



Coupling loss factors



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THE PREPARATION OF THIS DATA ITEM

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COUPLING LOSS FACTORS

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COUPLING LOSS FACTORS

1. NOTATION

		Units	
		<i>SI</i>	<i>British</i>
$\overline{a^2}$	spatial and time-averaged squared acceleration	m ² /s ⁴	in ² /s ⁴
a, b	lengths of plate in x- and y-directions	m	in
c_b	bending wave velocity	m/s	in/s
c_{b1}, c_{b2}	bending wave velocities in subsystems 1 and 2	m/s	in/s
c_g	group velocity	m/s	in/s
c_l	longitudinal wave velocity of beam	m/s	in/s
c_l'	longitudinal wave velocity of plate	m/s	in/s
c_{l1}, c_{l2}	longitudinal wave velocity in subsystems 1 and 2	m/s	in/s
c_0	velocity of sound	m/s	in/s
D	bending stiffness of plate $\left\{ = Eh^3/12(1-\nu^2) \right\}$	Nm	lbf in
D_y	bending stiffness of plate in y-direction	Nm	lbf in
D_1, D_2	bending stiffnesses of beams 1 and 2	Nm	lbf in
E	Young's modulus	Pa	lbf/in ²
E_y	Young's modulus in y-direction	Pa	lbf/in ²
E_1, E_2	vibrational energies of subsystems 1 and 2	Nm	lbf in
E_i	vibrational energy of plate i	Nm	lbf in
f	frequency	Hz	Hz
f_R	ring frequency	Hz	Hz
f_c	critical frequency	Hz	Hz
f_{c2}	critical frequency of orthotropic plate	Hz	Hz
f_{fun}	fundamental natural frequency	Hz	Hz
f_0	centre frequency of band	Hz	Hz

g_1, g_2	functions given by Equations (4.16) and (4.17)		
g_3	function in Equation (4.19)		
H	non-dimensional parameter, Equation (4.22)		
h	plate thickness	m	in
k_b	radius of gyration	kg	*
k_m	axial wavenumber	m ⁻¹	in ⁻¹
k_n	circumferential wavenumber	m ⁻¹	in ⁻¹
k_p	flexural wavenumber of plate	m ⁻¹	in ⁻¹
k_1, k_2	bending wavenumbers in plates 1 and 2	m ⁻¹	in ⁻¹
L	length of connection between plates, length of cylinder	m	in
M	modal overlap factor		
M_b	mass of beam	kg	*
M_s	mass of structure	kg	*
m_i	mass of plate i	kg	*
m_s	mass per unit area of panel	kg/m ²	*
m_1', m_2'	mass per unit length	kg/m	*
m, n	modal numbers		
N	number of bolts		
N_{AF}	number of acoustically fast modes		
N_{tot}	total number of modes		
N_1, N_2	numbers of modes		
n_s	modal density of structure	s	s
n_v	modal density of acoustic space	s	s
n_1, n_2	modal densities	s	s
P	perimeter	m	in
P_{ribs}	twice total length of ribs	m	in
P_{mn}	power radiated by plate in (m, n) mode	W	lbf in/s

R	transmission loss		
R_{mn}	radiation resistance of (m, n) mode		
R_r	radiation resistance		
R_{rib}	radiation resistance per unit length of rib	kg/s	*
R_c	radius of cylinder	m	in
$R_{12}, R_{13}, etc.$	transmission loss from subsystem 1 to 2, 1 to 3, etc.		
S	area of panel	m ²	in ²
S_c	surface area of cylinder	m ²	in ²
St_c	non-dimensional frequency parameter, Equation (4.9)		
S_1	area of plate 1, cross-sectional area of beam1	m ²	in ²
S_2	cross-sectional area of beam2	m ²	in ²
t	plate thickness	m	in
t_b	thickness of beam	m	in
t_p	thickness of plate	m	in
t_1, t_2	thicknesses of plates 1 and 2	m	in
V_s	volume of source room	m ³	in ³
$\overline{v_{mn}^2}$	mean-squared velocity	m ² /s ²	in ² /s ²
Z_b	moment impedance of semi-infinite beam	kgm ² /s	*
Z_p	moment impedance of infinite plate	kgm ² /s	*
Z_1, Z_2	characteristic impedances of beams or plates 1 and 2	kg/s	*
α	$h_2/h_1, S_2/S_1$		
β	angle of incidence		
β_b	boundary condition parameter		
χ	non-dimensional parameter		
δ	radius of circular area over which moment is applied	m	in
Λ	non-dimensional damping parameter, Equation (4.20)		
γ	$(f / f_c)^{1/2}$		

