



Mass flow rate of a leak into an external stream



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Mr J. Tweedie	— Short Brothers plc, Belfast
Dr M. West	— BAE Systems, Prestwick
Mr P. Woodrow	— Rolls-Royce plc, Derby

* Corresponding Member

The construction and subsequent development of the Data Item were undertaken by

Mr C.J. Batt — Senior Engineer.

The person with overall responsibility for the work in this subject area is Mr A.J. Clarke, Head of Aerodynamics Group.

MASS FLOW RATE OF A LEAK INTO AN EXTERNAL STREAM

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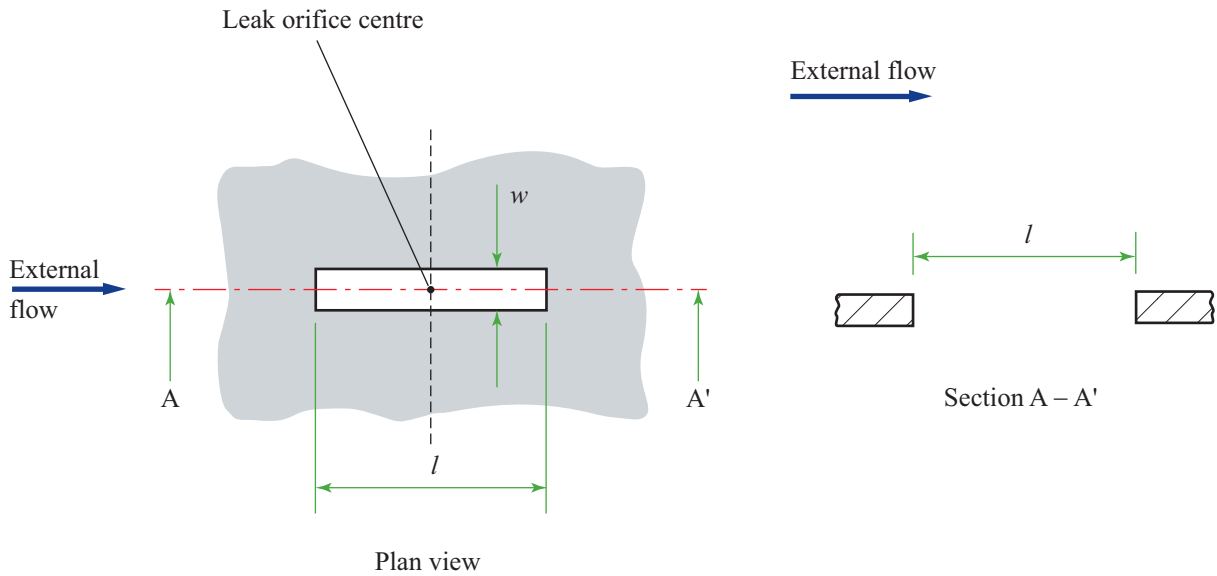
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MASS FLOW RATE OF A LEAK INTO AN EXTERNAL STREAM

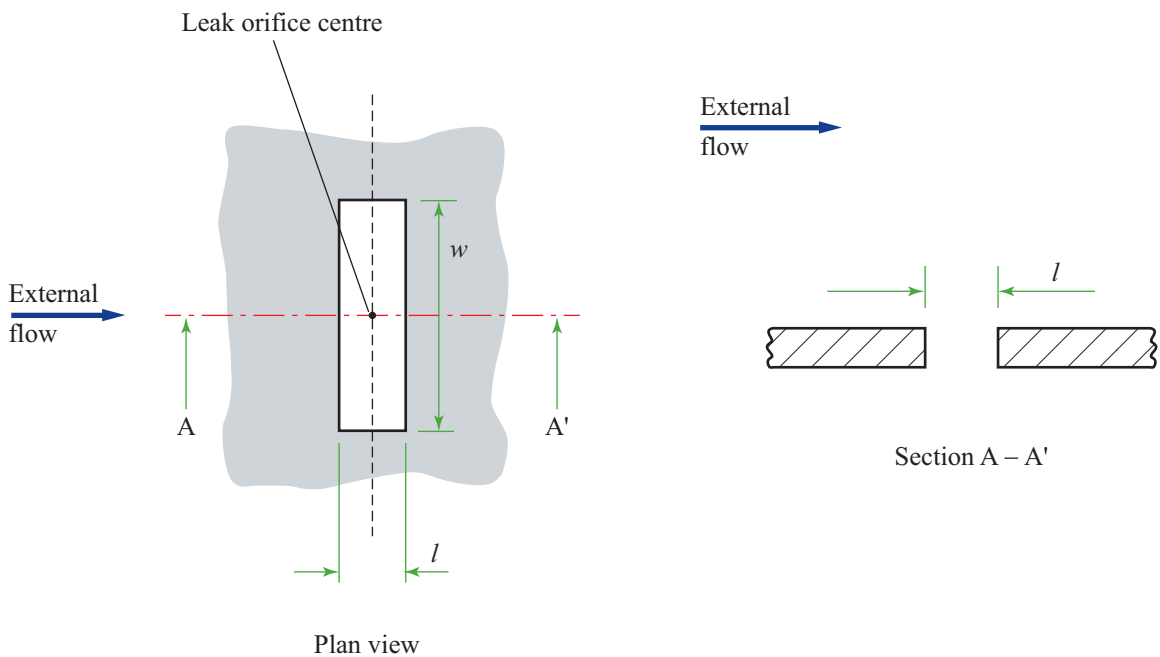
1. NOTATION

		<i>SI</i>	<i>British</i>
A_L	leak orifice area, see Section 3.1	m ²	ft ²
$(C_p)_{pl}$	plenum pressure coefficient, see Section 5.1.1 and Equation (5.1)		
$(C_p)_{pl,zf}$	value of $(C_p)_{pl}$ when there is no leakage flow, see Section 5.1.1		
$(C_p)_L$	leak exit pressure coefficient, see Section 5.1.3 and Equation (5.3)		
$(C_p^*)_{L, is}$	value of $(C_p)_L$ for choked isentropic leakage flow, see Section 5.1.3.2 and Equation (5.7)		
ΔC_p	leak differential pressure coefficient, see Section 5.1.4 and Equation (5.8)		
$(\Delta C_p^*)_{is}$	value of ΔC_p for choked isentropic leakage flow, see Section 5.1.4.2 and Equation (5.10)		
$[(C_p)_{pl}]_{is, crit}$	value of $(C_p)_{pl}$ at which isentropic leakage flow becomes choked, see Section 5.1.2 and Equation (5.2)		
F	composite factor, see Section 5.4.1 and Figure 2		
G	leak orifice geometry parameter, see Section 5.4.2 and Figure 3		
H	leak orifice geometry factor, see Section 5.4.3 and Figure 4		
K_d	discharge coefficient, ratio of actual mass flow rate to isentropic mass flow rate through leak orifice, see Sections 5.2 and 5.4 and Figure 1		
l	maximum leak orifice dimension in direction of external flow, see Section 3.1 and Sketches 1.1 and 1.2	m	ft
M_e	Mach number of external flow at edge of boundary layer at leak station with leak sealed, see Section 4.1.2		
$\dot{m}_e$	mass flow rate in external stream, <i>i.e.</i> through streamtube of cross-sectional area A_L , based on conditions at edge of boundary layer with leak sealed, see Section 4.1.2 and Equation (4.1)	kg/s	slug/s
$\dot{m}_L$	actual mass flow rate through leak orifice, see Section 5.2 and Sketch 1.3	kg/s	slug/s

