



# ESDU 14014

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## A theoretically-based method for predicting static-to-flight effects on coaxial jet noise

**Associated software: ESDUpac A1414  
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VIEWpac M1414  
VIEWpac MG1414**



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## A THEORETICALLY-BASED METHOD FOR PREDICTING STATIC-TO-FLIGHT EFFECTS ON COAXIAL JET NOISE

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## A THEORETICALLY-BASED METHOD FOR PREDICTING STATIC-TO-FLIGHT EFFECTS ON COAXIAL JET NOISE

### 1. NOTATION

|           |                                                                                             | Units           |                     |
|-----------|---------------------------------------------------------------------------------------------|-----------------|---------------------|
|           |                                                                                             | SI              | British             |
| $A_j$     | area of jet component                                                                       | m <sup>2</sup>  | ft <sup>2</sup>     |
| $A_p$     | geometric area of primary stream                                                            | m <sup>2</sup>  | ft <sup>2</sup>     |
| $A_s$     | geometric area of secondary stream                                                          | m <sup>2</sup>  | ft <sup>2</sup>     |
| $AR$      | area ratio, $A_s/A_p$                                                                       |                 |                     |
| $AR'$     | area ratio based on flow areas [see Equation (5.7)]                                         |                 |                     |
| $a_0$     | ambient speed of sound'                                                                     | m/s             | ft/s                |
| $C_d$     | nozzle discharge coefficient                                                                |                 |                     |
| $D_j$     | nozzle diameter of jet component                                                            | m               | ft                  |
| $D_p$     | equivalent primary jet nozzle diameter $\left( = \sqrt{\frac{4 A_p}{\pi}} \right)$          | m               | ft                  |
| $D_s$     | equivalent secondary jet nozzle diameter $= \left( \sqrt{\frac{4(A_p + A_s)}{\pi}} \right)$ | m               | ft                  |
| $F_D$     | spectral weighting for secondary jet                                                        | dB              | dB                  |
| $F_U$     | spectral weighting for mixed and interaction jets                                           | dB              | dB                  |
| $f$       | frequency                                                                                   | Hz              | Hz                  |
| $f_m$     | cut-off frequency for mixed jet                                                             | Hz              | Hz                  |
| $f_s$     | cut-off frequency for secondary jet                                                         | Hz              | Hz                  |
| $h'$      | modified ratio of secondary annulus height to the primary radius (see Equation (5.21))      |                 |                     |
| $k$       | ratio of dipole to quadrupole source strengths                                              | —               | —                   |
| $OASPL$   | overall sound pressure level                                                                | dB <sup>†</sup> | dB <sup>†</sup>     |
| $p_{ISA}$ | ambient pressure under ISA conditions                                                       | kPa             | lbf/ft <sup>2</sup> |
| $p_0$     | ambient pressure                                                                            | kPa             | lbf/ft <sup>2</sup> |
| $r$       | sound propagation distance (see Sketch 1.1)                                                 | m               | ft                  |

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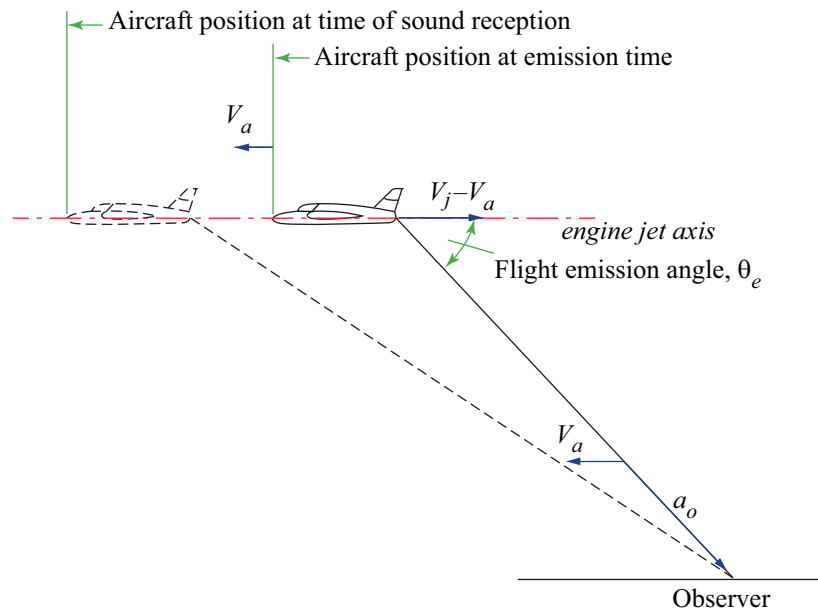
|            |                                                                       |                   |        |
|------------|-----------------------------------------------------------------------|-------------------|--------|
| $r'$       | ratio of turbulence levels                                            |                   |        |
| $S$        | Strouhal number, $S = fD_j/V_j$                                       |                   |        |
| $SPL$      | one-third octave band sound pressure level                            | dB                | dB     |
| $T_j$      | static temperature of jet component                                   | K                 | K      |
| $T_p$      | primary jet static temperature                                        | K                 | K      |
| $T_s$      | secondary jet static temperature                                      | K                 | K      |
| $T_0$      | ambient temperature                                                   | K                 | K      |
| $V_a$      | flight velocity                                                       | m/s               | ft/s   |
| $V_j$      | fully-expanded velocity of jet component                              | m/s               | ft/s   |
| $V_p$      | fully-expanded primary jet velocity                                   | m/s               | ft/s   |
| $V_s$      | fully-expanded secondary jet velocity                                 | m/s               | ft/s   |
| $VR$       | velocity ratio, $V_s/V_p$                                             |                   |        |
| $\Delta$   | correction for difference in turbulence levels                        | dB                | dB     |
| $\rho_p$   | density of primary jet flow                                           | kg/m <sup>3</sup> | ‡      |
| $\theta_e$ | flight emission angle (see Sketch 1.1)                                | degree            | degree |
| $\theta_j$ | angle between line to observer and engine exhaust axis for static jet | degree            | degree |