η	loss factor		
η_{bp}	coupling loss factor from beam to plate		
η_{sv}, η_{vs}	coupling loss factors between structure S and space V, and V and S		
η_1, η_2	loss factors for subsystems 1 and 2		
η_{12}, η_{21}	coupling loss factors for subsystems 1 and 2		
χ	ratio of bending wave numbers, Equation (3.15)		
ψ	non-dimensional parameter, Equation (3.7)		
ν	Poisson's ratio		
ν_{xy}	Poisson's ratio for load in x-direction and contraction in y-direction		
ν_{yx}	Poisson's ratio for load in y-direction and contraction in x-direction		
λ	wavelength	m	in
λ_c	wavelength at coincidence	m	in
Π_1, Π_2	power input to subsystems 1 and 2	W	lbf in/s
Π_{12}, Π_{21}	power from subsystem 1 to 2 and subsystem 2 to 1	W	lbf in/s
$\Pi_{1, diss}, \Pi_{2, diss}$	power dissipated in subsystems 1 and 2	W	lbf in/s
ρ	density	kg/m ³	*
ρ_b	density of beam	kg/m ³	*
ρ_p	density of plate	kg/m ³	*
ρ_1, ρ_2	density of plates or beams 1 and 2	kg/m ³	*
σ	radiation efficiency		
σ_{mn}	radiation efficiency for mode (m, n)		
τ	transmission efficiency		
τ_{sp}	transmission efficiency from source room to panel		
$\tau_{12}, \tau_{13}, \text{etc.}$	transmission efficiency from subsystem 1 to 2, 1 to 3, etc.		
ω	radian frequency, centre frequency ($\omega = 2\pi f$)	rad/s	rad/s

* In British units, for the system of units to be coherent the mass in lb must be divided by the constant 386.4.

Abbreviations

AF	acoustically fast
AS	acoustically slow
CLF	coupling loss factor
SEA	statistical energy analysis

2. INTRODUCTION

In Statistical Energy Analysis the coupling loss factor represents a dimensionless measure of the power transfer between weakly coupled subsystems. The concept of the coupling loss factor is unique to SEA. It has been introduced in ESDU 99009 and in the introduction to ESDU 11008, where it has been pointed out that the coupling loss factor is a vital part of the equation for the power transmission between the subsystems. The subsystems can be physical components of a larger structure. For example, an aircraft fuselage with its interior can be modelled as a system with two subsystems; a fuselage and an acoustic enclosure to which the vibrational energy is transmitted from the fuselage. The choice of the subsystems can also be associated with specific groups of response modes, which have comparable vibrational energy within the considered frequency band. The energy flow or power transmission between the subsystems, as discussed in the Item, are due to resonant structural or acoustic modes in the considered frequency band. SEA is particularly applicable at high frequencies where there are numerous resonant modes, which cannot be easily analysed individually by computational techniques, such as finite element analysis and boundary element analysis.

2.1 Energy Flow Relationships

SEA is based on several assumptions which are relevant to the energy flow between oscillators. The assumptions are (Lyon, 1975; ESDU, 1999; ESDUa, 2011, Norton and Karczub, 2003):

- (i) SEA is based on the energy conservation law applied to the structure which vibrates usually under steady-state conditions. The response of the structure is dominated by resonant modes, which in the subsystem should have comparable vibrational energies within the considered frequency band. In other words there is equipartition of energy between the modes of the subsystems. The subsystems should have at least several resonant modes within the frequency band.
- (ii) The time-averaged energy amount exchanged in unit time between a pair of modes each from different subsystems in the same frequency band is proportional to the difference in the time-averaged vibrational energies of both modes.
- (iii) Similarly, the coupling between subsystems is linear and conservative. The time-averaged energy amount exchanged in unit time between both subsystems is proportional to the difference between the time-averaged modal vibrational energies of each subsystem within a frequency band.
- (iv) The principle of reciprocity applies to coupling loss factors associated with the coupled subsystems.
- (v) The resonant modes results from broad-band random excitation. The excitation is not applied at a point. Note that in general random excitation means that less averaging of the system parameters is necessary (Lyon, 1975).