$(\dot{m}_L)_{is}$	isentropic mass flow rate through leak orifice, see Sections 5.2 and 5.3	kg/s	slug/s
$(\dot{m}_L^*)_{is}$	value of $(\dot{m}_L)_{is}$ for choked leakage flow, see Section 5.3.2.2 and Equation (5.22) or (5.23)	kg/s	slug/s
$(\dot{m}_L^*)_{is,crit}$	value of $(\dot{m}_L^*)_{is}$ when $(C_p)_{pl} = [(C_p)_{pl}]_{is,crit}$, see Equation (5.24)	kg/s	slug/s
p_e	static pressure of external flow at edge of boundary layer at leak station with leak sealed, see Section 5.1.1	N/m ²	lbf/ft ²
p_L	static pressure at leak orifice exit plane (L), see Section 5.1.3	N/m ²	lbf/ft ²
$p_{L,is}^*$	value of p_L for choked isentropic leakage flow, see Section 5.1.3.2	N/m ²	lbf/ft ²
p_{pl}	static pressure in plenum, see Section 5.1.1	N/m ²	lbf/ft ²
$p_{pl,zf}$	static pressure in plenum for zero leakage flow, see Section 5.1.1	N/m ²	lbf/ft ²
R_1	Reynolds number per unit length of external flow at edge of boundary layer at leak station with leak sealed, see Section 4.1.2	m ⁻¹	ft ⁻¹
R_δ	Reynolds number based on boundary-layer thickness, $R_1 \delta$		
V	local velocity in boundary layer, see Section 4.1.2 and Sketch 1.4	m/s	ft/s
V_e	external flow velocity at edge of boundary layer at leak station with leak sealed, see Section 4.1.2 and Sketch 1.4	m/s	ft/s
$V_{L,is}$	flow velocity at leak orifice exit plane (L) in isentropic stream, see Section 5.3	m/s	ft/s
w	maximum leak orifice dimension normal to external flow direction, see Section 3.1 and Sketches 1.1 and 1.2	m	ft
γ	ratio of specific heat capacities, taken as 1.4 for air		
δ	boundary-layer thickness at $V = 0.99 V_e$ at leak station of external flow with leak sealed, see Section 4.1.2 and Sketch 1.4	m	ft
μ_e	dynamic viscosity of external flow at edge of boundary layer at leak station with leak sealed, see Sections 8.1.1, 8.2.1 and 8.3.1	kg/m s	slug/ft s
ρ_e	external air density at edge of boundary layer at leak station with leak sealed, see Section 4.1.2 and Sketch 1.4	kg/m ³	slug/ft ³
$\rho_{L,is}$	air density at leak orifice exit plane (L) in isentropic flow, see Section 5.3	kg/m ³	slug/ft ³

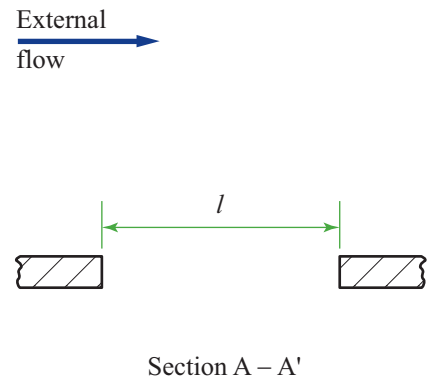
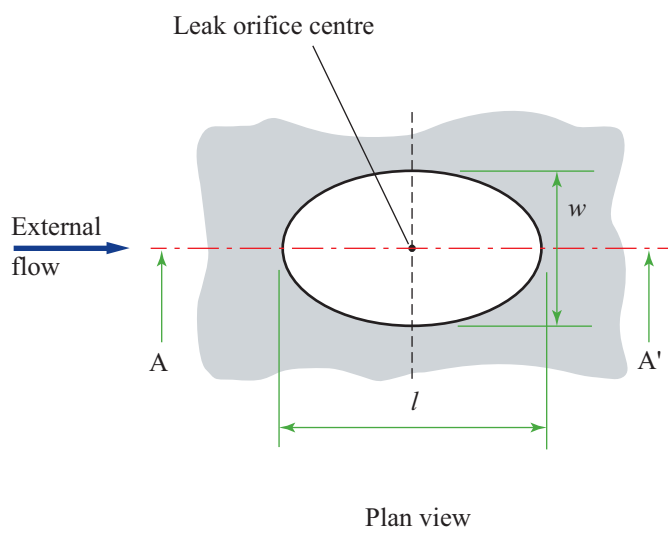


a) $w < l$

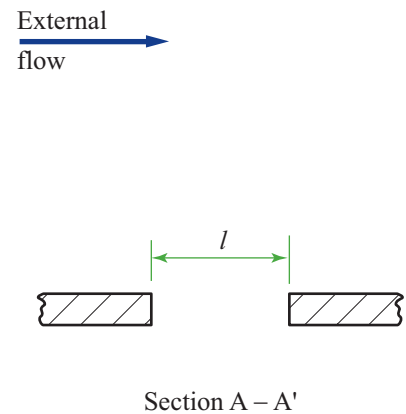
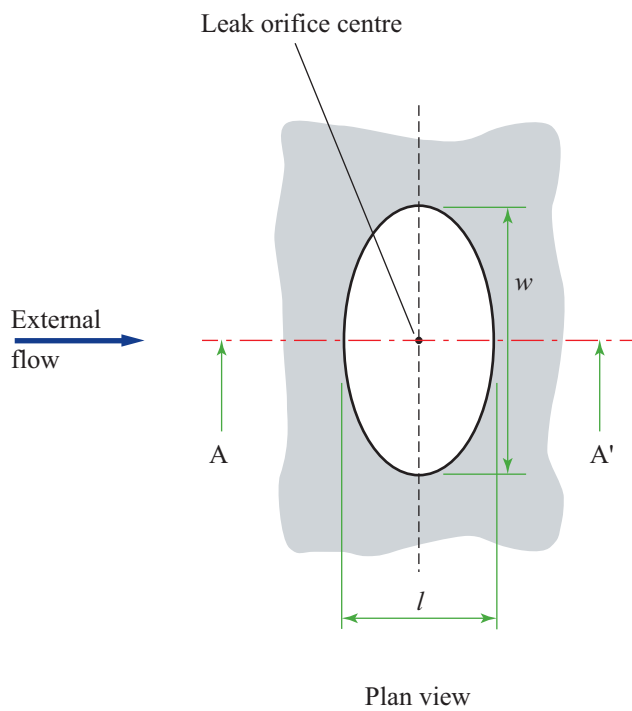


