



Drag due to a leak discharging into an external stream



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DRAG DUE TO A LEAK DISCHARGING INTO AN EXTERNAL STREAM

CONTENTS

	Page
1. NOTATION	1
2. INTRODUCTION	7
3. LEAK GEOMETRY	7
3.1 Leak Orifice	7
3.2 Plenum	8
4. LEAK FLOWFIELD	8
4.1 External Flow	8
4.1.1 Leak station	8
4.1.2 Mass flow rate in external stream, $\dot{m}_e$	8
4.2 Leakage Flow	9
5. METHOD	9
5.1 Leak Efflux Drag Coefficient, $C_{DL,E}$	10
5.1.1 $C_{DL,E}$ for incompressible leakage flow	10
5.1.1.1 Leak orifice geometries with $w/l > 1$	11
5.1.1.2 Leak orifice geometries with $w/l \leq 1$	11
5.1.2 $C_{DL,E}$ for compressible leakage flow	12
5.1.2.1 Critical leak mass flow ratio, $\frac{(\dot{m}_L^*)_{crit}}{\dot{m}_e}$	12
5.1.2.2 Choked flow	12
5.1.2.3 Unchoked flow	13
5.2 Boundary-layer Interference Drag Coefficient, $C_{DL,b}$	15
5.2.1 Leak orifice geometries with $w/l \leq 1$	15
5.2.2 Leak orifice geometries with $w/l > 1$	16
5.3 Calculation Flow Diagrams	16
6. ACCURACY AND APPLICABILITY	19
6.1 Wind-tunnel Test Data	19
6.1.1 Derivation 1	19
6.1.2 Derivation 2	19
6.2 Leak Drag Coefficient for Orifice Geometries with $w/l \geq 3.98$	19
6.2.1 Leak drag coefficient, $C_{DL,e}$	20
6.2.2 Leak efflux drag coefficient, $C_{DL,E}$	20
6.2.3 Boundary-layer interference drag coefficient, $C_{DL,b}$	20

6.3	Leak Drag Coefficient for Orifice Geometries with $w/l \leq 1$	22
6.3.1	Leak drag coefficient, $C_{DL,e}$	22
6.3.2	Leak efflux drag coefficient, $C_{DL,E}$	22
6.3.3	Boundary-layer interference drag coefficient, $C_{DL,b}$	22
6.4	Leakage flow exit angle, α_{VL}	24
6.5	Boundary-layer Drag Factor Due to Leakage Flow, $k_{w/l}$	25
6.6	Effect of Compressibility	26
6.6.1	Leakage flow exit angle, α_{VL}	26
6.6.2	Boundary-layer drag factor due to leakage flow, $k_{w/l}$	26
6.7	Ranges of Applicability	26
7.	DERIVATION AND REFERENCES	29
7.1	Derivation	29
7.2	References	29
8.	WORKED EXAMPLES	30
8.1	Example 1: Incompressible Leakage Flow	30
8.1.1	Leak efflux drag coefficient, $C_{DL,E}$	30
8.1.2	Drag coefficient due to interference of leakage flow with the boundary layer, $C_{DL,b}$	30
8.1.3	Leak drag coefficient based on external flow conditions, $C_{DL,e}$	31
8.1.4	Leak drag coefficient based on S_{ref} and freestream conditions, C_{DLA}	31
8.2	Example 2: Unchoked Compressible Leakage Flow	31
8.2.1	Leak efflux drag coefficient, $C_{DL,E}$	32
8.2.2	Drag coefficient due to interference of leakage flow with the boundary layer, $C_{DL,b}$	33
8.2.3	Leak drag coefficient based on external flow conditions, $C_{DL,e}$	33
8.2.4	Leak drag coefficient based on S_{ref} and freestream conditions, C_{DLA}	33
8.3	Example 3: Choked Leakage Flow	34
8.3.1	Leak efflux drag coefficient, $C_{DL,E}^*$	34
8.3.2	Drag coefficient due to interference of leakage flow with the boundary layer, $C_{DL,b}$	35
8.3.3	Leak drag coefficient based on external flow conditions, $C_{DL,e}$	35
8.3.4	Leak drag coefficient based on S_{ref} and freestream conditions, C_{DLA}	36
	FIGURES 1 AND 2	37-38
	APPENDIX A LEAK EFFLUX DRAG COEFFICIENT, $C_{DL,E}$	39
A1.	NOTATION	39
A2.	INTRODUCTION	40
A3.	DRAG ANALYSIS	40
A3.1	Streamforce, F	40
A3.2	Leak Exit Streamforce, F_L	40
A3.3	Leak Efflux Drag, $D_{L,E}$	41

