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Aircraft performance from flight test
Part 1: derivation of a mathematical model

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AIRCRAFT PERFORMANCE FROM FLIGHT TEST
PART 1: DERIVATION OF A MATHEMATICAL MODEL

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AIRCRAFT PERFORMANCE FROM FLIGHT TEST
PART 1: DERIVATION OF A MATHEMATICAL MODEL

1. NOTATION, DIMENSIONS AND DEFINITIONS

1.1 Notation and Dimensions

		Dimensions
A	cross-sectional area of stream tube	L^2
a_x, a_y, a_z	accelerometer readings along Ox, Oy, Oz directions	LT^{-2}
C_D	drag coefficient	–
C_L	lift coefficient	–
C_m	pitching moment coefficient	–
$\bar{c}$	mean chord of wing	L
D	drag	MLT^{-2}
F_{G0}	gauge stream force at engine Station 0; intake momentum or ram drag (see Appendix D)	MLT^{-2}
F_{G9}	gauge stream force at engine Station 9; standard gross thrust of engine (see Appendix D)	MLT^{-2}
g	acceleration due to gravity	LT^{-2}
I_X, I_Y, I_Z	moments of inertia of aircraft about Ox, Oy, Oz axes	ML^2
I_{XY}, I_{YZ}, I_{ZX}	products of inertia of aircraft about Ox and Oy; Oy and Oz; Oz and Ox	ML^2
J_X, J_Y, J_Z	components of angular momentum of engine rotors about Ox, Oy and Oz axes	ML^2T^{-1}
L	lift	MLT^{-2}
L, M, N	rolling, pitching and yawing moments about Ox, Oy, Oz axes (see Sketch 1.1)	ML^2T^{-2}
M	Mach number	–
M_0	pitching moment at zero lift	ML^2T^{-2}
M_q	pitching moment derivative due to rate of pitch, q	–
$M_{\dot{w}}$	pitching moment derivative due to acceleration in heave	–
m	mass of aircraft	M

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N_H	engine high pressure turbine shaft rotational speed	T^{-1}
N_L	engine low pressure turbine shaft rotational speed	T^{-1}
Ne	number of engine	–
NPR	nozzle pressure ratio $\left(= P_{T7}/P_{S0}\right)$	–
O	origin of axis system and centre of mass of aircraft	–
P_{T7}	total pressure at engine Station 7 (See Appendix D)	$ML^{-1}T^{-2}$
P_S	static pressure	$ML^{-1}T^{-2}$
p, q, r	rates of roll, pitch and yaw about Ox, Oy, Oz axes	T^{-1}
q_k	kinetic pressure of freestream $\left(0.5\rho V^2\right)_0$ or $\left(0.5\gamma P_S M^2\right)_0$	$ML^{-1}T^{-2}$
S_W	reference area of wing	L^2
t	time	T
U	velocity in engine stream tube	LT^{-1}
u, v, w	components of aircraft velocity along Ox, Oy, Oz axes (see Sketch 1.1)	LT^{-1}
V	velocity along the air-path (true airspeed of aircraft)	LT^{-1}
X, Y, Z	forces parallel to Ox, Oy, Oz body axes (see Sketch 1.1)	MLT^{-2}
x, y, z	co-ordinates in Ox, Oy, Oz body axes (see Sketch 1.1)	L
α	angle of attack	–
β	angle of sideslip	–
γ	ratio of specific heats for air (defined as 1.4)	–
Δ	small change of a quantity	–
ε	mean angle of downwash at the tailplane	–
ε_0	mean angle of downwash at the tailplane when $\alpha = 0$	–
η_T	angle of tailplane relative to aircraft longitudinal body axis	–
ρ	density	ML^{-3}
χ, τ, σ	angles of thrust vector as defined in Section 4.3.1	–
ψ, θ, ϕ	angles of azimuth, inclination (pitch), and bank	–

All angular measures are in radians

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Subscripts

A	aerodynamic
Ac	accelerometer
Aft	after-body force
a	air-path (or wind) axes
B	buoyancy
CG	centre of gravity
$Cowl$	intake cowl suction force
E	earth axes
$FG0$	intake momentum drag (gauge stream force at engine Station 0)
$FG9$	engine standard gross thrust (gauge stream force at engine Station 9)
g	gravity
Int	centre of plane of the intake face
$Meas$	measured
Noz	nozzle
P	propulsion
Pre	intake pre-entry force
q	pitch rate
Ref	moment reference point
T	tailplane
WB	wing/body
0	condition in free stream
9	condition at engine Station 9
19	condition at engine Station 19

See Appendix D for engine station designations.