b) $w > l$

Sketch 1.1 Rectangular leak orifice geometry in a thin plate

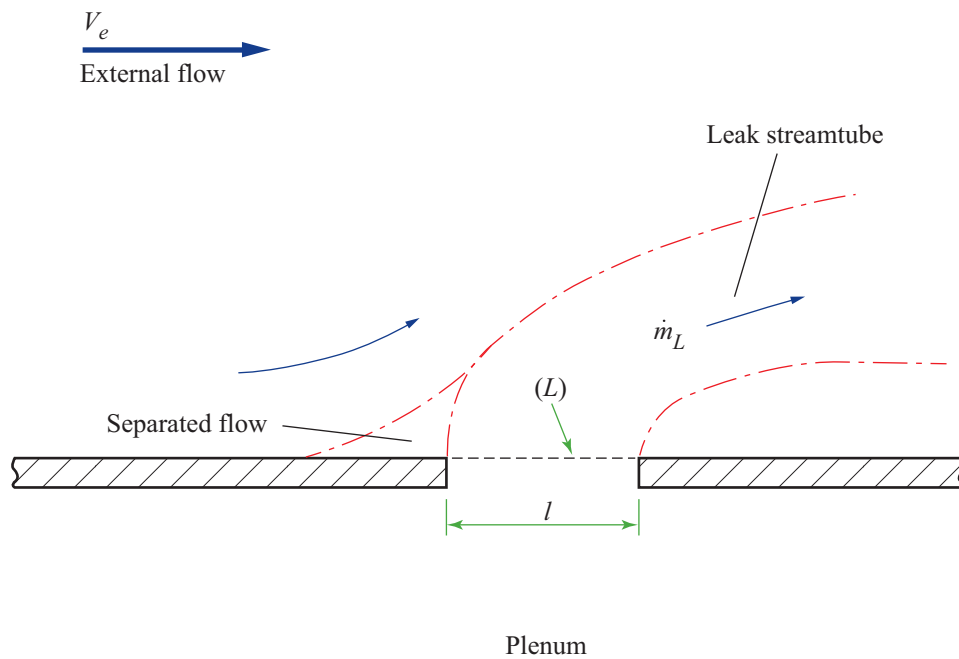


a) $w < l$

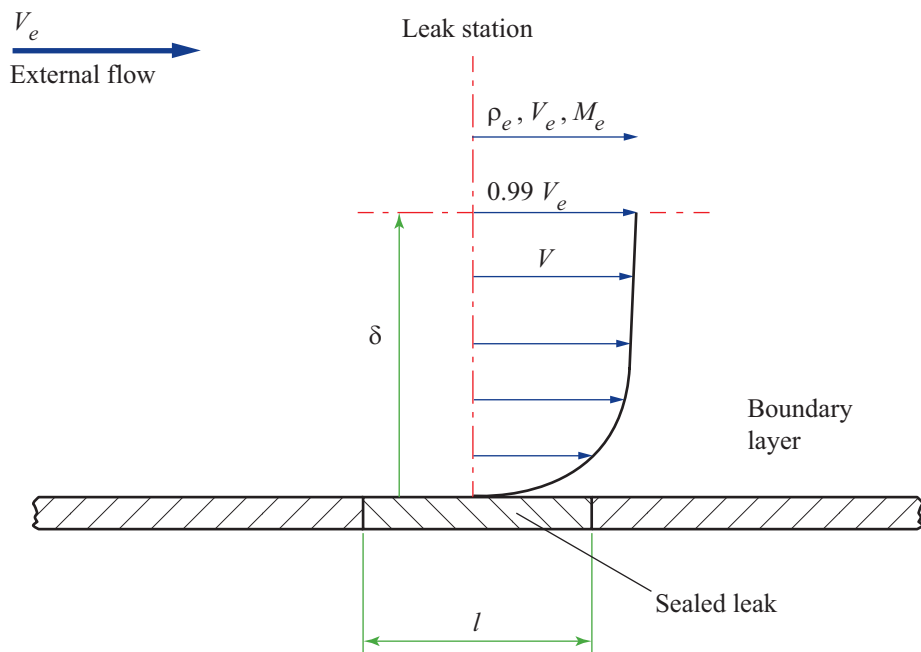


b) $w > l$

Sketch 1.2 Elliptical leak orifice geometry in a thin plate



Sketch 1.3 Section through flowfield due to a leak discharging into an external stream



Sketch 1.4 External flow when leak is sealed

2. INTRODUCTION

This Data Item provides a method of predicting the mass flow rate of a leak which discharges into an external stream. A discharge coefficient is used, which is an efficiency parameter enabling the prediction of the change in mass flow rate from its isentropic value, due to friction, viscous and real gas effects.

The mass flow rate of a leak discharging into an external stream is dependent on the pressure difference across the leak orifice, the geometry, size and orientation of the leak orifice and the external flow conditions.

The method is derived using flat plate test data for flows discharging into external streams through various rectangular, square, elliptical and circular orifice geometries in thin plates, see Sketches 1.1 and 1.2. These data usefully represent leakage flows into an external stream through well-defined orifice geometries. In addition to these geometries the method should also be applicable to other similar geometries. The method requires knowledge of the leak orifice area together with values of the maximum orifice dimensions in both the streamwise direction and perpendicular to it, see Section 3.

The leak geometry is described in Section 3. Section 4 describes the leak flowfield which for the purposes of this Item is divided into an external flow and a leakage flow. The prediction method is given in Section 5 and the accuracy and applicability are presented in Section 6. The Derivation is given in Section 7 and worked examples are presented in Section 8.

Appendices A and B derive equations giving the leak mass flow ratio for incompressible and compressible isentropic flows, respectively. Appendix C provides the derivation of various equations for choked isentropic flow. Appendix D investigates the interaction between the external flow and leakage flow and Appendix E discusses the likely effect of viscous entrainment on the leakage flow. Appendix F provides equations giving the curves in Figures 1 to 4.

3. LEAK GEOMETRY

The leak orifice geometry is described in Section 3.1 and the plenum is defined in Section 3.2.

3.1 Leak Orifice

The method of this Item has been derived using flat plate data for square, rectangular, circular and elliptical orifice geometries. In addition to these geometries the method should also be applicable to other similar geometries, see Section 6.1.