A4.	LEAK EFFLUX DRAG COEFFICIENT, $C_{DL,E}$	42
A5.	REFERENCE	42
APPENDIX B LEAKAGE VELOCITY RATIO, $\frac{V_{La}}{V_e}$		43
B1.	NOTATION	43
B2.	INTRODUCTION	44
B3.	LEAKAGE VELOCITY RATIO, $\frac{V_{La}}{V_e}$	44
B3.1	Incompressible Flow	45
B3.2	Compressible Flow	45
B3.2.1	Choked leakage flow	46
B3.2.2	Unchoked leakage flow	47
B4.	REFERENCE	48
APPENDIX C CRITICAL LEAK MASS FLOW RATIO, $\frac{(\dot{m}_L^*)_{crit}}{\dot{m}_e}$		49
C1.	NOTATION	49
C2.	INTRODUCTION	50
C3.	LEAK MASS FLOW RATIO, $\frac{\dot{m}_L}{\dot{m}_e}$	50
C4.	CHOKED LEAK MASS FLOW RATIO, $\frac{\dot{m}_L^*}{\dot{m}_e}$	52
C5.	CRITICAL LEAK MASS FLOW RATIO, $\frac{(\dot{m}_L^*)_{crit}}{\dot{m}_e}$	52
C6.	REFERENCE	52
APPENDIX D LEAK EXIT PRESSURE COEFFICIENT FOR CHOKED FLOW, $(C_p^*)_L$		53
D1.	NOTATION	53
D2.	INTRODUCTION	53
D3.	LEAK EXIT PRESSURE RATIO FOR CHOKED FLOW, $\frac{p_L^*}{p_e}$	53
D4.	LEAK EXIT PRESSURE COEFFICIENT FOR CHOKED FLOW, $(C_p^*)_L$	54

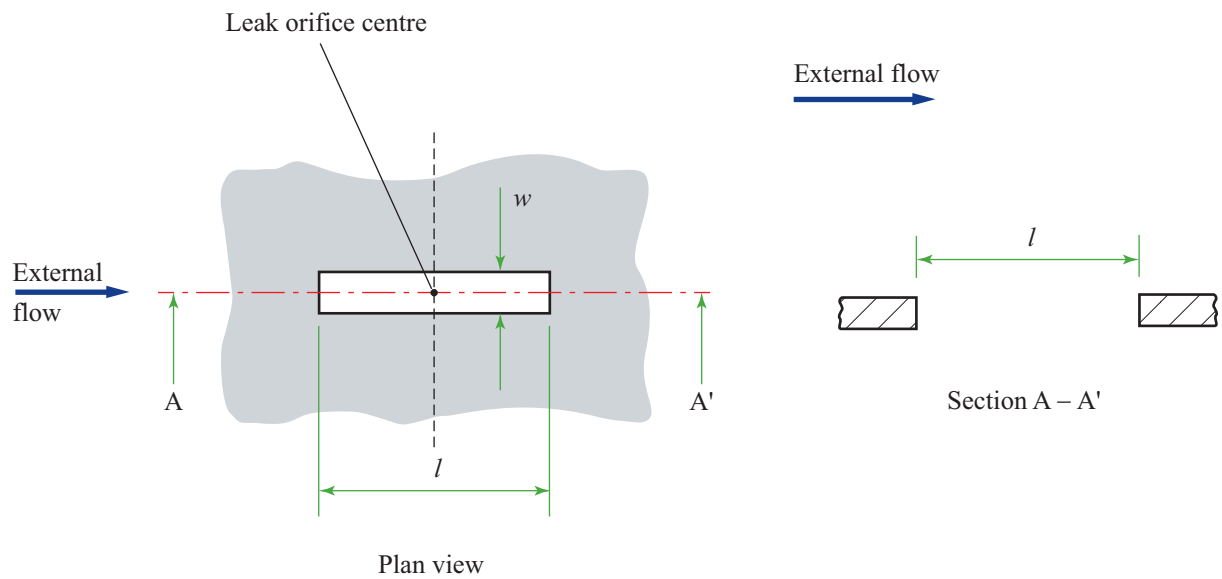
APPENDIX E	DUCTED FLUSH OUTLET EFFLUX DRAG COEFFICIENT, $C_{D,E}$	55
E1.	NOTATION	55
E2.	INTRODUCTION	59
E3.	EXTERNAL FLOW CONDITIONS WITH OUTLET SEALED	59
E4.	INCOMPRESSIBLE OUTLET FLOW	60
E5.	COMPRESSIBLE OUTLET FLOW	60
E5.1	Critical Outlet Mass Flow Ratio, $\frac{(\dot{m}_{out}^*)_{crit}}{\dot{m}_e}$	60
E5.2	Choked Outlet	61
E5.3	Unchoked Outlet	61
E6.	FLUSH OUTLET WITH 60° DUCT INCLINATION ANGLE	61
E6.1	Outlet Flow Exit Angle, α_{VL}	62
E6.2	Variation of Outlet Drag Coefficient Test Data with Outlet Mass Flow Ratio	62
E6.3	Variation of Efflux Drag Coefficient with Outlet Mass Flow Ratio	62
E6.4	Prediction of Outlet Drag Coefficient	62
E7.	FLUSH OUTLET WITH 90° DUCT INCLINATION ANGLE	69
E7.1	Outlet Flow Exit Angle, α_{VL}	69
E7.2	Variation of Outlet Drag Coefficient Test Data with Outlet Mass Flow Ratio	69
E7.3	Variation of Efflux Drag Coefficient with Outlet Mass Flow Ratio	69
E7.4	Prediction of Outlet Drag Coefficient	69
APPENDIX F	EQUATIONS DEFINING THE CURVES IN FIGURES 1 AND 2	72
F1.	NOTATION	72
F2.	INTRODUCTION	72
F3.	EQUATIONS DEFINING THE CURVES IN FIGURE 1	73
F4.	EQUATIONS DEFINING THE CURVE IN FIGURE 2	77