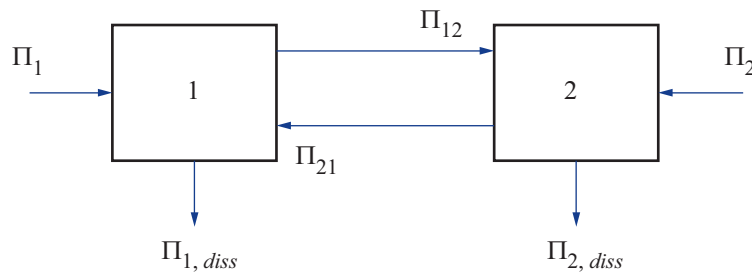
The SEA method is based on the energy conservation law, applied to the vibrating structure, which is represented by subsystems. The principle of energy conservation leads to the SEA power balance equations, formulated for each subsystem i in terms of the time-averaged vibrational energies of the subsystems E_i within the frequency band $\Delta\omega$. The parameters appearing in the power balance equations are:

- (1) the power input to the subsystem i , Π_i ,
- (2) the modal density n_i , which defines in subsystem i the average number of resonant modes per unit frequency band,

- (3) the loss factor η_i , connected with dissipation of the vibrational energy in the subsystem i , which are the mean of the loss factor of all the resonant modes within the considered frequency band,
- (4) the coupling loss factor η_{ij} , associated with the net transmission of energy between two subsystems i , and j , which are expressed as the degree of coupling between these subsystems,
- (5) the band centre frequency ω .

Once the subsystems and the parameters (1 to 5), associated with them are specified, the power balance equations can be solved to give the subsystem vibrational energies E_i . Knowledge of the distributions of the subsystems vibrational energies throughout the structure within the defined frequency band enables estimation of the space-averaged squared velocities over the structure. Alternatively, if the mean squared velocities are known, the coupling loss factors could be estimated.

For simplicity, the SEA power balance equations are formulated in this Item for two connected subsystems, 1 and 2, a diagram of which is shown schematically in Sketch 2.1.



Sketch 2.1 Two connected subsystems

The steady-state, time-averaged input power to subsystem 1 is Π_1 . There is a loss of time-averaged power due to energy dissipation in subsystem 1, $\Pi_{1, diss}$, and net transmission of power between the two subsystems. The power leaving subsystem 1 and entering 2 is Π_{12} and the transmitted power from 2 to 1 is Π_{21} . Hence, the power balance equation for subsystem 1 is:

$$\Pi_1 = \Pi_{12} + \Pi_{1, diss} - \Pi_{21} \quad (2.1)$$

A similar power balance equation may be written for subsystem 2, where the input power is Π_2 and the loss of power associated with energy dissipation is $\Pi_{2, diss}$.

$$\Pi_2 = \Pi_{21} + \Pi_{2, diss} - \Pi_{12} \quad (2.2)$$

The power terms in Equations (2.1) and (2.2), $\Pi_{1, diss}$ and $\Pi_{2, diss}$, are expressed by the loss factors of subsystems 1 and 2, η_1 and η_2 , respectively, and the time-averaged vibrational energies, E_1 and E_2 , within the frequency band with centre frequency ω . Hence, they are:

$$\Pi_{1, diss} = \omega \eta_1 E_1 \quad (2.3)$$

and

$$\Pi_{2, diss} = \omega \eta_2 E_2 \quad (2.4)$$

for subsystems 1 and 2, respectively.