In this Item l is the maximum dimension of the orifice aligned with the direction of the external flow and w is the maximum dimension perpendicular to the external flow. These dimensions are shown in Sketches 1.1 and 1.2 for rectangular and elliptical orifice geometries, respectively. For the purposes of the method the leak orifice geometry is defined by the ratio w/l and the leak orifice area, A_L .

The leak orifice area is

$$A_L = wl, \quad (3.1)$$

for a rectangular orifice and

$$A_L = \frac{\pi}{4}wl, \quad (3.2)$$

for an elliptical orifice.

3.2 Plenum

The plenum is a chamber within the vehicle structure which collects leakage inflows and distributes leakage outflows. It is assumed that the flow in the plenum is stationary relative to the plate before exiting via the leak orifice in the plate.

4. LEAK FLOWFIELD

It is useful to divide the leak flowfield into an external flow and a leakage flow. The external and leakage flows are described in Sections 4.1 and 4.2, respectively.

4.1 External Flow

An example external flow when a leakage flow is present is shown in Sketch 1.3. The external flow when the leak is sealed is shown in Sketch 1.4. Section 4.1.1 defines the leak station. The external flow conditions at the leak station when the leak is sealed are usually known. The mass flow rate in the external stream is defined in Section 4.1.2.

4.1.1 Leak station

Sketch 1.4 shows the external flow for the case when the leak has been sealed. The leak station is shown in this sketch and is located at the centre of the leak orifice (see Sketches 1.1 and 1.2).

4.1.2 Mass flow rate in external stream, $\dot{m}_e$

The external stream is defined here as the flow at the edge of the boundary layer at the leak station when the leak is sealed, see Sketch 1.4. In Sketch 1.4 ρ_e is the stream density, V_e is the flow velocity and M_e is the Mach number at the edge of the boundary layer at the leak station when the leak is sealed. The external mass flow rate, $\dot{m}_e$, is conveniently defined as the mass flow rate of the external stream through a streamtube with the same cross-sectional area as the leak orifice, so that

$$\dot{m}_e = \rho_e A_L V_e . \quad (4.1)$$

The boundary-layer thickness, δ , is the height of the boundary layer at $V = 0.99V_e$, where V is the local velocity in the boundary layer, see Sketch 1.4. The unit Reynolds number, R_1 , is also based on the flow conditions at the edge of the boundary layer at the leak station when the leak is sealed.

4.2 Leakage Flow

The leakage flow is the flow in the streamtube passing through the leak orifice. The flow is dependent primarily on the pressure difference between the plenum and the external flow adjacent to the leak orifice.

A cross-sectional view of a leak orifice with a leakage flow discharging into an external stream is shown in Sketch 1.3, in which the leak orifice exit plane is denoted by (L) . The leak mass flow rate, $\dot{m}_L$, is the mass flow rate through the leak orifice.

5. METHOD

The leak mass flow rate, $\dot{m}_L$, is obtained via a leak mass flow ratio. The leak mass flow ratio is obtained by applying a discharge coefficient to a calculated leak mass flow ratio under isentropic conditions.

Various leakage flow pressure coefficients are defined in Section 5.1. The leak mass flow ratio involving the discharge coefficient is defined in Section 5.2 and equations giving the mass flow ratio under isentropic conditions are provided in Section 5.3. The discharge coefficient is dealt with in Section 5.4.

5.1 Leakage Flow Pressure Coefficients

For a given external flow, the pressure in the plenum determines the leak mass flow rate. A plenum pressure coefficient and a critical plenum pressure coefficient are defined in Sections 5.1.1 and 5.1.2, respectively. A leak exit pressure coefficient and a leak differential pressure coefficient are defined in Sections 5.1.3 and 5.1.4, respectively.

5.1.1 Plenum pressure coefficient, $(C_p)_{pl}$

The plenum pressure coefficient, $(C_p)_{pl}$, is given by

$$(C_p)_{pl} = \frac{p_{pl} - p_e}{\frac{1}{2}\rho_e V_e^2}, \quad (5.1)$$

where p_{pl} is the static pressure in the plenum and p_e is the static pressure of the external flow at the edge of the boundary layer at the leak station when the leak is sealed.

It is useful to define $(C_p)_{pl,zf}$ as the value of $(C_p)_{pl}$ when there is no leakage flow, defined using $p_{pl,zf}$. For a given leak orifice geometry, $(C_p)_{pl,zf}$ has a fixed value. For a leak orifice opening normal to the external flow direction (Sketches 1.1 and 1.2), $p_{pl,zf} \approx p_e$ so that $(C_p)_{pl,zf} \approx 0$.

5.1.2 Critical plenum pressure coefficient, $[(C_p)_{pl}]_{is,crit}$

For a given external flow the leakage flow rate is primarily dependent upon the plenum pressure coefficient, $(C_p)_{pl}$. When sonic conditions are reached at the leak orifice the flow is said to be choked.

The critical plenum pressure coefficient, $[(C_p)_{pl}]_{is,crit}$, is defined here as the value of $(C_p)_{pl}$ at which the isentropic leakage flow becomes choked.

For an isentropic flow Equation (C4.2) in Appendix C gives

$$\boxed{[(C_p)_{pl}]_{is,crit} \approx \frac{2}{\gamma M_e^2} \left[\left(\frac{1+\gamma}{2} \right)^{\frac{\gamma}{\gamma-1}} - 1 \right]}, \quad (5.2)$$

where γ is the ratio of specific heat capacities, taken as 1.4 for air.

5.1.3 Leak exit pressure coefficient, $(C_p)_L$

The leak exit pressure coefficient, $(C_p)_L$, is the pressure coefficient at the leak orifice exit plane, *i.e.* (L) in Sketch 1.3, and is given by

$$(C_p)_L = \frac{p_L - p_e}{\frac{1}{2}\rho_e V_e^2}, \quad (5.3)$$

where p_L is the static pressure at the leak orifice exit plane.

The leak exit pressure coefficient for unchoked and choked isentropic leakage flows is given in Sections 5.1.3.1 and 5.1.3.2, respectively.

5.1.3.1 Unchoked isentropic leakage flow, $(C_p)_{pl} < [(C_p)_{pl}]_{is,crit}$

The leakage flow under isentropic conditions is unchoked for subcritical values of plenum pressure coefficient, $(C_p)_{pl} < [(C_p)_{pl}]_{is,crit}$. When the leakage flow is unchoked it is assumed that p_L remains constant and is therefore equal to the static pressure in the plenum for the case where there is no leakage flow, so that

$$(C_p)_L = (C_p)_{pl,zf}. \quad (5.4)$$

For a leak orifice opening normal to the external flow (Sketches 1.1 and 1.2) Section 5.1.1 gives $(C_p)_{pl,zf} \approx 0$, so that from Equation (5.4)

$$\boxed{(C_p)_L \approx 0}. \quad (5.5)$$

5.1.3.2 Choked isentropic leakage flow, $(C_p)_{pl} \geq [(C_p)_{pl}]_{is,crit}$

The leakage flow under isentropic conditions is choked for critical and supercritical values of plenum pressure coefficient, $(C_p)_{pl} \geq [(C_p)_{pl}]_{is,crit}$. For a choked isentropic leakage flow, the leak exit pressure coefficient, $(C_p^*)_{L,is}$, is

$$(C_p^*)_{L,is} = \frac{p_{L,is}^* - p_e}{\frac{1}{2}\rho_e V_e^2}, \quad (5.6)$$

where $p_{L,is}^*$ is the value of p_L for a choked isentropic leakage flow. The value of $p_{L,is}^*$ varies with plenum pressure.

