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Stress concentration factors for optimised notches and fillets in flat plate

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STRESS CONCENTRATION FACTORS FOR OPTIMISED NOTCHES AND FILLETS IN FLAT PLATE

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STRESS CONCENTRATION FACTORS FOR OPTIMISED NOTCHES AND FILLETS IN FLAT PLATE

1. INTRODUCTION

This Item reviews some contemporary publications in the field of optimising typical notch and fillet shapes as part of the design of structural components. The data may also be used to guide remedial work carried out on such parts to significantly reduce the peak elastic stresses and to increase the fatigue life, inspection interval and availability of structural parts containing stress concentrations (Derivation 8). ESDU 15005*, which accompanies this Item, considers the optimisation of holes in flat plate.

In general, notches that are present in many structural components are there as part of the initial design or as a result of modifications to an ageing structure. Usually, the design of a notch is based on a circular or elliptical profile and, although the stress concentration factors for these standard geometries are readily available (see Reference 36, for example), such shapes can be improved in terms of stress concentration (Derivation 5). The same can be said for fillets which are also a standard feature of loaded components. It has been reported that for cases when an optimised notch or fillet shape was used to replace one with a customary circular or elliptical profile, there was an increase in the fatigue life of the components (Derivation 1).

This Item discusses the results from a variety of 2D finite element analyses (FEA) that have been carried out to optimise fillet and notch shapes, to reduce the peak stresses and to generate a near constant distribution of tangential stress around a significant portion of the boundary. The FEA to optimise the shapes has been carried out on:

- notches on opposite edges of finite width plates subject to tensile loading, Section 2, and
- fillets under tensile loading and bending, as well as on fillets with a stress concentration close to unity, see Section 3.

Some worked examples are given in Section 5.

The shape optimisation approach that was used to obtain the data for this Item is based on an iterative gradientless method to produce a free-form optimal shape in a fully automated manner. This method has been used to create a body of transferable optimal shapes that can be readily applied to many different types of common engineering problems involving stress concentrations (Derivations 1, 3, 4 and 5). This method was originally proposed by Heller *et al.* (Reference 38), and over the intervening years has been significantly enhanced to:

- handle multiple stress peaks,
- achieve stress solutions in the presence of minor misalignments of the loading direction for multiple load cases, and
- consider geometric constraints that may be placed on the shape of the optimised boundary (Derivations 7 and 8 and Reference 41).

* ESDU 15005, 'Stress concentration factors for optimised holes in flat plates'.

The gradientless shape optimisation algorithm works by adding material to local regions of high stress and/or removing material from local regions of low stress on the stress boundary. It is used because it is easy to implement, readily amenable to extension, and has been created to achieve very good solutions for many different types of practical stress minimisation problems, some of which involve geometrical constraints (which have not been a focus of other investigators). The underlying approach that is used is based on the aim of achieving a minimum peak stress by directly attempting to make the stresses uniform along the longest possible portion of a stress-concentrating boundary, as uniformity of stress is an essential requirement for optimality. This idea is well known, and has been successfully used in various analytical studies (References 10, 11, 17, 20, 21 and 33), numerical studies (Derivation 8 and References 27, 35, 38 and 41), and experimental studies (Reference 12, 23 and 25).

Optimised 'ideal' shapes for notches and fillets tend to have sharp corners. This may lead to unintentional stress concentrations if the loading direction changes from the ideal design condition. These shapes should not be used in practice unless some rounding is applied to their corners to make them more robust. The worked examples in Section 5 are based on the cases of the optimised 'ideal' shapes when a minimum radius of curvature has been used in the FEA to numerically round off the sharp corners producing more usable notch and fillet shapes.

During the typical lifetime of an aircraft, certain stress concentration locations may become fatigue critical and an effective repair option is required. To extend the life of the aircraft component, reshaping is often carried out to repair an area where damaged material has had to be removed, or it may be used as a pre-emptive measure to avoid cracking. One practical use of optimised shape geometries has been reported in the development of robust, optimal free-form shapes in four of the most fatigue prone locations in stiffener run-outs on a General Dynamics F-111 wing pivot fitting. These optimal shapes were able to produce a 30 to 40 per cent reduction in the peak elastic stress concentration compared to shapes with circular radii at the ends, whilst maintaining the buckling strength of the component (Derivation 2).