Loss factors are used extensively, for example, in the estimation of stresses in structures under acoustic loading. Values of loss factors for structures are provided in ESDU Items (ESDU, 1973; ESDU, 2007).

The transmitted power Π_{12} from subsystem 1 to subsystem 2, as well as the transmitted power Π_{21} from subsystem 2 to subsystem 1, can be written in terms of the coupling loss factors, η_{12} and η_{21} , and the vibrational energies of the subsystems, E_1 and E_2 , within the frequency band $\Delta\omega$ as:

$$\Pi_{12} = \omega \eta_{12} E_1 \quad (2.5)$$

and

$$\Pi_{21} = \omega \eta_{21} E_2, \quad (2.6)$$

where $\eta_{12} \neq \eta_{21}$. Both the coupling loss factors, η_{12} and η_{21} , are not independent magnitudes, as they fulfil the consistency relationship. The reciprocal behaviour of the coupling loss factors can be expressed in the form (Lyon, 1975; Manning, 1997; Norton and Karzucub, 2003):

$$\eta_{12} N_1 = \eta_{21} N_2. \quad (2.7)$$

Since N_1 and N_2 are the number of resonant modes within the frequency band $\Delta\omega$ with centre frequency ω , $N_1 = n_1 \Delta\omega$ and $N_2 = n_2 \Delta\omega$, the reciprocity law of Equation (2.7) provides the relationship between the coupling loss factors, η_{12} and η_{21} , and the modal densities of subsystems 1 and 2, n_1 and n_2 , respectively,

$$\eta_{12} n_1 = \eta_{21} n_2. \quad (2.8)$$

Modal densities have been discussed in some detail in ESDU Items (ESDUa, 2011; ESDUb, 2011; ESDU, 2013).

Equations (2.5) and (2.6) introduce the concept of the coupling loss factors. The coupled system is said to be weakly coupled, if $\eta_{12} \ll \eta_1$. The SEA method is said to work better, if the coupling is weak. Weak coupling implies that, if one particular subsystem is subjected to excitation, then the response of that subsystem should be significantly greater than that of any other subsystem (ESDU, 1999).

Hence, Equations (2.1) and (2.2) may be expressed in terms of the loss factors, the coupling loss factors and the averaged vibrational energy of subsystems 1 and 2:

$$\Pi_1 = \omega \eta_{12} E_1 + \omega \eta_1 E_1 - \omega \eta_{21} E_2 \quad (2.9)$$

and

$$\Pi_2 = \omega \eta_{21} E_2 + \omega \eta_2 E_2 - \omega \eta_{12} E_1. \quad (2.10)$$

Equations (2.9) and (2.10) may be rewritten as:

$$\Pi_1 = (\omega \eta_{12} + \omega \eta_1) E_1 - \omega \eta_{21} E_2 \quad (2.11)$$

and

$$\Pi_2 = -\omega\eta_{12}E_1 + (\omega\eta_{21} + \omega\eta_2)E_2, \quad (2.12)$$

or in the matrix form as:

$$\begin{bmatrix} \Pi_1 / \omega \\ \Pi_2 / \omega \end{bmatrix} = \begin{bmatrix} \eta_1 + \eta_{12} & -\eta_{21} \\ -\eta_{12} & \eta_2 + \eta_{21} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}. \quad (2.13)$$

The set of power balance equations (Equation (2.13)) could be expressed in the form:

$$\begin{bmatrix} \Pi_1 / \omega \\ \Pi_2 / \omega \end{bmatrix} = \begin{bmatrix} (\eta_1 + \eta_{12})N_1 & -\eta_{21}N_2 \\ -\eta_{12}N_1 & (\eta_2 + \eta_{21})N_2 \end{bmatrix} \begin{bmatrix} E_1 / N_1 \\ E_2 / N_2 \end{bmatrix}, \quad (2.14)$$

where E_1/N_1 and E_2/N_2 are the modal energies of the subsystems 1 and 2, respectively. Since the consistency relationship is fulfilled, the square matrix in Equation (2.14) is symmetrical. If excitation is assumed to be applied at a point, space-averaging is essential. In such a case input powers as well as vibrational energies of the subsystems in the power balance Equations (2.14) are time and space-averaged.