Equation (C3.12) in Appendix C gives

$$(C_p^*)_{L,is} = \left[(C_p)_{pl} + \frac{2}{\gamma M_e^2} \right] \left(\frac{1+\gamma}{2} \right)^{\frac{-\gamma}{\gamma-1}} - \frac{2}{\gamma M_e^2}, \quad (5.7)$$

where $(C_p^*)_{L,is}$ is the value of $(C_p)_L$ for a choked isentropic leakage flow. The equation shows that $(C_p^*)_{L,is}$ varies with $(C_p)_{pl}$ for a given external flow Mach number.

5.1.4 Leak differential pressure coefficient, ΔC_p

The pressure difference between the plenum and the leak orifice exit plane, $p_{pl} - p_L$, determines the leakage flow rate. Hence the leak differential pressure coefficient, ΔC_p , given by

$$\Delta C_p = (C_p)_{pl} - (C_p)_L, \quad (5.8)$$

is a useful parameter that can be used to predict the leakage flow rate. Equations giving the leak differential pressure coefficient for unchoked and choked isentropic leakage flows are given in Sections 5.1.4.1 and 5.1.4.2, respectively.

5.1.4.1 Unchoked isentropic leakage flow, $(C_p)_{pl} < [(C_p)_{pl}]_{is,crit}$

The leakage flow under isentropic conditions is unchoked for subcritical values of plenum pressure coefficient, $(C_p)_{pl} < [(C_p)_{pl}]_{is,crit}$. For an unchoked leakage flow, substituting Equation (5.5) into Equation (5.8) gives

$$\Delta C_p \approx (C_p)_{pl}, \quad (5.9)$$

for a leak orifice opening normal to the external flow (Sketches 1.1 and 1.2).

5.1.4.2 Choked isentropic leakage flow, $(C_p)_{pl} \geq [(C_p)_{pl}]_{is,crit}$

The leakage flow under isentropic conditions is choked for critical and supercritical values of plenum pressure coefficient, $(C_p)_{pl} \geq [(C_p)_{pl}]_{is,crit}$. From Equation (5.8) the leak differential pressure coefficient for a choked isentropic flow, $(\Delta C_p^*)_{is}$, is given by

$$(\Delta C_p^*)_{is} = (C_p)_{pl} - (C_p^*)_{L,is}. \quad (5.10)$$

5.2 Leak Mass Flow Ratio, $\frac{\dot{m}_L}{\dot{m}_e}$

The leak mass flow rate, $\dot{m}_L$, is the mass flow rate through the leak orifice, see Sketch 1.3. Values of $\dot{m}_L$ are predicted using

$$\dot{m}_L = K_d(\dot{m}_L)_{is}, \quad (5.11)$$

where $(\dot{m}_L)_{is}$ is the mass flow rate in an isentropic stream (see Section 5.3) and K_d is a discharge coefficient (see Section 5.4). The discharge coefficient is the ratio of actual mass flow rate to the isentropic mass flow rate.

The leak mass flow ratio is the ratio of the mass flow rate through the leak orifice, $\dot{m}_L$, to the mass flow rate in the external stream, $\dot{m}_e$ (see Section 4.1.2). The leak mass flow ratios associated with subcritical plenum pressure coefficients are considered in Section 5.2.1. Similarly, leak mass flow ratios associated with critical and supercritical plenum pressure coefficients, are considered in Section 5.2.2.

5.2.1 Subcritical plenum pressure coefficient, $(C_p)_{pl} < [(C_p)_{pl}]_{is,crit}$

From Equation (5.11) the leak mass flow ratio accounting for friction, viscous and real gas effects is

$$\frac{\dot{m}_L}{\dot{m}_e} = K_d \frac{(\dot{m}_L)_{is}}{\dot{m}_e}. \quad (5.12)$$

This equation is applicable for incompressible flows and those compressible flows where $(C_p)_{pl} < [(C_p)_{pl}]_{is,crit}$.

5.2.2 Critical and supercritical plenum pressure coefficients, $(C_p)_{pl} \geq [(C_p)_{pl}]_{is,crit}$

For critical and supercritical plenum pressure coefficients the leak mass flow rate in the choked isentropic stream is $(\dot{m}_L^*)_{is}$, see Section 5.3.2.2.

From Equation (5.12) with $(\dot{m}_L)_{is} = (\dot{m}_L^*)_{is}$,

$$\frac{\dot{m}_L}{\dot{m}_e} = K_d \frac{(\dot{m}_L^*)_{is}}{\dot{m}_e}. \quad (5.13)$$

5.3 Leak Isentropic Mass Flow Ratio, $\frac{(\dot{m}_L)_{is}}{\dot{m}_e}$

The mass flow rate, $(\dot{m}_L)_{is}$, through the leak orifice at the exit plane (L) in an isentropic stream is

$$(\dot{m}_L)_{is} = \rho_{L,is} A_L V_{L,is}, \quad (5.14)$$

where $\rho_{L,is}$ is the density and $V_{L,is}$ is the velocity of the isentropic stream at (L).

Combining Equations (4.1) and (5.14) the leak isentropic mass flow ratio is

$$\frac{(\dot{m}_L)_{is}}{\dot{m}_e} = \frac{\rho_{L,is} A_L V_{L,is}}{\rho_e A_L V_e} = \frac{\rho_{L,is} V_{L,is}}{\rho_e V_e}. \quad (5.15)$$

Sections 5.3.1 and 5.3.2 present $(\dot{m}_L)_{is}/\dot{m}_e$ for incompressible and compressible flows, respectively.

5.3.1 Incompressible flow

For low leakage flow velocities the flow field may be considered to be incompressible and Equation (A4.3) in Appendix A gives

$$\frac{(\dot{m}_L)_{is}}{\dot{m}_e} = \sqrt{\Delta C_p}. \quad (5.16)$$

For an incompressible flow field the leakage flow is unchoked, therefore substituting Equation (5.9) into Equation (5.16) gives

$$\boxed{\frac{(\dot{m}_L)_{is}}{\dot{m}_e} \approx \sqrt{(C_p)_{pl}}}, \quad (5.17)$$

for a leak orifice opening normal to the external flow (Sketches 1.1 and 1.2).