DRAG DUE TO A LEAK DISCHARGING INTO AN EXTERNAL STREAM

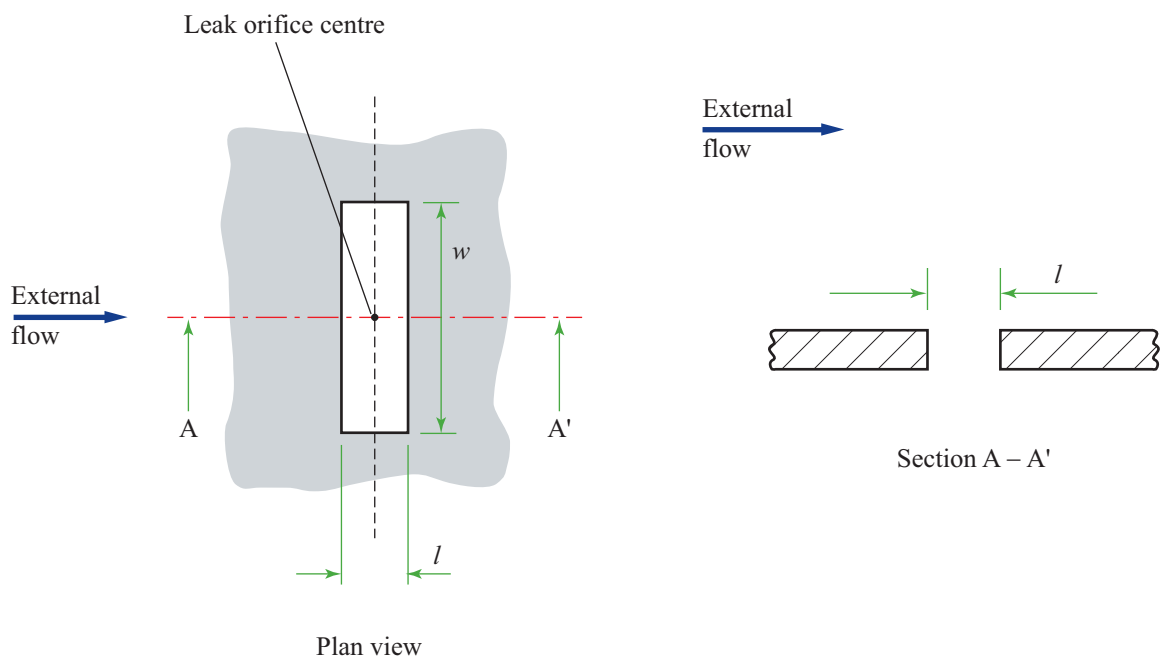
1. NOTATION

		<i>SI</i>	<i>British</i>
A_L	leak orifice area, see Section 3.1	m ²	ft ²
C_{DLA}	leak drag coefficient based on S_{ref} and freestream kinetic pressure, see Equations (5.1) and (5.2)		
$C_{DL,b}$	component of C_{DLe} due to interference of leakage flow with boundary layer, see Section 5.2 and Equations (5.4), (5.25) and (5.26)		
C_{DLe}	leak drag coefficient, based on conditions at edge of boundary layer at leak station with leak sealed and A_L , see Section 5 and Equations (5.3) and (5.4)		
$C_{DL,E}$	component of C_{DLe} due to leakage efflux momentum at leak exit plane (L), see Section 5.1 and Equation (5.5)		
$C_{DL,E}^*$	value of $C_{DL,E}$ for choked leakage flow, see Section 5.1.2.2 and Equations (5.14) and (5.15)		
$(C_p)_L$	leak exit pressure coefficient, see Section 5.1 and Equation (5.6)		
$(C_p^*)_L$	value of $(C_p)_L$ for choked leakage flow, see Section 5.1.2.2 and Equation (5.16)		
D_L	drag due to individual leak, measured in freestream direction, see Section 5 and Equations (5.1) and (5.3)	N	lbf
d	outer diameter of annulus, see inset sketch in Sketch 6.1	m	ft
$k_{w/l}$	factor in Equation (5.26) accounting for effect of w/l on $C_{DL,b}$, see Figure 2		
l	maximum leak orifice dimension in direction of external flow, see Section 3.1 and Sketches 1.1 to 1.4	m	ft
M_e	Mach number of external flow at edge of boundary layer at leak station with leak sealed, see Section 4.1.2 and Sketch 1.4		
$\dot{m}_e$	mass flow rate in external stream, <i>i.e.</i> through streamtube of cross-sectional area A_L , based on conditions at edge of boundary layer at leak station with leak sealed, see Section 4.1.2 and Equation (4.1)	kg/s	slug/s
$\dot{m}_L$	mass flow rate through leak orifice, see Section 4.2 and Sketch 1.3	kg/s	slug/s
$\dot{m}_L^*$	value of $\dot{m}_L$ for choked leakage flow, see Section 5.1.2.2	kg/s	slug/s
$(\dot{m}_L^*)_{crit}$	value of $\dot{m}_L$ when leakage flow becomes choked, see Section 5.1.2.1 and Equation (5.13)	kg/s	slug/s