## Subscripts

|     |                 |
|-----|-----------------|
| $i$ | interaction jet |
| $m$ | mixed jet       |
| $p$ | primary jet     |
| $s$ | secondary jet   |

† The reference pressure for all decibel levels is 20  $\mu$ Pa (*i.e.*  $2 \times 10^{-5}$  N/m<sup>2</sup>) or  $0.42 \times 10^{-6}$  lbf/in<sup>2</sup>.

‡ A density value expressed in British units as lb per cubic foot has to be divided by 32.2 before the calculation for  $A_p$  shown in Equation (5.2) is carried out. (A force of 1 lbf acting on a mass of 1 lb produces an acceleration of 32.2 ft/s<sup>2</sup>.)



Sketch 1.1

## 2. INTRODUCTION

ESDU 14014 provides a computerized method for estimating static-to-flight effects on coaxial jet noise. It is intended that this method should be applicable to aircraft turbofan engines with separate core and bypass exhaust nozzles and with a hot core flow. The method is a theoretically-based one which reflects the known aerodynamic characteristics of both single-stream and coaxial jets. An account of the development of the prediction method, which is based on the coaxial jet modelling described in References 2 and 3, is given in Derivation 1 which is an internal ESDU technical memorandum available on request.

In essence, the method breaks up the coaxial jet mixing noise into three single-stream jet noise sources which can be predicted using Reference 4, modified appropriately and then summed. To each of these sources, the static-to-flight effect estimation procedure for single-stream jets described in Reference 1 is applied. The three spectra are then recombined to provide a flight spectrum. The difference between this flight spectrum and the static spectrum provides the static-to-flight effects for a coaxial jet.

It is postulated that the prediction accuracy of the changes in going from static to flight conditions should be substantially better than the prediction accuracy of the absolute spectral levels. Thus when predicting a flight spectrum:

- for the greatest accuracy, these calculated static-to-flight effects should be applied to a measured static spectrum,
- alternatively the method given in ESDU 11002 (Reference 5) may be used to predict a static spectrum,
- if neither of these options is available the flight spectrum generated here may be used, but the errors will be greater.

As stated above, the computer programs that accompany this Item can be used to predict flight spectra either directly or from static spectra input by the user. In the first case, flight spectra are predicted at angles

specified by the user, whereas in the second, flight spectra are predicted at the same angles as those of the static spectra.

The prediction method has been developed for a coplanar exhaust configuration but is considered to be applicable to jets with short centre-bodies and for cases where the secondary stream emerges a little upstream of the primary nozzle exit plane, as, for example, in a “three-quarter cowl” configuration of an engine exhaust system.

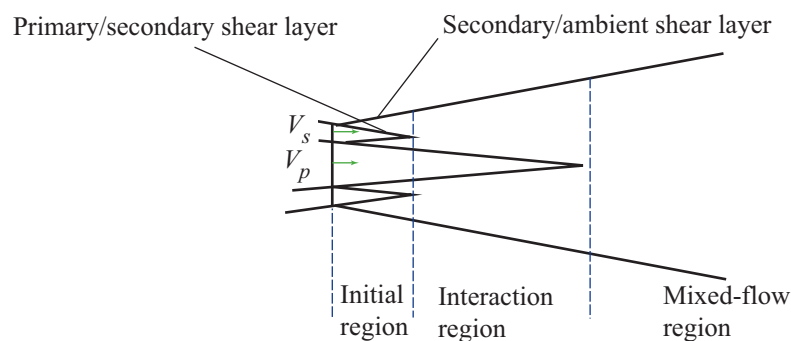
The method is not applicable to mixed exhaust configurations or non-circular nozzles and does not take into account any acoustic or aerodynamic installation effects.