5.3.2 Compressible flow

For those cases where the leakage flow cannot be considered incompressible, Equation (B7.4) in Appendix B gives

$$\frac{(\dot{m}_L)_{is}}{\dot{m}_e} = \frac{p_L}{p_e} \frac{1}{M_e} \sqrt{\frac{4}{(\gamma-1) \left[2 + (\gamma-1) M_e^2 \right]}} \left[\left(\frac{p_{pl}}{p_L} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \left(\frac{p_{pl}}{p_L} \right)^{\frac{\gamma-1}{\gamma}}. \quad (5.18)$$

Section 5.3.2.1 presents $(\dot{m}_L)_{is}/\dot{m}_e$ for subcritical values of plenum pressure coefficient. Section 5.3.2.2 presents $(\dot{m}_L^*)_{is}/\dot{m}_e$ for critical and supercritical values of plenum pressure coefficient. The variations of $(\dot{m}_L)_{is}/\dot{m}_e$ with ΔC_p and $(\dot{m}_L^*)_{is}/\dot{m}_e$ with $(\Delta C_p^*)_{is}$ are described in Section 5.3.2.3 for an example flow field when $M_e = 0.7$.

5.3.2.1 Subcritical plenum pressure coefficient, $(C_p)_{pl} < [(C_p)_{pl}]_{is,crit}$

For subcritical plenum pressure coefficients and a leak orifice opening normal to the external flow (Sketches 1.1 and 1.2) Equation (5.5) gives $(C_p)_L \approx 0$. Therefore from Equation (5.3)

$$p_L \approx p_e. \quad (5.19)$$

Substituting Equation (5.19) into Equation (5.18) gives

$$\frac{(\dot{m}_L)_{is}}{\dot{m}_e} \approx \frac{1}{M_e} \sqrt{\frac{4}{(\gamma - 1)[2 + (\gamma - 1)M_e^2]}} \left[\left(\frac{p_{pl}}{p_e} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \left(\frac{p_{pl}}{p_e} \right)^{\frac{\gamma-1}{\gamma}}, \quad (5.20)$$

in which, from Equation (C3.10) in Appendix C,

$$\frac{p_{pl}}{p_e} = \frac{1}{2} \gamma M_e^2 (C_p)_{pl} + 1. \quad (5.21)$$

In the limit as $M_e \rightarrow 0$, Equation (5.20) reduces to Equation (5.17).

5.3.2.2 Critical and supercritical plenum pressure coefficients, $(C_p)_{pl} \geq [(C_p)_{pl}]_{is,crit}$

For choked leakage flows Equation (C5.2) in Appendix C gives

$$\frac{(\dot{m}_L^*)_{is}}{\dot{m}_e} = \frac{\frac{1}{2} \gamma M_e^2 (C_p^*)_{L,is} + 1}{M_e} \sqrt{\frac{1 + \gamma}{2 + (\gamma - 1)M_e^2}}, \quad (5.22)$$

where $(\dot{m}_L^*)_{is}$ is the choked mass flow rate in an isentropic stream.

Alternatively, Equation (C5.3) in Appendix C gives

$$\frac{(\dot{m}_L^*)_{is}}{\dot{m}_e} = \frac{\frac{1}{2} \gamma M_e^2 (C_p)_{pl} + 1}{M_e} \left[\left(\frac{1 + \gamma}{2} \right)^{\frac{-\gamma}{\gamma-1}} \right] \sqrt{\frac{1 + \gamma}{2 + (\gamma - 1)M_e^2}}, \quad (5.23)$$

for $(C_p)_{pl} \geq [(C_p)_{pl}]_{is,crit}$.

It is assumed that $(C_p^*)_{L,is} \approx 0$ when $(C_p)_{pl} = [(C_p)_{pl}]_{is,crit}$ for a leak orifice opening normal to the external flow (Sketches 1.1 and 1.2), so that Equation (5.22) with $(C_p^*)_{L,is} \approx 0$ gives

$$\boxed{\frac{(\dot{m}_L^*)_{is,crit}}{\dot{m}_e} \approx \frac{1}{M_e} \sqrt{\frac{1+\gamma}{2+(\gamma-1)M_e^2}}}, \quad (5.24)$$

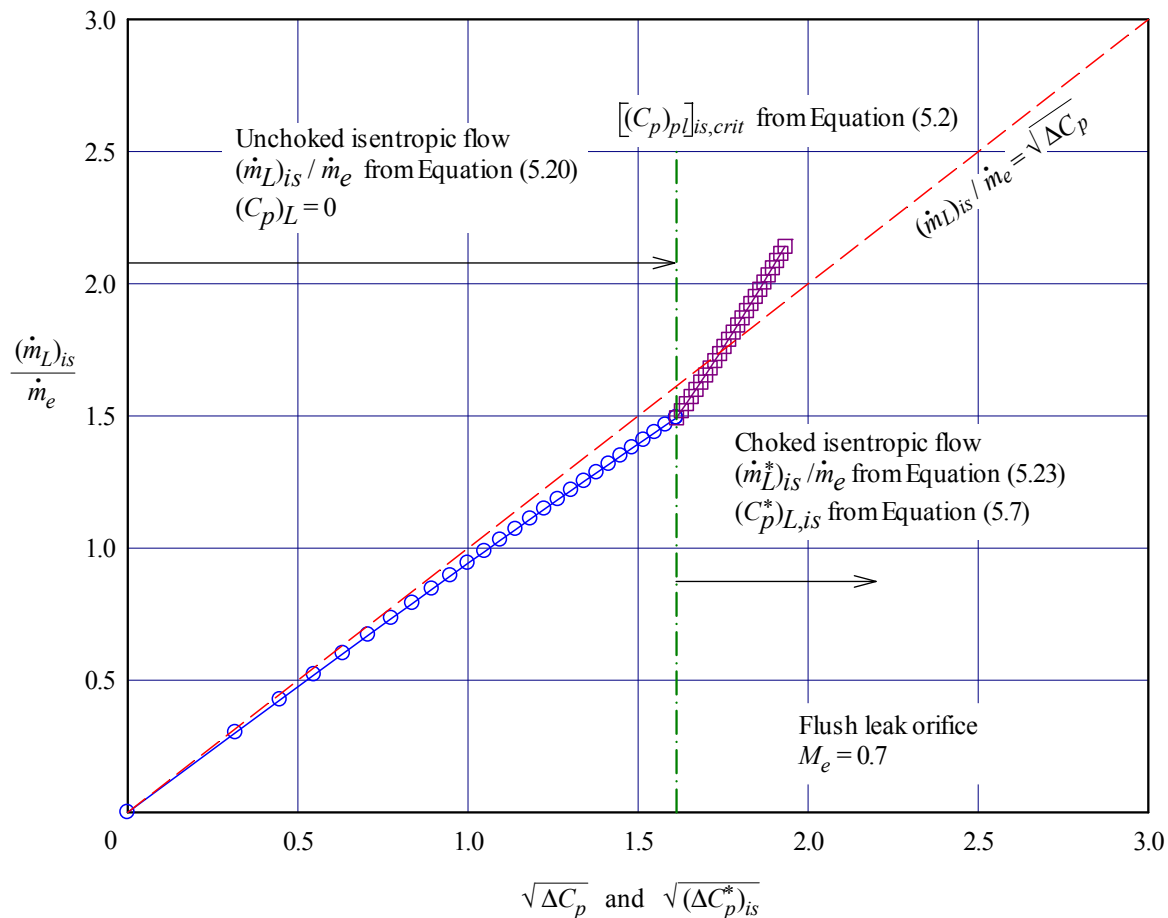
where $(\dot{m}_L^*)_{is,crit}$ is the value of $(\dot{m}_L)_{is}$ when $(C_p)_{pl} = [(C_p)_{pl}]_{is,crit}$. It should be noted that Equation (5.24) is only applicable for leakage flows where the leak orifice opens normal to the external flow (Sketches 1.1 and 1.2).

