



Stress concentration factors in keyways

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Dr J.W.H. Price* — Integrity Partners, Australia.

Mr A. Serna — Meggitt Polymers and Composites, Loughborough

Mr T. Spencer — Lloyd's Register Global Technology Centre, Southampton

* Corresponding Member

The construction and subsequent development of the Data Item were undertaken by

Dr D.L. Downs — Senior Engineer.

The person with overall responsibility for the work in this subject area is Mr A.C. Quilter, Head of Strength Analysis Group and Technical Director – Aerospace..

STRESS CONCENTRATION FACTORS IN KEYWAYS

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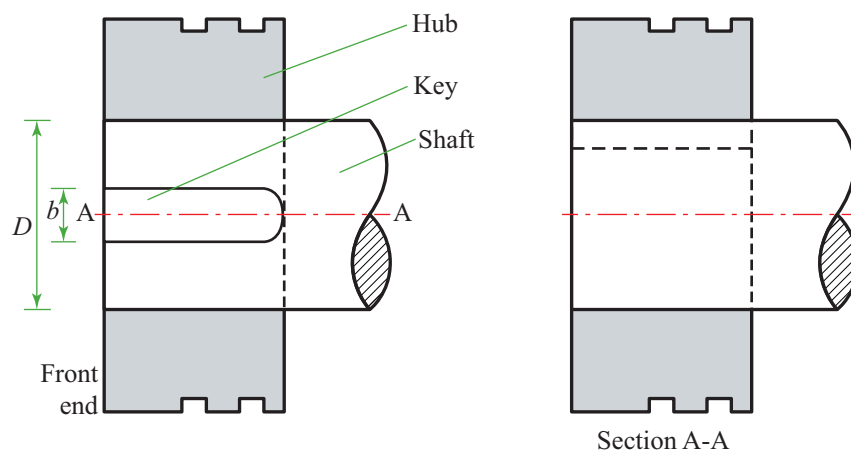
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STRESS CONCENTRATION FACTORS IN KEYWAYS

1. INTRODUCTION

Many shaft and hub designs include arrangements for transmitting torque from one element to another along the shaft. Keys and keyways are common torque transfer elements. This Item discusses the stresses that are present in keyways and suggests ways to optimise the shape of the keyway to improve the fatigue life of the shaft.

Since the review of stress concentration data in keyways by Peterson (Derivation 6), more data has been published on this subject. Almost 20 years after the publication of Peterson's work, a survey of the later developments in the field of stress concentrations ("post-Peterson") was carried out (Derivation 9). This survey cited work to determine the stress concentration in end-milled keyways in hubs under torsion using frozen-stress photoelastic models (Derivation 8). The greatest stresses in keys were found to occur very near the front ends of their contact with the hub, see Sketch 1.1. It was reported that the stress concentrations in the shaft could be reduced by chamfering and rounding the ends of the keys, by maximizing the lengths of the hub and the keys, and by moving the position of the torque applied by the key as far as possible from the front of the hub by extending the length of the keyway. It has also been found that a hole drilled in the key perpendicular to the shaft axis near to the key end would reduce the peak forces in the key end and so reduce the stress concentrations in the shaft near the key end (Derivation 7). More recently, Pedersen (Derivation 12) published details of using a numerical finite element analysis (FEA) to determine the stress concentration factors in keyways and ways to optimise keyway design.



Sketch 1.1

The general aspects of common keys and keyway designs such as end milled and sled runner are presented in Section 2. More information on keyway design can be found in References 13, 14 and 15 and in ESDU 86028*.

Section 3 discusses the stress concentrations in end-milled keyways during bending, torsion and combined bending and torsion of a shaft. These were estimated using photoelastic techniques and a numerical FEA. In Section 3.1, the stress concentrations determined from photoelastic investigations of keyways during bending of a shaft with a diameter less than 165 mm (6.5 in) are given. In Section 3.2, the results from a photoelastic investigation of stresses in a keyway in a shaft subject to torsion with diameter of less than 165 mm (6.5 in) are discussed along with the results from the FEA of stresses in a keyway designed for a

* ESDU 86028, 'Fatigue strength of keyed assemblies'.

parallel key (as per DIN 6885-1) in a shaft with a diameter between 6 mm (0.24 in) and 500 mm (19.7 in). The stress concentrations in keyways in shafts under combined bending and torsion loading are presented in Section 3.3.