By using fully-expanded jet velocities, the method can be applied to the mixing noise of supercritical jets although this has not been validated with experimental data. The noise arising from shock-associated sources is not included.

### 3. VISUALIZATION OF THE COAXIAL JET

The representation of the noise creating mechanism of a coaxial jet is extremely complex. The three-source model (References 2 and 3) is an attempt to simplify matters by representing the noise from a coaxial jet by the sum of three separate single-stream jets, each with specific operating conditions and dimensions.

The noise-producing regions of a coaxial jet for an aero-engine are illustrated in Sketch 3.1.



**Sketch 3.1 Noise-producing regions**

An overview of the noise characteristics of each of these regions is given here before the governing equations for the different parameters are presented.

In the outer part of the initial region, the mixing of the secondary jet with the ambient air can be represented as the high-frequency part of a single-stream jet with the secondary jet velocity, temperature and nozzle diameter. This region is subjectively important because it generates high-frequency noise and because it contributes substantially to the total jet noise at other than low secondary-to-primary velocity ratios. This jet is referred to as the secondary jet.

In the far downstream region, where the primary and secondary flows become well mixed, the noise generated is at low frequencies and characterized by the low-frequency part of a single-stream jet at the mixed-flow jet velocity, temperature and diameter. This jet is referred to as the mixed jet.

A third region is the interaction zone where the primary jet generates noise by its interaction with the secondary jet shear layer. This is the most difficult region to characterize. Its noise is deemed to depend on

the primary jet velocity but at a level lower than that of an isolated single-stream jet. This jet is referred to as the interaction jet.

In addition to the noise sources from these three regions, there is a fourth noise source arising from the shear layer between the primary and secondary flows in the initial region of the jet. However, this source has been found to be of little significance in the context of aero-engines and has been neglected in the modelling.

## 4. THE STRUCTURE OF THE PREDICTION METHOD

The three main components of the prediction method are:

- (i) the specification of the single-stream jet parameters describing the individual components of the coaxial jet as three single-stream jets, together with the spectral cut-off and other modifications required to each,
- (ii) the prediction of the static noise spectra for each of the three sources and
- (iii) the calculation and application of the static-to-flight effects for each of the three sources.

Each of these stages is detailed in the following Sections. There are several complex issues involved in the modelling which, for the sake of brevity, are specified below without explanation. The reader is referred to Derivation 1 for elucidation.

## 5. THE THREE-SOURCE MODEL COMPONENTS

The three-source model represents the coaxial jet by three single-stream jets, each with its own characteristics, viz.

- (i) the secondary jet,
- (ii) the mixed jet and
- (iii) the interaction jet.

The specific parameters of each single-stream jet required for predicting its noise spectra are the fully-expanded jet velocity,  $V_j$ , the jet static temperature  $T_j$ , the jet nozzle diameter  $D_j$  and the area  $A_j$ .

It is then necessary to specify what spectral or other modifications should be applied to the predicted spectra.

Such details for each of the three jet components are given in the sections below.

### 5.1 Secondary Jet

The mixing of the secondary jet with the ambient air can be represented by a single-stream jet at the secondary jet flow conditions and nozzle diameter. This jet is cut off at low frequencies. Somewhat arbitrarily, the cut-off frequency is taken to be that at a Strouhal number of unity, which corresponds roughly to the end of the potential core of the jet.

## Jet Parameters

$$V_j = V_s \quad (5.1)$$

$$T_j = T_s \quad (5.2)$$

$$D_j = D_s \quad (5.3)$$

$$A_j = A_s \quad (5.4)$$

## Spectral Modification

To cut off the single-stream jet spectrum calculated with the above parameters, the predicted sound pressure levels,  $SPL_s(f)$ , are modified by the addition of a frequency weighting to give

$$SPL_s(f) + 10 \log F_u(f)$$

where

$SPL_s(f)$  is the estimated noise spectrum of the secondary jet,

$$F_u(f) = 1 - e^{-4f/f_s} \cdot \left[ 1 + \frac{4f}{f_s} + \frac{1}{2} \left( \frac{4f}{f_s} \right)^2 + \frac{1}{6} \left( \frac{4f}{f_s} \right)^3 \right] \quad (5.5)$$

and where the cut-off frequency corresponding to a Strouhal number of unity is given by

$$f_s = \frac{V_s}{D_s} \quad (5.6)$$

## 5.2 Mixed Jet

Downstream of the potential core of the primary jet, where the flows from the primary and the secondary jets are deemed to have combined, the noise generated is at low frequencies and is characterized by the mixed-flow jet conditions. This jet is cut off at high frequencies.

