 Member thicknesses in a bolted assembly

For assemblies containing bolts with shorter gripped lengths, the bolt stiffness increases and the area which the bolt load diffuse into is smaller. These assemblies show increased non-linearity due to

- the reduction in the bending stiffness of the member, and
- a greater portion of the load being carried by the bolt (because the bolt is relatively stiffer).

As the gripped length of the bolt gets longer, the stiffness of the members does not increase by as much as the stiffness of the bolt decreases suggesting that a stage is reached where the portion of the external load carried by the bolt becomes small.

4.3.3 Member thickness ratio

Figure 12 shows the effect of unequal member thicknesses on the amount of external load carried by the bolt in an M10 bolted assembly, see also Table 8.2. The member thickness ratio on Figure 12 is the ratio of the thickness of the upper member below the bolt head, t_{m1} , to the thickness of the lower member above the nut, t_{m2} . The values of the ratios investigated were $t_{m1}/t_{m2} = 0.25, 0.5, 0.75$ and 1.0 . Note that $t_{m1} + t_{m2} = l'$, see Sketch 4.3, and the value of l' remains constant in Figure 12 for all the data presented.

As the upper member gets thinner, and the member thickness ratio decreases, the bending stiffness of the upper member also decreases. This leads to

- a greater non-linear effect, and
- a greater bolt load, which is largely because the member-interface contact pressure is reduced at a much earlier stage.

4.4 Effect of Material Properties

Figure 13 shows the portion of the applied external load carried by the M10 bolt when different combinations of materials are used in the bolted assembly, see also Table 8.3. (Note that on Figure 13, 'Steel Al' indicates a steel bolt allied to an aluminium member.)

- If the same materials are used for the bolt and the member then load sharing in the assembly is the same for each of the metals used, apart from a slight change due to different coefficients of friction.
- If different materials are used for the bolt and the member then more of the load is taken by the component made from the material with the larger value of Young's modulus.

4.5 Effect of Thread Connection Flexibility at the Nut Face

In Derivation 13, a detailed elastic model of the threaded end of the bolt in a bolted assembly has been developed. Note that the threads in the model are represented as a homogeneous stack of concentric discs rather than as an inclined thread. The axisymmetric assumption in the solid integral head model (used in Sections 4.2, 4.3 and 4.4) is still considered to be accurate for the model containing the threads.

Figure 14 shows the amount of applied external load carried by the M10 bolt calculated using the solid integral head model compared to the model that considers the threads. The bolted assemblies had gripped lengths of either 25 or 50 mm and external load was applied at a radial position of 22.5 or 50 mm.

From Figure 14, a slight proportionate drop in the bolt load is seen when the thread flexibility has been incorporated into the model. This is thought to be due to the reduced stiffness of the bolt. The nut face also conforms more readily to the member face due to expansion at the most stressed threads however this makes little difference to the pattern of non-linear behaviour.

4.6 Inter-member Contact Pressure – Non-linear Finite Element Analysis Data

In a bolted assembly, the inter-member contact pressure is important from the point of view of maintaining a seal or some other securing function. Figure 15 shows the effect of the magnitude of the applied external load on the inter-member contact pressure in terms of a dimensionless pressure parameter for a reference joint (M10 steel bolt, steel members, $r_p = 50$ mm, $l' = 50$ mm).

On Figure 15, the dimensionless pressure parameter is:

$$p_c / \sigma_i, \quad (4.2)$$

where p_c is the pressure at a given radius, r , and σ_i is the pre-load value of stress in the bolt.

On Figure 15, the dimensionless radius parameter is:

$$\left(r^2 - r_{mi}^2 \right) / r_{mo}^2, \quad (4.3)$$

where r_{mi} is the inner radius of the member (*i.e.* the bolt-hole radius), and r_{mo} is the outer radius of the member.

From Figure 15, when the applied external load, P , is zero and only the pre-load is considered, the members lose contact at a dimensionless radius of 0.34 corresponding to an actual radius of about 29.5 mm. Derivation 14 defines outer diameter of the member contact area, d_{od} as

$$d_{od} \approx \beta d + l' \tan 30^\circ. \quad (4.4)$$

Equation (4.4) can be used to estimate the outer diameter of the member contact area at pre-load. For the case on Figure 15, this corresponds to a radius of 23 mm.

During the pre-loading of a bolted assembly that contains a steel bolt and members, the variation of the member contact outer diameter with the member thickness ratio (see Section 4.3.3) was examined using a FEA including gap elements (Derivation 9).

Derivation 9 showed that increasing the thickness of the lower member resulted in a decrease in the outer diameter of the member contact. In addition, the stiffness of the lower member was found to have a direct relationship to the member contact outer diameter due to the presence of the bolt threads. As the member thickness ratio increased, the total stiffness of the members in the bolted assembly reduced gradually.

If it were simply necessary to prevent separation of members in a multi-bolt assembly then a maximum bolt spacing of $2d_{od}$ is needed. Closer spacing of the bolts would achieve the desired minimum compressive stress (Derivation 9).

If the applied load is raised to half the maximum external load, *i.e.* $P = 0.475F_i$, the member contact area in the bolted assembly becomes substantially smaller and the maximum pressure is only slightly greater, Derivation 13. This is due to a bending rotation of the members applying larger compressive forces nearer to the edge of the bolt hole. (Note that the pressure values at the member inner radius, *i.e.* at the edge of the bolt-hole, may not be accurate due to approximation used in the evaluation of forces at the inner nodes.)

If the applied load is increased up to the full-load level of 95 per cent of the pre-load level ($P = 0.95F_i$) then more of the member contact pressure is lost.

Figure 16 shows the distribution of contact pressure around M5, M10, M16 and M24 bolts subject to the full-load level of $P = 0.95F_i$.

- Large diameter bolts maintain contact over wider areas and maintain greater pressure at the member interface. (Note that the curves on Figure 16 are the same for all values at pre-load).

If the member interface contact pressure is an important consideration in the design of the joints then large diameter bolts are suitable but they will take up more of the bolt load if the applied load increases.

Figure 17 shows the variation of member interface contact pressure around an M10 bolt when the external load is at the full load level ($P = 0.95F_i$) and the radial position of the load is between 10 mm and 50 mm.

The bending effect that comes from the external load is responsible for

- the separation of the member faces, and
- the concentration of contact pressure close to the inner radius of the members.

Figures 18 and 19 show the effect of the gripped length, l' , on member contact pressure around an M10 bolt when the external load is at the pre-load level, $P = 0$, and the full load level, $P = 0.95F_i$, respectively. The isolated marker points on Figure 18 identify the limiting contact points according to the approximate equation for the dimensionless radius given by Equations (4.4) and (4.2). This approximation is accurate