This Item complements ESDU 09014* that covers stress concentrations for notches, fillets and holes (of standard geometry with a circular or elliptical profile) in bars and strips and a wider range of geometric parameters. Note that in ESDU 09014, the early photoelastic investigations of stress concentration tended to give results up to 10 per cent lower than the finite element analyses. In most cases, the current FEA to optimise shapes gives lower stress concentrations than previously reported and in some cases are closer to the exact analytical solution (Derivation 6).

* ESDU 09014, "Elastic stress concentration factors. Geometric discontinuities in flat bars or strips of isotropic material".

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<i>Section number and description</i>	<i>Number</i>	<i>Title</i>	<i>Variables</i>
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	Table 6.2	Gross stress concentration factors for symmetrical optimised notches under tensile stress (Derivation 5)	$L/2h = 1-8$, $W/2h = 2.222-20$, $\rho'_{\min} = 0.3$
	Table E1*	Non-dimensional coordinates for optimal notch shapes under tensile stress with $L/2h = 1$ and 2 and $\rho'_{\min} = 0.1$ (Derivation 5)	$L/2h = 1$ and 2, $W/2h = 2-20$, $\rho'_{\min} = 0.1$
	Table E2*	Non-dimensional coordinates for optimal notch shapes under tensile stress with $L/2h = 3$ and 5 and $\rho'_{\min} = 0.1$ (Derivation 5)	$L/2h = 3$ and 5, $W/2h = 2-20$, $\rho'_{\min} = 0.1$
	Table E3*	Non-dimensional coordinates for optimal notch shapes under tensile stress with $L/2h = 8$ and 12 and $\rho'_{\min} = 0.1$ (Derivation 5)	$L/2h = 8$ and 12, $W/2h = 2-20$, $\rho'_{\min} = 0.1$
	Table E4*	Non-dimensional coordinates for optimal notch shapes under tensile stress with $L/2h = 16$ and $\rho'_{\min} = 0.1$ (Derivation 5)	$L/2h = 16$, $W/2h = 2-20$, $\rho'_{\min} = 0.1$
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	Figure 2	Gross stress concentration factor for circular and optimal notch shapes with $\rho'_{\min} = 0.1$ and various $L/2h$ and $W/2h$ ratios (Derivation 5)	$L/2h = 1-16$, $W/2h = 2-20$, $\rho'_{\min} = 0.1$

* See accompanying Excel Spreadsheet.

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Section number and description	Number	Title	Variables
Section 2.2 Notches without stress concentration on opposite edges of flat plate	Figure 3	Variation of $W/2h$ and $L/2h$ for circular and optimal notches with $K' = 1.005$ (Derivation 5)	Optimal notch, $L/2h = 2.3 - 13.8$, $W/2h = 1.5 - 5$, $\rho'_{\min} = 0.1$ Circular notch, $L/2h = 1 - 15.4$, $W/2h = 1 - 1.8$, $\rho'_{\min} = 0.1$ Limiting case, $L/2h = 1$ and 20 , $W/2h = 1$
Section 3.1 Fillets in flat plate under applied tensile stress	Table 6.3	Net stress concentration factors for optimised fillets under tensile stress (Derivation 1)	$W/w = 1.5 - 2$, $l/h = 0.825 - 5$
	Table 6.4	Comparison of net stress concentration factors for optimal fillets with circular fillets under tensile stress (Derivation 1)	$W/w = 1.33$, $l/h = 1$ and 2.642
	Table E5*	Non-dimensional coordinates for optimal fillet shapes under tensile stress when $l/h = 0.5$, 1 and 2 (Derivation 1)	$W/w = 2$, 1.67 and 1.33 , $l/h = 0.5$, 1 and 2
	Table E6*	Non-dimensional coordinates for optimal fillet shapes under tensile stress when $l/h = 3$, 4 and 5 (Derivation 1)	$W/w = 2$, 1.67 and 1.33 , $l/h = 3$, 4 and 5
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* See accompanying Excel Spreadsheet.