$\dot{m}_{out}$	mass flow rate through outlet, see Section 6.6.1 and Appendix E	kg/s	slug/s
p_e	static pressure at edge of boundary layer at leak station with leak sealed, see Section 5.1	N/m ²	lbf/ft ²
p_L	static pressure at leak orifice exit plane (L), see Section 5.1	N/m ²	lbf/ft ²
p_L^*	value of p_L for choked leakage flow, see Section 5.1.2.2	N/m ²	lbf/ft ²
R_1	Reynolds number per unit length of external flow at edge of boundary layer at leak station with leak sealed, see Section 4.1.2	m ⁻¹	ft ⁻¹
R_δ	Reynolds number based on boundary-layer thickness, $R_1 \delta$		
S_{ref}	air-vehicle reference area	m ²	ft ²
s	annular gap, see inset sketch in Sketch 6.1	m	ft
V	local velocity in boundary layer, see Section 4.1.2 and Sketch 1.4	m/s	ft/s
V_e	flow velocity at edge of boundary layer at leak station with leak sealed, see Section 4.1.2 and Sketches 1.4 and 1.5	m/s	ft/s
V_{La}	average flow velocity at leak orifice exit plane (L), see Section 5.1 and Sketch 1.3	m/s	ft/s
V_{La}^*	value of V_{La} for choked leakage flow, see Section 5.1.2.2	m/s	ft/s
V_∞	freestream velocity	m/s	ft/s
w	maximum leak orifice dimension normal to external flow direction, see Section 3.1 and Sketches 1.1 and 1.2	m	ft
α_{VL}	angle of leakage efflux at leak orifice exit plane (L) relative to external flow direction, see Sections 5.1, 5.1.2.3, 6.4 and Sketch 1.3	deg	deg
α_{VL}^*	value of α_{VL} for choked leakage flow, see Section 5.1.2.2	deg	deg
$(\alpha_{VL})_{Fig1}$	value of α_{VL} obtained from Figure 1 for $w/l \geq 4$, see Sections 5.1.1.1 and 5.1.2.3	deg	deg
β_s	angle between freestream direction and V_e , see Section 5.1 and Sketch 1.5	deg	deg
γ	ratio of specific heat capacities, taken as 1.4 for air		
δ	boundary-layer thickness at $V = 0.99 V_e$ at leak station of external flow with leak sealed, see Section 4.1.2 and Sketch 1.4	m	ft
ρ_e	external air density at edge of boundary layer at leak station with leak sealed, see Section 4.1.2 and Sketch 1.4	kg/m ³	slug/ft ³
ρ_∞	freestream air density	kg/m ³	slug/ft ³