5.3.2.3 Variation of leak isentropic mass flow ratio with ΔC_p and $(\Delta C_p^*)_{is}$

Sketch 5.1 shows the variation of $(\dot{m}_L)_{is}/\dot{m}_e$ with $\sqrt{\Delta C_p}$ and $(\dot{m}_L^*)_{is}/\dot{m}_e$ with $\sqrt{(\Delta C_p^*)_{L,is}}$ for a leak orifice opening normal to the external flow, when $M_e = 0.7$. The critical plenum pressure coefficient, $[(C_p)_{pl}]_{is,crit}$, is obtained using Equation (5.2), see Section 5.1.2.

At subcritical values of plenum pressure coefficient, values of $(\dot{m}_L)_{is}/\dot{m}_e$ for a leak orifice opening normal to the external flow are given by Equation (5.20), see Section 5.3.2.1. From Equation (5.9) $\Delta C_p \approx (C_p)_{pl}$ so that in the construction of Sketch 5.1 it is assumed that $\Delta C_p = (C_p)_{pl}$ when $(C_p)_{pl} \leq [(C_p)_{pl}]_{is,crit}$. The leak mass flow rate increases as the value of $(C_p)_{pl}$ increases.

At critical and supercritical values of plenum pressure coefficient, $(C_p)_{pl} \geq [(C_p)_{pl}]_{is,crit}$, the leakage flow is choked and values of $(\dot{m}_L^*)_{is}/\dot{m}_e$ for a leak orifice opening normal to the external flow are given by Equation (5.22) or (5.23). Values of $(C_p^*)_{L,is}$ are obtained using Equation (5.7) and values of $(\Delta C_p^*)_{is}$ are obtained using Equation (5.10). The density of the leakage flow increases as the value of $(C_p)_{pl}$ increases, so that the leak mass flow rate also increases with $(C_p)_{pl}$.



Sketch 5.1 Variation of $\frac{(\dot{m}_L)_{is}}{\dot{m}_e}$ with $\sqrt{\Delta C_p}$ and $\sqrt{(\Delta C_p^*)_{is}}$ for flush leak orifice with $M_e = 0.7$

5.4 Discharge Coefficient, K_d

From Equation (5.11), the discharge coefficient, K_d , is the ratio of actual mass flow rate to the isentropic mass flow rate. It therefore enables the prediction of the change in leak mass flow rate due to friction, viscous and real gas effects. For an unchoked leakage flow discharging into an external stream the discharge coefficient is greatly influenced by the interaction between the external and leakage flows.

Figure 1 gives the discharge coefficient in terms of $F\sqrt{\Delta C_p}$ and $F\sqrt{(\Delta C_p^*)_{is}}$ for various values of M_e , where F is a composite factor accounting for the effects of leak orifice geometry and boundary-layer thickness. Linear interpolation for M_e is acceptable. The effect of the external flow on leak mass flow rate is greatly influenced by the leak orifice geometry.

The composite factor, F , is introduced in Section 5.4.1. A leak orifice geometry parameter, G , and leak orifice geometry factor, H , are used to obtain values of $F = fn[H(G + \delta/w)]$ and are introduced in Sections 5.4.2 and 5.4.3, respectively.

5.4.1 Composite factor, F

The composite factor, F , is obtained from Figure 2 which was constructed using rectangular orifice flat plate test data from Derivation 4. The dashed line in Figure 2 is tentative due to a paucity of usable test data for orifice geometries where $w/l < 0.056$. Values of a leak orifice geometry parameter, G , (see Section 5.4.2) and a leak orifice geometry factor, H , (see Section 5.4.3) together with the ratio δ/w are required in order to obtain F . Values of G and H are obtained from Figures 3 and 4, respectively.

5.4.2 Leak orifice geometry parameter, G

When $w/l < 1$ the leak orifice has its longest dimension in the direction of the external flow (Sketches 1.1 and 1.2). For a given value of ΔC_p the discharge coefficient generally has a larger value for those orifice geometries where $w/l < 1$. (This can be seen by comparing Sketches D3.1 and D3.2 in Appendix D.) The leak orifice geometry parameter, G , is used to account for this effect. Values of G are obtained from Figure 3 for $w/l < 1$, and $G = 0$ for $w/l \geq 1$. The dashed line in Figure 3 is tentative due to a paucity of usable test data for orifice geometries where $w/l < 0.056$.

5.4.3 Leak orifice geometry factor, H

When $w/l > 1$ the leak orifice has its shortest dimension in the direction of the external flow (Sketches 1.1 and 1.2). The leak orifice geometry factor, H , accounts for those orifice geometries where $w/l > 1$. Values of H are obtained from Figure 4 for $w/l > 1$, and $H = 1$ for $w/l \leq 1$. The dashed line in Figure 4 is tentative due to a paucity of usable test data for orifice geometries where $w/l > 17.68$.

6. ACCURACY AND APPLICABILITY

Sketches 6.1 and 6.2 indicate that the majority of the mass flow ratio test data are predicted within ± 0.05 for the rectangular, square, elliptical and circular orifice geometries identified. The orifice geometries and flow field conditions used to obtain the test data are summarised in Tables 6.2a to 6.6. The limits of applicability of leak orifice geometry, unit Reynolds number and external flow Mach number are discussed in Sections 6.1, 6.2 and 6.3, respectively. The ranges of applicability for various geometric and flow parameters are summarised in Section 6.4.

6.1 Leak Orifice Geometry

The Derivation 4 flat plate test data indicate that rectangular and elliptical orifice geometries sharing the same values of w/l and A_L have similar values of discharge coefficient. The method has been derived using rectangular, square, elliptical and circular orifice geometries. Hence, in addition to these geometries it is suggested that the method could also be applicable to other geometries provided that values of w , l and leak orifice area, A_L , are known.