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Section number and description	Number	Title	Variables
Section 3.3 Fillets without stress concentration in flat plate	Table 6.6	Previously reported and optimised net stress concentration factors for fillets (with $\rho'_{\min} \geq 0.15$) under tensile stress (Derivation 4)	$2h/w = 0.25$ and 3 , $2l/w = 3.47$ and 4.25 , $\rho'_{\min} \geq 0.15$
	Table 6.7	Comparison of net stress concentration factors for optimised fillets (with $\rho'_{\min} \geq 0.15$) under tensile or bending stress (Derivation 4)	$2h/w = 0.33 - 2.5$, $2l/w = 3.66$ and 4.25 , $\rho'_{\min} \geq 0.15$
	Table 6.8	Fillet height, $2h/w$, and fillet length, $2l/w$, of optimised fillet shape under tensile stress with $K' = 1.005$ and when $\rho'_{\min} = 0.1$ (Derivation 4)	–
	Table E9*	Non-dimensional coordinates for fillet shapes with $K' = 1.005$ and $\rho'_{\min} \geq 0.15$ under tensile stress for $2h/w$ between 0.07 and 0.333 (Derivation 3)	$2h/w = 0.07 - 0.333$, $2l/w = 2.58$ and 3.66 , $\rho'_{\min} \geq 0.15$
	Table E10*	Non-dimensional coordinates for fillet shapes with $K' = 1.005$ and $\rho'_{\min} \geq 0.15$ under tensile stress for $2h/w$ between 0.5 and 1.3 (Derivation 3)	$2h/w = 0.5 - 1.3$, $2l/w = 3.91$ and 4.2 , $\rho'_{\min} \geq 0.15$
	Table E11*	Non-dimensional coordinates for fillet shapes with $K' = 1.005$ and $\rho'_{\min} \geq 0.15$ under tensile stress for $2h/w$ between 1.6 and 3 (Derivation 3)	$2h/w = 1.6 - 3$, $2l/w = 4.22$ and 4.25 , $\rho'_{\min} \geq 0.15$
	Figure 8a	Comparison of optimal and published fillet shapes under tensile stress when $2h/w = 0.250$ and $2l/w = 3.47$ (Derivation 4)	$2h/w = 0.25$, $2l/w = 3.47$, $\rho'_{\min} \geq 0.15$
	Figure 8b	Comparison of optimal and published fillet shapes under tensile stress when $2h/w = 3$ and $2l/w = 4.25$ (Derivation 4)	$2h/w = 3$, $2l/w = 4.25$, $\rho'_{\min} \geq 0.15$
	Figure 9	Minimum fillet lengths required to optimise fillet shapes under tensile stress with $K' = 1.005$ for any given shoulder height when $\rho'_{\min} \geq 0.15$ (Derivation 4)	$\rho'_{\min} \geq 0.15$

* See accompanying Excel Spreadsheet.

1.1 Notation

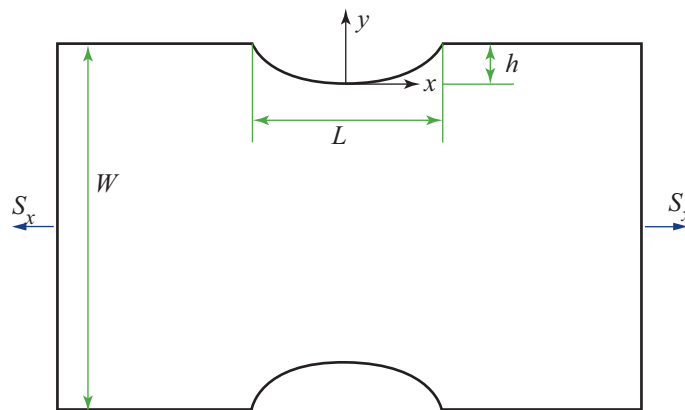
Both SI and British units are quoted but any coherent system of units may be used.