One dot above a quantity indicates the first derivative of that quantity with respect to time, t.

Square brackets indicate a function of the quantity or quantities within the brackets.

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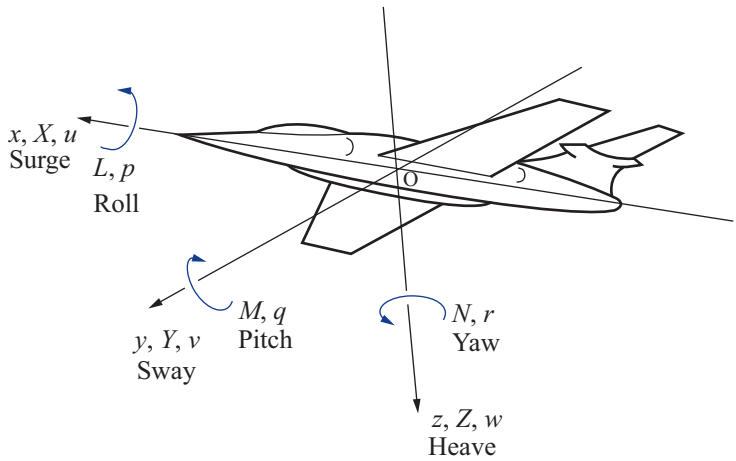
1.2 Definitions

The six degrees of freedom in aircraft body axes are as follows:

surge	linear motion along the x-axis,
sway	linear motion along the y-axis,
heave	linear motion along the *z*-axis,
roll	rotation about the *x*-axis,
pitch	rotation about the *y*-axis,
yaw	rotation about the *z*-axis.



Sketch 1.1 Body Axis System

Sketch 1.1 shows the motions in the body axis system. The arrows indicate the positive direction of each parameter.

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2. INTRODUCTION

This Data Item considers the derivation, from flight test, of an aerodynamic model for the purpose of aircraft performance prediction. The analysis is limited to rigid fixed-wing aircraft (aeroplanes) of a conventional "aft-tailed" configuration with an all moving tail and one, or more, turbojet/fan engines, in a stable non-moving atmosphere. However, the principles are readily extended to less conventional configurations and propeller driven aircraft.

The analysis requires a test aircraft fitted with three accelerometers mounted orthogonally within the airframe and a means of determining rotational rates and accelerations about the three axes. Precise air data are also required so that Mach number, airspeeds, altitude and angles of attack and sideslip can be determined. It is assumed here that these derived air data are available (see References 1 and 2 for a discussion on the calibration of air data sensors).

Aircraft performance is generally concerned with the efficient use of the aircraft and hence it is assumed to be operating in flight regimes where there is negligible motion normal to the plane of symmetry, that is, sideslip angles are zero. Consequently, the aerodynamic coefficients of interest are those of drag, lift and pitching moment; these are to be derived from appropriate flight manoeuvres within a consistent and coherent modelling framework.

Flight dynamics deals in general with the motion of an aircraft under the influence of many forces. These forces include those due to propulsive, gravitational and inertial effects in addition to those due to aerodynamics. In principle aircraft performance is concerned primarily with the study of these forces due to linear velocities. However, when considering manoeuvre performance, account has to be taken of longitudinal rotary effects when modelling the aerodynamic coefficients. Also, due to the limitations in the accuracy with which test manoeuvres can be performed, sufficient instrumentation is required in the test aircraft to enable adjustments to be made for deviations from the planned manoeuvres. It is, therefore, necessary that the test aircraft instrumentation system is able to record velocities and accelerations in all six degrees of freedom, control deflections and those engine parameters required by the propulsion system simulation.