When one of the two subsystems is excited by external forces and the other is driven through the coupling ($\Pi_2 = 0$), the steady-state energy ratio becomes:

$$\frac{E_2}{E_1} = \frac{\eta_{12}}{\eta_2 + \eta_{21}}. \quad (2.15)$$

Equation (2.15) represents a possible basis for the measurement of coupling loss factors.

The accuracy of the SEA expressions depends on the modal overlap factor $M(\omega)$, which is an important property of a subsystem. The modal overlap factor is related to the loss factor and the modal density in the form

$$M(\omega) = \omega\eta n(\omega). \quad (2.16)$$

The modal overlap factor is equal to the ratio of the width of the resonant peak to the average modal spacing (ESDU, 1999). If the factor is of the order of unity or greater, the resonant peaks tend to merge together.

Coupling loss factors are related to parameters, such as transmission loss and radiation efficiency, which are extensively applied in other fields. An important application of SEA is the prediction of noise levels inside internal spaces, such as an aircraft cabin. In this case the required coupling loss factor is between a structure and a space, which can be regarded as an acoustic volume. The radiation efficiency, or radiation ratio, of the structure is an important parameter in computing CLFs. Further information on CLFs is given in reviews and texts (Craik, 1996; Cremer *et al.*, 1973; ESA, 1996; Lyon and deJong, 1995; Norton and Karczub, 2003; Wijker, 2009). Software for SEA is available from Cambridge Collaborative (www.seam.com), wave⁶ (www.wavesix.com), ESI Group (previously AutoSEA), GSSEA-Light from Gothenburg Sound and LMS Virtual Laboratory.

3. COUPLING BETWEEN STRUCTURAL SUBSYSTEMS

Section 3 is concerned with the coupling loss factors between two structures, which may be a plate and a beam, fastened together by rivets, welding, or some other method. Power may be transmitted by longitudinal waves, torsional waves or bending waves, with each type subjected to a separate analysis. In many applications bending waves are the most important. Provided the plates are thin, bending waves will dominate over the shear and longitudinal waves. In this Section it is assumed that free bending waves in a plate produce similar waves in a second, connected, plate. In the case of bending waves the power transmission between the two structures is normally analysed in terms of the interaction of the waves at the structural junction and the appropriate equations are confined to the transmission of bending waves. The parameter that describes transmission across a joint is called the transmission coefficient (Craig, 1996; Norton and Karzucub, 2003), transmissibility (Lyon, 1975) or transmission efficiency (Cremer *et al.*, 1973) and is given the symbol τ . Following Cremer, the term transmission efficiency is used in this Item. The term transmission coefficient is applied to a ratio of wave amplitudes and is a complex quantity (Cremer *et al.*, 1973). Transmission efficiency is defined for structural elements as the ratio of the power transmitted through a joint to the power incident on the joint. The transmission efficiency is sometimes expressed logarithmically as a transmission loss, R , where

$$R = 10 \log \left(\frac{1}{\tau} \right) \text{ dB.} \quad (3.1)$$

A primary application of the theory in relation to coupling between structural subsystems is in relation to buildings (Craig, 1996), but can have general applications.

3.1 Structures Connected by Line Junction

Beams and plates connected by a line junction are considered in this Section. The structures may be welded or bolted together.

3.1.1 Coupling loss factors for beams and plates

For two beams connected together the coupling loss factor η_{12} is related to the transmission efficiency τ_{12} by the expression (Wijker, 2009)

$$\eta_{12} = \frac{c_g \tau_{12}}{\omega L}, \quad (3.2)$$

where c_g is the group velocity of the wave type in the first beam, ω is the centre frequency of the band of interest and L is the length of the line junction. The group velocity for bending waves is twice the bending wave phase velocity c_b , *i.e.*,

$$c_g = 2 c_b. \quad (3.3)$$

The bending wave velocity for beams of thickness h is given by:

$$c_b = \sqrt{\frac{\omega h c_l}{\sqrt{12}}} \quad (3.4)$$