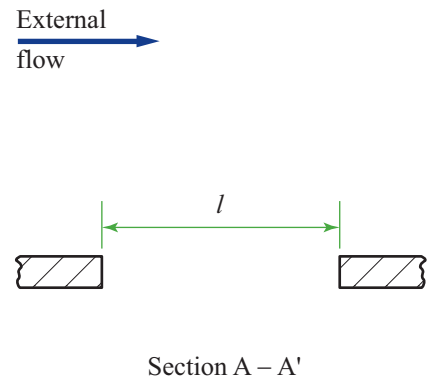
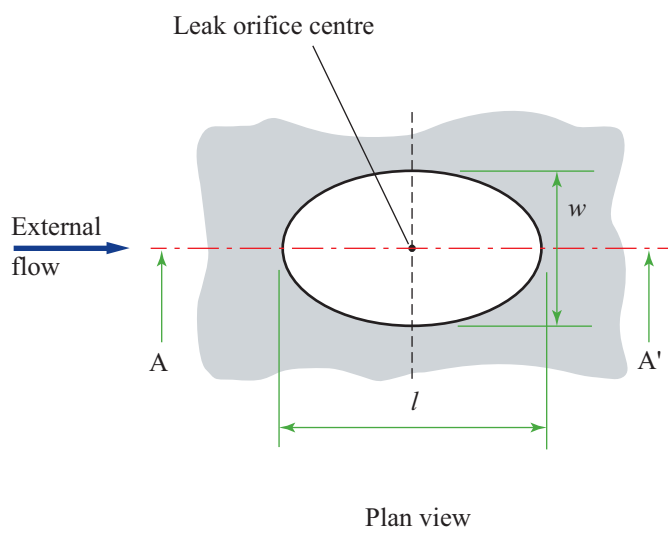


a) $w < l$

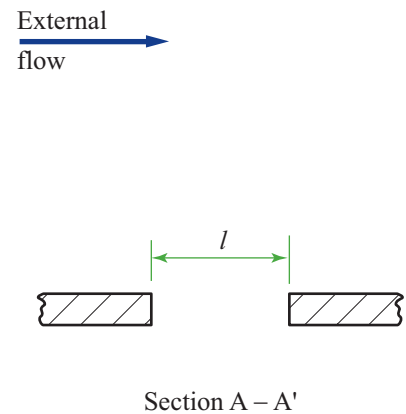
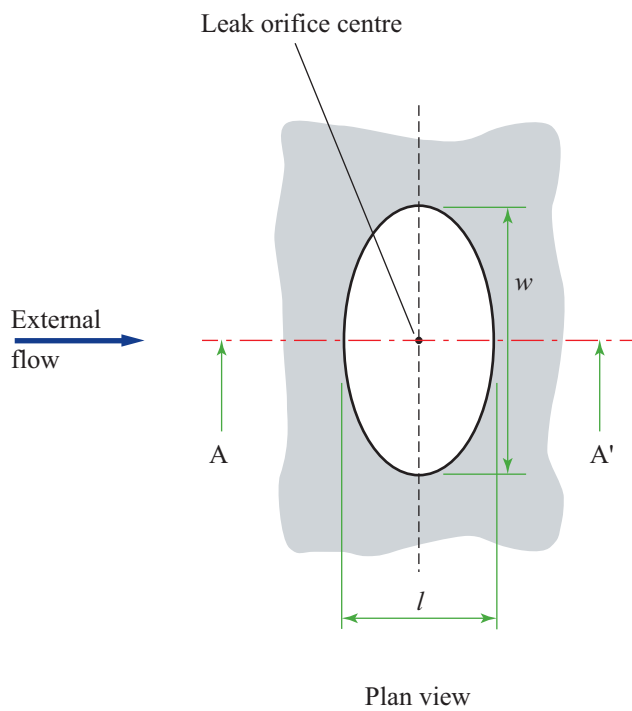