Section 4 presents the stress concentration factors for a keyed assembly when torque is transmitted by the key for a sled runner type keyway in a shaft with a diameter of between 10 mm (0.39 in) and 305 mm (12 in).

The results from an optimisation of keyway design which has been investigated using an FEA to alter the keyway dimensions and the shape of the end of the keyway are given in Section 5.

A worked example considering the design of a shaft with a keyway is discussed in Section 7.

1.1 Notation

Both SI and British units are quoted but any coherent system of units may be used.

A	position		
<i>A</i>	length of semi-minor axis of ellipse	m	in
B	position		
<i>B</i>	length of semi-major axis of ellipse	m	in
<i>b</i>	keyway width	m	in
<i>D</i>	shaft diameter	m	in
<i>D_k</i>	nominal shaft diameter at the bottom of the keyway	m	in
<i>d</i>	key diameter		
<i>e</i>	offset (in a Woodruff key)	m	in
<i>h</i>	key height	m	in
<i>h'</i>	Woodruff key depth in shaft	m	in
<i>K</i>	stress concentration factor based on gross cross-sectional stress		
<i>K_s</i>	stress concentration factor for shear stress		
<i>L</i>	dimension in optimised keyway	m	in
<i>l</i>	key length	m	in
<i>M</i>	moment	N m	lbf in
<i>r</i>	fillet radius	m	in
<i>T</i>	torque	N m	lbf in
<i>t</i>	keyway depth	m	in
<i>w</i>	key width	m	in

η	super-elliptical power (design parameter)		
σ	reference stress	N/m ²	lbf/in ²
τ	reference shear stress	N/m ²	lbf/in ²

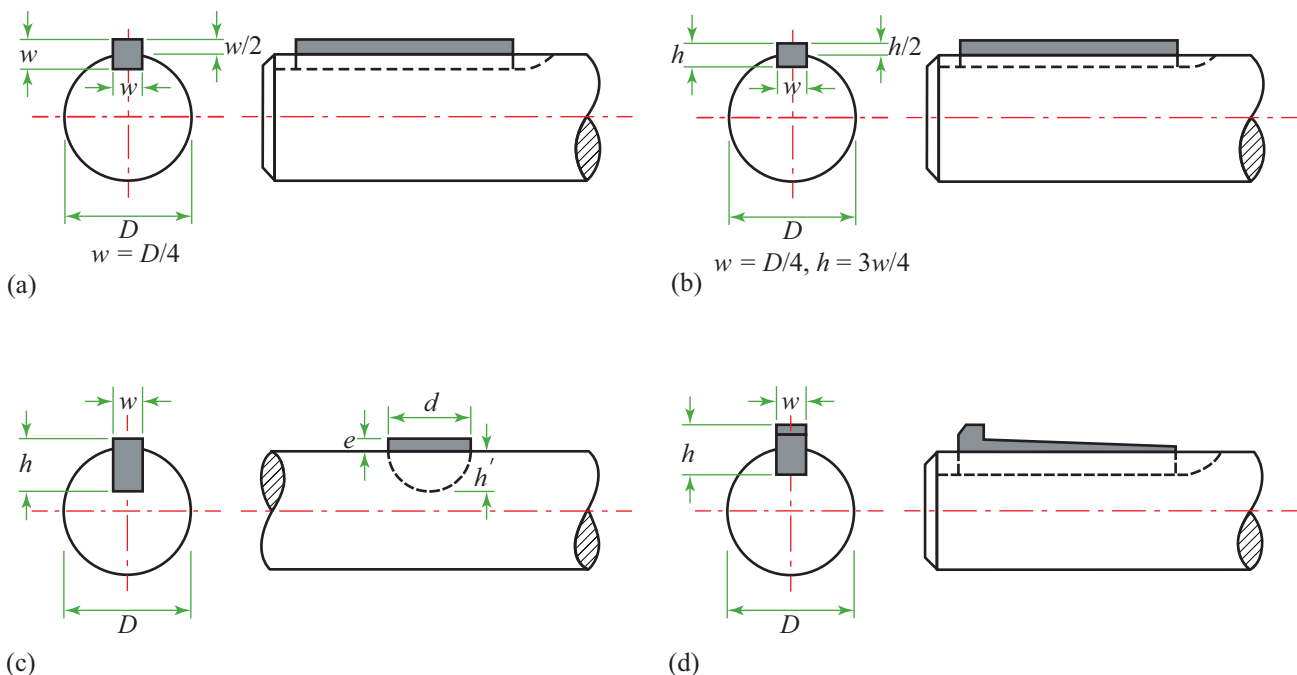
2. GENERAL

2.1 Types of Key

When power is to be transmitted to, or supplied from, a rotating shaft then components such as pulleys, sprockets, or gears can be attached to the shaft. To prevent any relative rotation between the shaft and the attached component, the connection between the components and the shaft must be secure. Retention devices, such as keys, can be used to prevent such relative motion.