Sketch 6.1 compares predicted values of leak mass flow ratio with flat plate test data from Derivations 3 and 4 for rectangular (including square) leak orifice geometries. Sketch 6.2 compares predicted values of leak mass flow ratio with flat plate test data from Derivations 2 and 4 for elliptical (including circular) leak orifices. The test data used to derive the method cover rectangular leak orifice geometries over the range $0.0056 \leq w/l \leq 176.8$ (including square) and elliptical geometries over the range $0.25 \leq w/l \leq 4$ (including circular). The rectangular orifice data for $w/l = 176.8$ in Sketch 6.1 exhibit a bias; the method underpredicts the leak mass flow ratio for this leak orifice geometry.

6.2 Unit Reynolds Number, R_1

The range of unit Reynolds numbers at which the Derivation 4 test data were obtained is $4.71 \times 10^6 \text{ m}^{-1} \leq R_1 \leq 4.92 \times 10^6 \text{ m}^{-1}$, see Tables 6.5a, 6.5b and 6.6. For the Derivation 4 test data $\delta/w \leq 440$ (see Tables 6.5a, 6.5b and 6.6) and $\delta/l \leq 430$ so that in a large number of the test cases the leakage flow is likely to be wholly immersed in the boundary layer. It is assumed that for larger values of unit Reynolds number the effect of viscosity on the interaction between the leakage flow and external flow will remain largely unchanged. Hence for $R_1 \geq 4.7 \times 10^6 \text{ m}^{-1}$ the method is applicable for $\delta/w \leq 440$ and $\delta/l \leq 430$.

The range of unit Reynolds numbers at which the Derivation 1 test data were obtained is estimated as $1.59 \times 10^6 \text{ m}^{-1} \leq R_1 \leq 2.49 \times 10^6 \text{ m}^{-1}$, see Tables 6.2a and 6.2b. For the Derivation 1 test data $\delta/w \geq 1.92$ for $w/l < 1.0$ (see Table 6.2a) and $\delta/l \geq 1.91$ for $w/l > 1.0$ (see Table 6.2b). Hence for these test cases the leakage flows are likely to be wholly immersed in the boundary layer. Values of discharge coefficient obtained from the Derivation 1 test data are significantly larger than those obtained from Derivation 4, see Sketches D3.1 and D3.2 in Appendix D. It is thought that this is due to greater viscous effects on the wholly immersed leakage flows, associated with the much lower values of unit Reynolds number. Appendix E indicates that at the low unit Reynolds number ($R_1 = 2.45 \times 10^6 \text{ m}^{-1}$) for the wind-tunnel test discussed, the effect of the leakage flow being wholly immersed in the external flow boundary layer is to increase the value of discharge coefficient.

The unit Reynolds numbers at which the Derivation 2 flat plate test data for a circular orifice were obtained are estimated to be $R_1 = 1.23 \times 10^6 \text{ m}^{-1}$ and $2.45 \times 10^6 \text{ m}^{-1}$, see Table 6.3. For the Derivation 2 circular orifice data $\delta/w = 0.3$ and 0.45 , so that the leakage flows are likely to be partially immersed in the boundary layer. Values of discharge coefficient obtained from the Derivation 2 test data are similar to those obtained from Derivation 4.

6.2.1 Leakage flow partially immersed in boundary layer

The flat plate test data in Derivations 2, 3 and 4 indicate that for leakage flows partially immersed in a boundary layer the viscous effects in the boundary layer do not greatly influence the leakage flow. For those leak orifice geometries where $w/l \geq 1.0$ the leakage flow is estimated in this Item to be partially immersed in the boundary layer when $\delta/l \leq 1.0$. Similarly, for those leak orifice geometries where $w/l \leq 1.0$ the leakage flow is taken to be partially immersed in the boundary layer when $\delta/w \leq 1.0$. For leak orifice geometries with $w/l \leq 1.0$ the method is applicable for $R_1 \geq 1.0 \times 10^6 \text{ m}^{-1}$ for leakage flows where $\delta/w \leq 1.0$. For leak orifice geometries with $w/l \geq 1.0$ the method is applicable for $R_1 \geq 1.0 \times 10^6 \text{ m}^{-1}$ for leakage flows where $\delta/l \leq 1.0$.

6.2.2 Leakage flow wholly immersed in boundary layer

The test data in Derivations 1 and 4 indicate that for leakage flows wholly immersed in a boundary layer, external flows having low values of unit Reynolds number have the effect of greatly increasing the leak mass flow rate. For those leak orifice geometries where $w/l \geq 1.0$ the leakage flow is estimated in this Item to be wholly immersed in the boundary layer when $\delta/l \geq 1.0$. Similarly, for those leak orifice geometries where $w/l \leq 1.0$ the leakage flow is taken to be wholly immersed in the boundary layer when $\delta/w \geq 1.0$.

For leak orifice geometries with $w/l \leq 1.0$ the method is applicable for $R_1 \geq 4.7 \times 10^6 \text{ m}^{-1}$ for leakage flows where $\delta/w \geq 1.0$. For leak orifice geometries with $w/l \geq 1.0$ the method is applicable for $R_1 \geq 4.7 \times 10^6 \text{ m}^{-1}$ for leakage flows where $\delta/l \geq 1.0$.

The method does not attempt to account for the increased leak mass flow rate of wholly immersed leakage flows, which occurs at low values of unit Reynolds number. Extreme caution should therefore be employed when using the method for $R_1 < 4.7 \times 10^6 \text{ m}^{-1}$ when $\delta/l \geq 1.0$ for leak orifice geometries where $w/l \geq 1.0$ and when $\delta/w \geq 1.0$ for $w/l \leq 1.0$.

6.3 External Flow Mach Number, M_e

The method is constructed using flat plate test data in which the external flow Mach number is in the range $0.1 \leq M_e \leq 1.1$. The Derivation 4 test data in Sketches D3.1 and D3.2 in Appendix D indicate that K_d increases in value as the external flow Mach number increases, for the larger values of $\sqrt{\Delta C_p}$.

6.4 Ranges of Applicability

The dashed lines in Figures 2 to 4 are tentative and indicate ranges of w/l where usable test data are scarce, see Tables 6.4 to 6.6. The dashed line for $M_e \leq 0.1$ in Figure 1 is an extrapolation of the discharge coefficient data obtained from Derivation 2.

Table 6.1 gives the ranges of applicability of various parameters identified above. Also given is the lower boundary of applicability for Reynolds number based on boundary-layer thickness, R_δ . The ranges of applicability were determined by the orifice geometries and flow conditions at which the available test data were obtained. The lower boundaries of applicability for δ/w and δ/l are provided by the Derivation 3 flat plate test data where $\delta/w \geq 0.079$ and $\delta/l \geq 0.079$, see Table 6.4.