a, b, c, d	constants		
e, f, g			
A	position		
B	position		
h	depth of notch or height of fillet	m	in
K	stress concentration factor based on gross cross-sectional stress		
K'	stress concentration factor based on net cross-sectional stress		
L	length notch	m	in
l	transition length of fillet	m	in
l_W	length of maximum plate width, W	m	in
l_w	length of minimum plate width, w	m	in
P	point on a circle		
r	radius of a circular hole	m	in
r'	radius of end of fillet, or bottom of notch (see ESDU 09014)	m	in
S	gross stress	N/m ²	lbf/in ²
W	maximum width of plate	m	in
w	minimum width of plate	m	in
x	global Cartesian coordinate		
y	global Cartesian coordinate		
β	ratio $w/2h$ in a fillet		
λ	ratio l/h in a fillet		
ρ	radius of curvature	m	in
ρ'	normalised radius of curvature (equal to $\rho_{\min}/(L/2)$ for notches and $\rho_{\min}/l$ for fillets)		
$\rho_{\min}$	minimum radius of curvature constraint	m	in
$\rho'_{\min}$	normalised minimum radius of curvature		
ξ	x/l or y/l		

2. NOTCHES ON OPPOSITE EDGES OF FLAT PLATE

Sketch 2.1 shows the geometry and notation of a large plate of width W containing symmetrical notches with a notch area aspect ratio, $L/2h$, on opposite edges of the plate subject to a uniaxial tensile stress, S_x .

An FEA was used to optimise the notch shapes and to reduce the stress concentration, subject to the geometric constraints of the plate. Note that there is symmetry in the stresses applied to the plate and also in the profile of the notch. The optimised notches have a minimum radius of curvature constraint that results in a 'robust' optimal shape that has no sharp corners (Derivation 5).



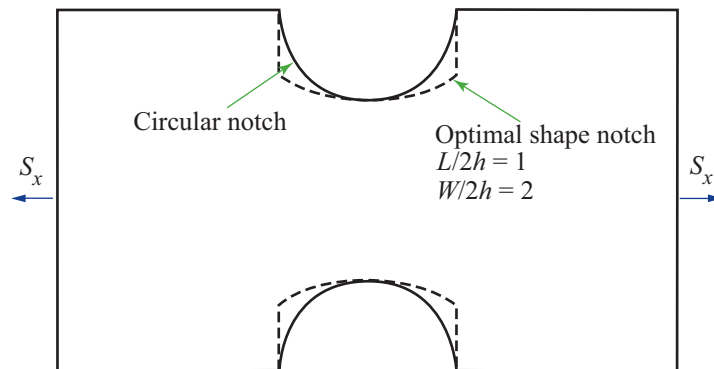
Sketch 2.1 Geometry and notation showing notches on opposite edges of a flat plate

2.1 Applied Tensile Stress

Table 6.1 presents the gross stress concentration factors for optimised shape notches with $\rho'_{\min} = 0.1$, elliptical and circular arc notches on opposite edges of plates that have a width to notch height ratio, $W/2h$, of between 2 and 20. Note that the minimum normalised radius of curvature of the optimised notch, $\rho'_{\min}$, is a function of the minimum radius of curvature of the notch, $\rho_{\min}$, divided by half the notch length, $L/2$ (Derivation 5).

Longer notches typically have lower stress concentrations and the optimal notches also have lower peak stresses. Figure 1 shows that for a plate with a width of $W/2h = 5$, the optimised notch shape becomes more rectangular as $L/2h$ increases – this change also takes place for fixed values of $L/2h$ as the $W/2h$ ratio decreases.

Sketch 2.2 shows circular arc notches and optimised shape notches on opposite edges of a plate when $L/2h = 1$ and $W/2h = 2$. The shape of the optimised notches is more rectangular than the usual elliptical notches, more so as $L/2h$ increases and $W/2h$ decreases. The biggest reduction in peak tangential stress has been found when optimised notches are used in plates that have a large width. For deep notches in a large plate, *i.e.* when $L/2h = 1$ and $W/2h = 20$ (not shown on Table 6.1), a 26 per cent reduction in the peak stress in the optimised notch compared to an elliptical notch was reported in Derivation 5. For the case when the plate is narrow and the notch is long, such as $L/2h = 16$ and $W/2h = 2$, the peak tangential stress of the optimal notch was found to be only 0.5 per cent less than for elliptical notches.