b) $w > l$

Sketch 1.1 Rectangular leak orifice geometry in a thin plate

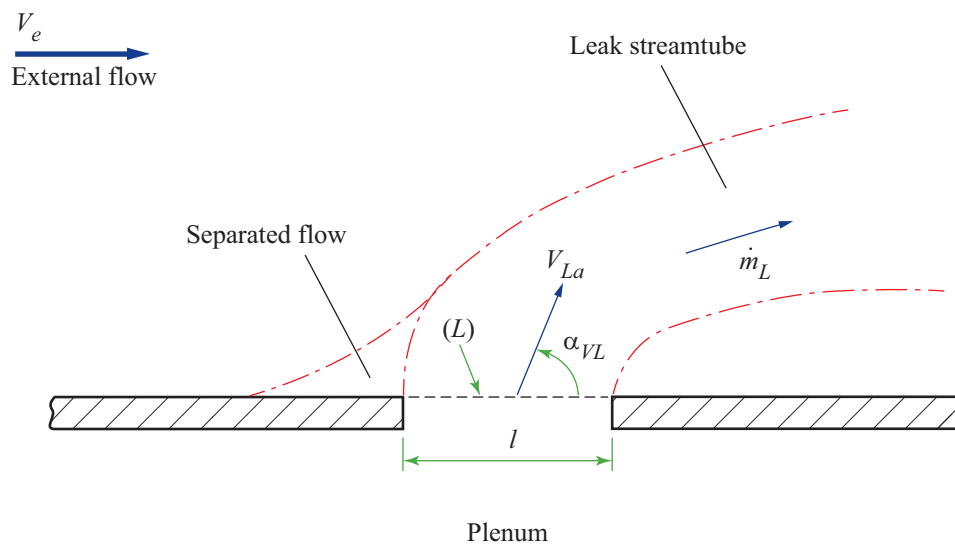


a) $w < l$

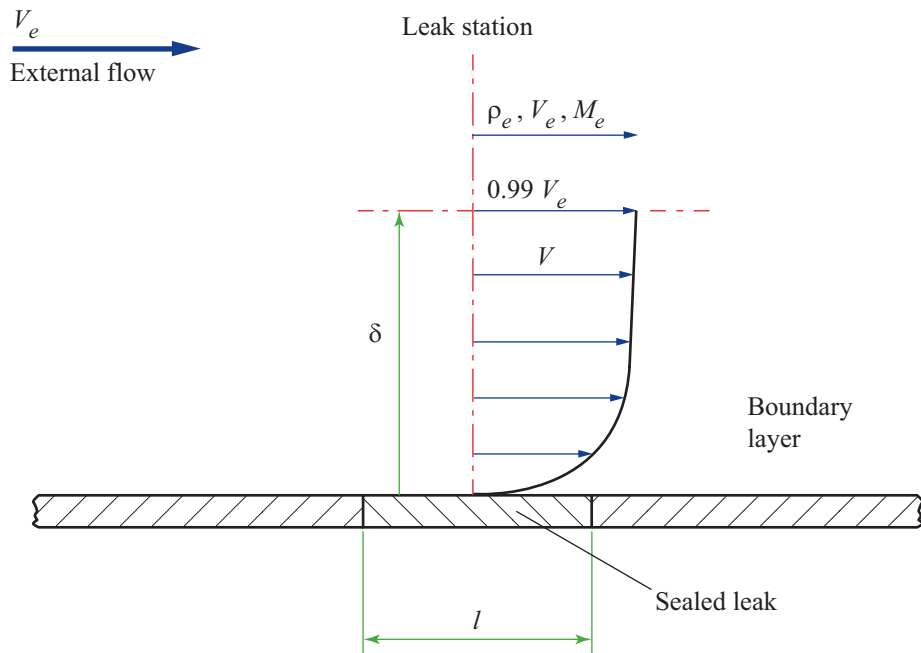


b) $w > l$

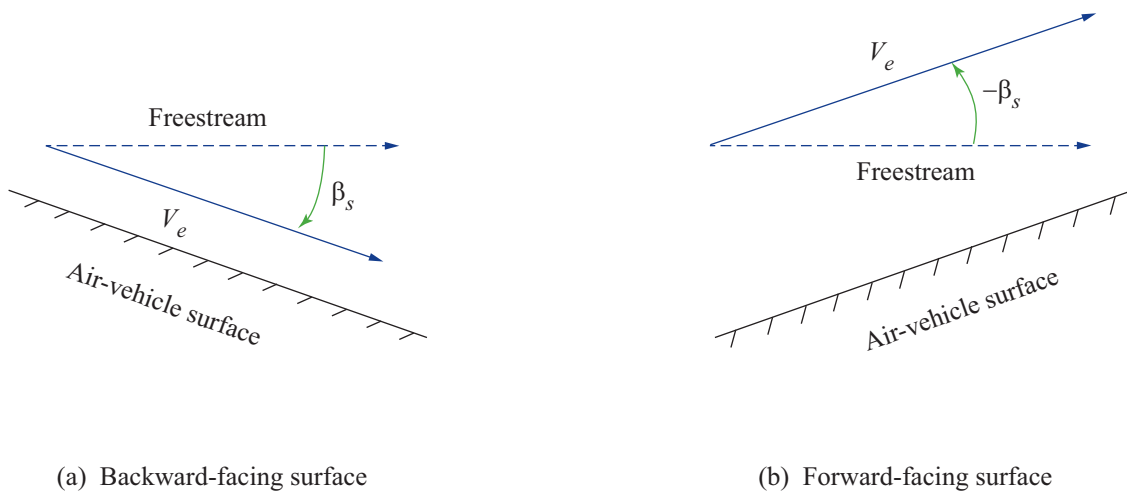
Sketch 1.2 Elliptical leak orifice geometry in a thin plate



Sketch 1.3 Section through flowfield due to a leak discharging into an external stream



Sketch 1.4 External flow when leak is sealed



Sketch 1.5 Angle, β_s , between freestream direction and V_e

2. INTRODUCTION

This Data Item provides a method of predicting the drag due to a leak which discharges into an external stream. A leakage flow discharging into an external stream affects drag in two main ways. First, the leakage efflux momentum exiting the leak orifice contributes directly to the drag. Second, the interference effects of the emerging leakage flow on the external flow boundary layer are expected, in some cases, to influence drag.

The drag due to the leakage efflux momentum is the component of force due to the efflux momentum measured in the freestream direction. It may have a positive value (*i.e.* drag) or a negative value (*i.e.* a thrust) depending on the location of the leak on the air vehicle, the geometry, size and orientation of the leak orifice and the external flow conditions.

The drag due to the interference of the leakage flow with the external flow boundary layer is dependent on the geometry and orientation of the leak orifice, as well as the leak mass flow rate. Wind-tunnel test data indicate that for some leak orifice geometries this drag component is small enough to be neglected.