In this region, it is required on theoretical grounds to take into account the discharge coefficients of both the primary and the secondary nozzle flows. Consequently an area ratio based on the nozzle flow areas, viz.

$$AR' = AR \cdot \frac{C_{ds}}{C_{dp}} \quad (5.7)$$

has been used in determining the jet parameters. This is not a factor which is of significance in the aero-engine context and, if the discharge coefficients are not known, a value of unity should be entered.

## Jet Parameters

$$V_j = V_m = V_p \cdot \frac{1 + VR^2 \cdot AR' \cdot TR}{1 + VR \cdot AR' \cdot TR} \quad (5.8)$$

$$T_j = T_m = T_p \cdot \frac{1 + VR \cdot AR'}{1 + VR \cdot AR' \cdot TR} \quad (5.9)$$

$$D_j = D_m = D_p \sqrt{\frac{(1 + VR \cdot AR') \cdot (1 + VR \cdot AR' \cdot TR)}{(1 + VR^2 \cdot AR' \cdot TR)}} \quad (5.10)$$

$$A_j = A_m = \pi D_j^2 / 4 \quad (5.11)$$

## Spectral Modification

The spectral correction to the mixed jet is applied in a similar manner to that of the secondary jet, the predicted spectral levels,  $(SPL_m(f))$ , being modified to

$$SPL_m(f) + 10 \log F_D(f)$$

where

$SPL_m(f)$  is the estimated noise spectrum of the mixed jet,

$$F_D(f) = e^{-4f/f_m} \cdot \left[ 1 + \frac{4f}{f_m} + \frac{1}{2} \left( \frac{4f}{f_m} \right)^2 + \frac{1}{6} \left( \frac{4f}{f_m} \right)^3 \right] \quad (5.12)$$

and where the cut-off frequency is given by

$$f_m = \frac{V_m}{D_m} \quad (5.13)$$

## 5.3 Interaction Jet

The region in which the primary jet flow interacts with the secondary shear layer is complex both aerodynamically and acoustically. However, two key observations have enabled the noise from this region to be modelled. Firstly, experimental data from isothermal coaxial jet tests showed that (after making allowances for the noise from the secondary and the mixed jet components) the interaction noise correlated well with the eighth power of the primary velocity at 90° to the jet axis – as would be expected for an isolated jet at the primary velocity. But, secondly, turbulence measurements in this region of coaxial jets have shown a lower level than would be expected for an isolated jet. Hence the interaction noise is modelled by a jet at the primary flow conditions but with the predicted noise levels reduced as a consequence of the reduced turbulence intensity.

## Jet Parameters

$$V_j = V_i = V_p, \quad (5.14)$$

$$T_j = T_i = T_p, \quad (5.15)$$

$$D_j = D_i = D_p \sqrt{1 + VR^2 \cdot AR} \quad \text{when } D_i < D_s$$

and

$$D_j = D_i = D_s \quad \text{when } D_i \geq D_s \quad (5.16)$$

$$A_j = A_i = \pi D_j^2 / 4 \quad (5.17)$$

## Spectral Modifications

In addition to the correction for the reduced turbulence levels, the method in this Item includes both low and high frequency modifications and convective corrections to the spectral levels predicted for the interaction jet.

(a) The reduced-turbulence modification

For a hot jet, the correction for the reduced turbulence level of the interaction jet requires taking into account the balance between the dipole and the quadrupole noise sources. If the turbulence is reduced by a factor,  $r'$ , then the noise reduction is given by

$$\Delta = 10 \log r'^2 \left[ \frac{k + r'^2}{k + 1} \right] \text{dB}, \quad (5.18)$$

where

$$k = 7 \cdot \frac{\left\{ \frac{T_p/T_0 - 1}{V_p/a_0} \right\}^2}{1 + 0.65 \left( \frac{T_p}{T_0 - 1} \right)} \quad (5.19)$$

The turbulence factor  $r'$  has been found to depend on the ratio of the secondary annulus height to the primary diameter modified to take account of the change in the secondary spreading rate in flight in the form

$$r' = 0.6 [\exp(-0.7h')] + 0.4 \quad (5.20)$$

where

$$h' = \left( \frac{1 + V_a/V_s}{1 - V_a/V_s} \right) \cdot (D_s/D_p - 1). \quad (5.21)$$