Sketch 2.2 A circular notch and an optimised shape notch with $L/2h = 1$ and $W/2h = 2$ (Derivation 5)

Figure 2 shows the decrease in the gross stress concentration factor for optimal notches as the $L/2h$ ratio increases when $W/2h$ varies between 2 and 20. For a given $L/2h$ notch ratio, the stress concentration factor decreases when $W/2h$ increases. For comparison, Figure 2 also shows a similar decrease in stress concentration factor for elliptical notches when $L/2h$ increases and $W/2h$ increases.

The stress concentration factors and the coordinates of non-dimensionalised optimised notch shapes under tensile stress when the $W/2h$ ratio varies between 2 and 20, and when $\rho'_{\min} = 0.1$, are given in the accompanying Excel spreadsheet; see **Table E1** for notches with $L/2h = 1$ and 2, **Table E2** with $L/2h = 3$ and 5, **Table E3** with $L/2h = 8$ and 12 and **Table E4** with $L/2h = 16$.

In this Data Item, the gross stress concentration factors for circular notches are presented for larger plates ($0.025 \leq r'/(W-2h) \leq 64.25$) compared to the finite element results and closed form solutions discussed in ESDU 09014 where ($0.005 \leq r'/(W-2h) \leq 0.2$) and ($1.01 \leq W/(W-2h) \leq 2$). Also note that the present stress concentration factors are lower than those found in other published work on circular arcs and elliptical shapes, see Table 6.2 for a comparison (Derivation 5). In Table 6.2, the results for the circular arcs obtained from Reference 36 are not very accurate, so they were recalculated to obtain more accurate results.

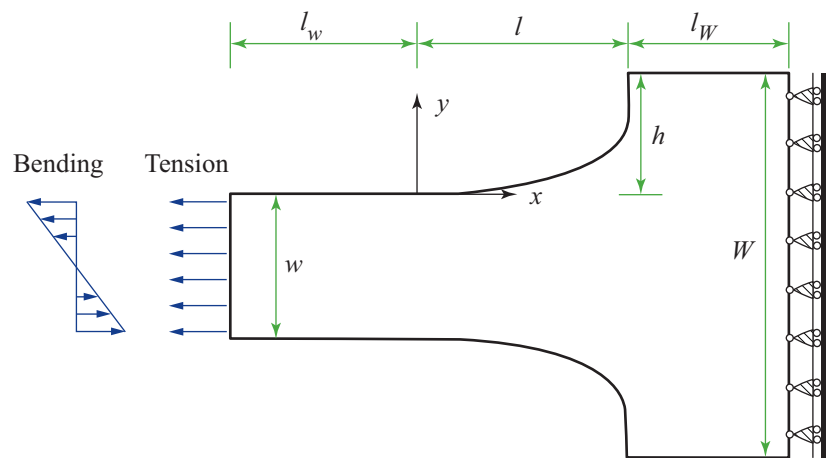
In the context of reshaping existing components, the overall length of the optimised notch used to remove the damaged material is shorter than the required length of an equivalent circular arc notch. Optimal notches are also useful in critical situations where the lowest possible stress concentration is required. In more complex loading situations, optimal notches can be used for the initial design of shapes in plates.

2.2 Notches Without Stress Concentrations

For practical applications such as removing damaged material, it is possible to obtain an optimised notch shape that has a net stress concentration factor, K' , close to unity. Figure 3 shows that for optimised notches where $K' = 1.005$, the notch length is considerably shorter than for comparable circular arc notches with $K' = 1.005$ (calculated using Neuber's formula, Reference 15 cited in Reference 31). Figure 3 also shows that the minimum required notch aspect ratio ($L/2h$) for any notch shape giving $K' = 1.005$ increases as $W/2h$ gets larger.

3. FILLETS IN FLAT PLATES

Sketch 3.1 shows the general notation for a symmetric flat plate with shoulder fillets loaded by a uniform stress or a bending stress. An FEA was used to determine the optimum fillet shape that has a constant, or near constant, distribution of tangential stress around its boundary. A roller constraint was applied to the right-hand side of the fillet so that displacements in the x -direction are zero but the plate was free to move in the y -direction. Note that for the FE analysis, finite width effects were present and both l_w and l_W are equal to $5l$ so as to keep the effects of plate length on the state of stress in the vicinity of the fillet at a negligible level (Derivation 1). For the case of combined loading, the maximum stress for each type of load need to be evaluated separately, and then the individual stresses superimposed.