Sketch 2.1 shows some common types of keys. The square key (Sketch 2.1(a)), the flat key (Sketch 2.1(b)) and the Woodruff key (Sketch 2.1(c)) are probably used more widely than others. Recommendations for key size and keyway depth, as a function of shaft diameter, are provided in Standards (see References 13, 14 and 15). The standard sizes of square and rectangular keys are given on Table 8.1. The length of the key, l , is based on the length of the hub and the amount of torsional load to be transferred.

It is often desirable to size a key so that it will fail in shear in the event of overloading. This means that using an inexpensive shaft key as a safety device protects the shaft and other expensive machine elements from damage. Key materials tend to be soft, ductile low-carbon steels that have been cold rolled to standard key sizes then cut to length. The key design must also ensure that the range of operational torques are transferred without failure whilst the torques produced by overload in situations, like jamming or seizure of the component, do cause the key to shear off across the shaft hub interface.



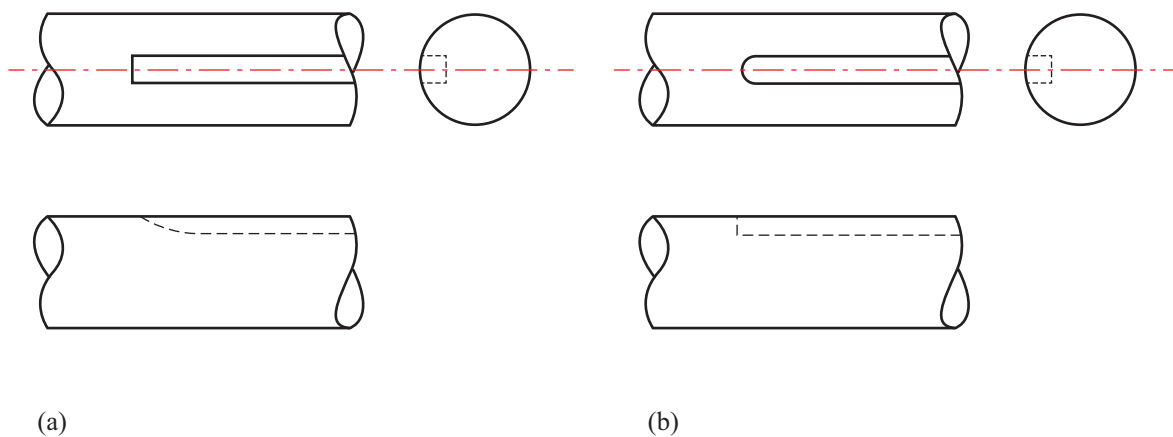
Sketch 2.1 Examples of keys: (a) square key, (b) flat key, (c) Woodruff key, (d) gib-head tapered key (Derivation 11)

Tapered keys are used during assembly of the shaft to ensure that the hub position can be adjusted for the best axial location and when they have been firmly driven into place they also act to prevent relative axial motion. A typical inclination of a tapered key is 1:100. A tapered driving key with a thicker end, a feature referred to as a nose, is called a gib-head key (Sketch 2.1(d)). These types of keys are used if driving in or out can only be done from one side. The head makes it possible to remove the key without access to the other end of the shaft but the end projection may be classed as an operational hazard.

Woodruff keys (Sketch 2.1(c)) can also assume the function of taper sunk keys because, due to their rotatable mounting in the keyway, they are able to adapt themselves to the taper in a hub keyway. The Woodruff key can be used for locating a hub in position against a shaft shoulder since the end of the keyseat can be positioned away from the shoulder stress concentration region. The Woodruff key also yields better concentricity after assembly of, say, a hub and a shaft. This is especially important for shafts that operate at high speeds, such as a turbine wheel and shaft. Dimensions for some standard Woodruff keys are given in Table 8.2 and sizes of Woodruff keys suitable for various shaft diameters are given in Table 8.3.