In the derivation of aerodynamic coefficients from flight test, the data are obtained at various discrete conditions. These conditions include the engine intake mass flow ratio, exhaust nozzle pressure ratio, centre of gravity, Reynolds number and aircraft geometric configuration for each test point, all of which may have an effect on forces and moments experienced by the airframe. In order to normalise the derived aerodynamic coefficients it is necessary to adjust them to reference conditions. These adjustments to reference conditions will need to allow for variations of engine power setting, trim (due to centre of gravity and manoeuvre requirements) and Reynolds number. Trim effects are dealt with by separately accounting the tailplane and its environment, the reference condition being the aircraft wing/body without the tailplane. Other effects may need to be considered depending on the aircraft configuration. Sometimes these adjustments may be derived from the flight testing if appropriate manoeuvres are flown, at other times they may need to be deduced from wind tunnel testing or determined analytically. The model so obtained will, therefore, include incremental forces and moments as well as the normalised aerodynamic coefficients.

To maintain coherence within the model it is important that a comprehensive thrust/drag accounting procedure is observed. The incremental forces and moments should be allocated to either the thrust or drag account in a manner that is theoretically consistent and practical (see Reference 3).

The methods documented here are not intended to be prescriptive but are meant to provide guidance on how an aircraft performance model might be determined from flight test. It is essential that any subsequent synthesis performed to predict aircraft performance must be consistent with the analysis carried out in the derivation of the model.

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3. FLIGHT DYNAMICS

In principle, the motion of an aircraft is influenced by seven types of force:

- gravitational,
- propulsive,
- aerodynamic (static) depending on linear velocity,
- aerodynamic (rotary) depending on angular velocity,
- aerodynamic (static) depending on control deflections,
- inertia arising from the mass distribution and combinations of linear and angular velocities and accelerations (including engines),
- buoyancy.

By considering the motion of a body of constant mass with reference to a set of axes fixed at the centre of gravity of that body and thus moving with it, a set of force and moment equations in six degrees of freedom can be formulated. From d'Alembert's principle at constant mass it follows that:

- the sum of the inertia forces and the external forces is zero
- the sum of the moments, about any axis, due to the inertia forces and the external forces is zero.

This leads to the following equations of motion for a rigid aircraft in a stable non-moving atmosphere.

$$m(\dot{u} - r\,v + q\,w) = X_A + X_P + X_g + X_B\ , \tag{3.1}$$

$$m(\dot{v} - p\,w + r\,u) = Y_A + Y_P + Y_g + Y_B\ , \tag{3.2}$$

$$m(\dot{w} - q\,u + p\,v) = Z_A + Z_P + Z_g + Z_B\ , \tag{3.3}$$

$$I_X\dot{p} - (I_Y - I_Z)\,qr - I_{YZ}\,(q^2 - r^2) - I_{ZX}\,(\dot{r} + pq) - I_{XY}\,(\dot{q} - rp) + J_Z q - J_Y r = L_A + L_P + L_B\ , \tag{3.4}$$

$$I_Y\dot{q} - (I_Z - I_X)\,rp - I_{ZX}\,(r^2 - p^2) - I_{XY}\,(\dot{p} + qr) - I_{YZ}\,(\dot{r} - pq) + J_X r - J_Z p = M_A + M_P + M_B\ , \tag{3.5}$$

$$I_Z\dot{r} - (I_X - I_Y)\,pq - I_{XY}\,(p^2 - q^2) - I_{YZ}\,(\dot{q} + rp) - I_{ZX}\,(\dot{p} - qr) + J_Y p - J_X q = N_A + N_P + N_B\ . \tag{3.6}$$

If Oxz is a plane of symmetry for the aircraft then I_{XY} and I_{YZ} are zero.

The buoyancy due to the air displaced by the aircraft, seen in terms X_B, Y_B, Z_B, L_B, M_B and N_B in Equations (3.1) to (3.6), will form a very small part of the force and moment balances and in practice can usually be ignored. For aircraft of large volume this may be significant although is unlikely to represent more than 1% of aircraft weight (see Section 8 of ESDU 85029, Reference 4, and Section 3.6 of ESDU 79018, Reference 5).