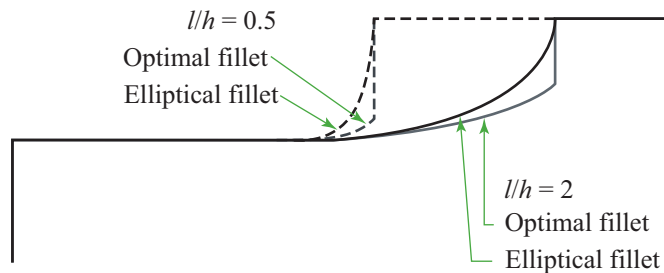
Sketch 3.1 Geometry and notation of fillet in a flat plate

3.1 Applied Tensile Stress

Table 6.3 presents the net stress concentration factor, K' , for optimised fillet shapes in plates when W/w is between 1.5 and 2.167 and for a range of fillet geometries under applied tensile stress (Derivation 1). When compared to previously published results which were thought to underestimate the true value of stress concentration (also shown on Table 6.3), the present method for optimising fillet shapes gives lower values of stress concentration.

The coordinates of the non-dimensionalised optimal fillets under applied tensile stress are given on **Table E5** in the accompanying Excel spreadsheet when $l/h = 0.5, 1$ and 2 , and on **Table E6** when $l/h = 3, 4$ and 5 .

Figure 4 shows the shape of non-dimensional optimised fillets when $W/w = 1.33$ and the shape of the fillets, l/h , is between 0.5 and 5 . As the fillet gets longer (l/h gets larger), the optimal fillet profile gets flatter, see Sketch 3.2. On Figure 5 for a given value of W/w , the net stress concentration along the optimised fillet shape boundary decreases when the l/h ratio increases. For a given value of l/h , the stress concentration also decreases as the ratio of W/w gets bigger, which demonstrates the finite width effects present in the plate. The lowest values of stress concentration are obtained when W/w and l/h ratios in the fillet are large, and in some cases the stress concentration reduces to close to unity.



Sketch 3.2 Elliptical and optimal fillets loaded in tension when $W/w = 1.33$ and $l/h = 0.5$ or 2 (Derivation 1)

Table 6.4 shows the lower net stress concentration factors for optimal fillets compared to circular fillets. Further stress concentration data for circular fillets in a bar or strip loaded in tension determined using FEA and photoelastic methods, as well as closed form equations, can be found in ESDU 09014 for geometries in the range $1.01 \leq W/w \leq 2$ and $0.02 \leq r'/w \leq 0.2$.

3.2 Bending Stress

A bending moment can be applied to the fillet in Sketch 3.1 using a linearly varying stress distribution on the loaded edge of the plate. The net stress concentration factors and the coordinates of the non dimensionalised optimal fillets under bending stress are given on **Table E7** when $l/h = 0.5$ and 1 and on **Table E8** when $l/h = 2, 3$ and 4 , see the accompanying Excel spreadsheet.

Under bending stress, the shapes of the non dimensional optimal fillets when $W/w = 1.33$ and l/h between 0.5 and 4 shown in Figure 6 are similar to those for tensile stress; as the fillet gets longer the optimal shape under bending stress becomes flatter. Note, however, that the optimal shapes of the bending and tension fillets are geometrically different meaning that the resulting stress cannot be superimposed. For loading cases involving simultaneous tension and bending loading, a new optimal fillet shape would be derived, appropriate to the particular combination of tension and bending.

Figure 7 shows the net stress concentration factor in optimised fillets as a function of l/h for different W/w ratios under bending stress. Their behaviour is similar to that seen for the optimised fillets under tension (Section 3.1). It is also possible to reduce the stress concentration factor to close to unity under bending.

Table 6.5 shows the lower net stress concentration factors for optimised fillet shapes subject to bending compared to the best possible circular fillets. ESDU 09014 presents further stress concentration data for circular fillets in a bar or strip loaded under in-plane bending determined using analytical and photoelastic methods, as well as closed form equations, in the range $1.01 \leq W/w \leq 2$ and $0.01 \leq r'/w \leq 0.2$.