2.2 Keyway Designs

To assess the stress concentrations in a keyed shaft, the shape of the keyway end must be taken into account. Sketch 2.2 shows two standard designs of keyway machined into the shaft to capture the key. Note that keyways may also run out at the end of the shaft (this is not shown on Sketch 2.2). The end-milled keyway is cut by an end-milling cutter and the sled runner keyway is cut by a circular side-face cutter. The end-milled keyway is more widely used owing to its compactness and definite key positioning longitudinally. In bending, the stress concentration in a sled-runner keyway is less than for the end-milled keyways so with respect to fatigue, the sled runner keyway is the better design. ESDU 86028 gives further details of fatigue loading in different keyway designs.



Sketch 2.2 Examples of keyways: (a) sled-runner keyway and (b) end-milled, profiled or semi circular end keyway

3. STRESSES IN END-MILLED KEYWAYS

3.1 Shafts Loaded in Bending

Figure 1 shows the stress concentration factors in an end-milled keyway in a shaft with a diameter less than 165 mm (6.5 in) under bending obtained from a photoelastic investigation (Derivations 5 and 6). In bending, the reference stress is defined as the maximum bending stress in the shaft *without the keyway*. The positions of maximum stress at the surface and in the fillet, position **A** and **B**, respectively, are shown on Figure 1. The stress concentration factor at the surface, position **A**, is $K_A = 1.6$ and it is not significantly affected by moderate changes in the keyway depth ratio and is independent of the r/D ratio. The maximum stress at the surface, position **A**, occurs at an angle within about 10 degrees of the tangent at that point.

The maximum stress in the fillet is located at the end of the keyway, position **B**, about 15 degrees up on the fillet.

The stress concentrations in end-milled keyways (Figure 1) have been found for shafts with a diameter less than 6.5 in (0.165 m) for which $t/D = 0.125$. For larger diameter shafts with $t/D = 0.09$, the stress concentration does not differ significantly, taking into account the different values of t/D and r/D . It has been suggested that stress concentration values obtained when $t/D = 0.125$ and $r/D = 0.0208$ can be used for other designs incorporating any shaft diameter (Derivation 6).

Note that some of the results on Figure 1 were obtained from tensile loading tests and the stress concentration factors were then scaled down to compensate for the stress gradient that occurs for the bending case. This method was found to give good agreement when compared to those results obtained directly using bending (see ESDU 86028).

3.2 Shafts Loaded in Torsion

Figure 2 shows the stress concentration at the fillet of an end-milled keyway under torsion using a photoelastic investigation (Derivations 1, 3 and 6). At the surface of the keyway, position **A**, the stress concentration is $K_A = \sigma_{\max}/(16T/\pi D^3) \approx 3.4$ (Derivation 1). The maximum normal stress is tangential to the semi circle and occurs at 50 degrees to the axial direction and is independent of r/D . The maximum shear stress is at 45 degrees to the maximum normal stress and $K_{sA} = \tau_{\max}/\tau \approx 1.7$.

In the fillet, the maximum shear stress is at position **B** and is in the longitudinal and perpendicular direction. The maximum normal stresses are of the same magnitude and at 45 degrees to the shear stress meaning that $K_{sB} = \tau_{\max}/\tau$ is equal to $K_B = \sigma_{\max}/\sigma = \sigma_{\max}/\tau$ where $\tau = 16T/\pi D^3$.

On Figure 2, the dashed curve shows the stresses determined using a photoelastic investigation in the keyway when torque is transmitted through a key of length $2.5D$. Note that in practice, the degree of press fit pressure is an important factor and during the tests the shaft friction was held to a minimum so that all of the torque was being transmitted via the key during the photoelastic investigations.

A finite element analysis of the stresses found in a keyway loaded in torsion without the key inserted has been investigated (Derivation 12). The stress concentration in the prismatic part of the keyway can be estimated from a study without the torsion coming from the key by adding 12 per cent to the stresses in the prismatic part.

The stresses in the fillet at the bottom of the prismatic part of a keyway design based on the DIN 6885-1 Standard were determined for a shaft under pure torsion and with a diameter between 6 mm (0.24 in) $\leq D \leq 38$ mm (1.5 in) and 38 mm (0.15 in) $\leq D \leq 500$ mm (19.7 in) as shown in