Also, for conventional turbo-jet powered aircraft, considered in this Item, the angular momentum terms of the engine rotors, as defined by the terms involving J_X, J_Y and J_Z in the same equations, will normally be very small and can be ignored. This is particularly true for large turbo-jet powered aeroplanes that will almost certainly employ contra-rotating engine shafts. (These engine momentum terms are included in the equations of motion of a rigid aircraft given in ESDU 67003, see Reference 6.)

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Ignoring buoyancy and engine momentum terms, Equations (3.1) to (3.6) become

$$m(\dot{u} - r\,v + q\,w) = X_A + X_P + X_g\,, \tag{3.7}$$

$$m(\dot{v} - p\,w + r\,u) = Y_A + Y_P + Y_g\,, \tag{3.8}$$

$$m(\dot{w} - q\,u + p\,v) = Z_A + Z_P + Z_g\,, \tag{3.9}$$

$$I_X\dot{p} - (I_Y - I_Z)\,qr - I_{YZ}\,(q^2 - r^2) - I_{ZX}\,(\dot{r} + pq) - I_{XY}\,(\dot{q} - rp) = L_A + L_P\,, \tag{3.10}$$

$$I_Y\dot{q} - (I_Z - I_X)\,rp - I_{ZX}\,(r^2 - p^2) - I_{XY}\,(\dot{p} + qr) - I_{YZ}\,(\dot{r} - pq) = M_A + M_P\,, \tag{3.11}$$

$$I_Z\dot{r} - (I_X - I_Y)\,pq - I_{XY}\,(p^2 - q^2) - I_{YZ}\,(\dot{q} + rp) - I_{ZX}\,(\dot{p} - qr) = N_A + N_P\,. \tag{3.12}$$

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4. PERFORMANCE MODEL

4.1 The Model and Reference Conditions

Aircraft performance is concerned with energy efficient flight in which there is negligible motion normal to the plane of symmetry (Oxz-plane). For performance modelling it is, therefore, usual to reduce the six equations of motion to three. These three longitudinal equations are the two orthogonal forces in the Oxz-plane (Equations (3.7) and (3.9)) and the moment about the Oy-axis, the pitching moment (Equation (3.11)). To solve these equations for the aerodynamic forces and moment it is necessary to model the components in a manner which is both representative of the physical interactions and useful for performance determination.

The performance model consists of forces and moments due to both the airframe aerodynamics and the propulsion system. It is important that an appropriate accounting system is used to differentiate between airframe and propulsion forces so no components are overlooked and none are duplicated.

Fuel usage during flight results in a more or less continuous change in the position of the centre of gravity making this an inconvenient point about which to define moments. It is, therefore, useful to refer the pitching moment to a fixed point in the airframe, the moment reference point. It is usual, but not essential, to use a point that approximates to the aircraft aerodynamic centre, for example the quarter mean chord point for a conventional subsonic aircraft.

The aircraft's drag and lift will also vary with these changes in centre of gravity position due to different tailplane angles, and hence different tailplane forces and moments, being required to trim the aircraft. These trim effects may also be functions of engine throttle setting and variations in longitudinal control setting during dynamic manoeuvres.

In order to normalise the aerodynamic forces it is useful to model the airframe aerodynamics as a wing and body (fuselage) and a separate tailplane. These wing/body aerodynamic coefficients are essentially independent of the trim requirements and therefore the "tail-off" case is suitable as a reference condition. Consequently the longitudinal aerodynamics of a conventionally configured aircraft is partitioned into the following components (see Sketch 4.1):

- wing/body aerodynamics,
- tailplane aerodynamics,
- a downwash field which connects the wing/body to the tailplane,
- interference effects between propulsion system and airframe.

Due to the interactions of the propulsion system with the airframe it is necessary to adjust the derived aerodynamic coefficients to reference engine conditions; these are addressed in Section 4.3.2. Also some aerodynamic coefficients vary significantly with Reynolds number; a reference value should be defined to which all the appropriate data are adjusted.

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