The method is derived using test data for flows discharging into external streams through various well-defined orifice geometries in thin plates. The method is applicable to rectangular, square, elliptical and circular orifice geometries see Sketches 1.1 and 1.2. In addition to these geometries the method should also be applicable to other similar geometries. The method requires knowledge of the leak orifice area together with values of the maximum orifice dimensions in both the streamwise direction and perpendicular to it, see Section 3. Leak mass flow ratio is also required and is obtained using ESDU 16005 (Derivation 4).

The leak geometry is described in Section 3. Section 4 describes the leak flowfield which for the purpose of this Item is divided into an external flow and a leakage flow. The prediction method is given in Section 5 and the accuracy and applicability are presented in Section 6. The Derivation and References are given in Section 7 and three worked examples are presented in Section 8.

Appendix A derives an equation giving the drag coefficient due to leakage efflux momentum. Appendix B provides the derivation of an equation giving leakage velocity ratio. Appendices C and D derive equations giving the critical leak mass flow ratio and the leak exit pressure coefficient for choked flow, respectively.

Appendix E presents wind-tunnel test data for flush outlets with ducts for a range of external flow Mach numbers. The available compressible flow test data are currently limited to ducted outlets. Appendix E shows that the drag coefficient due to leakage efflux can be predicted with confidence for compressible flows. Appendix F provides equations giving the curves in Figures 1 and 2.

3. LEAK GEOMETRY

The leak orifice geometry is described in Section 3.1 and the plenum is defined in Section 3.2.

3.1 Leak Orifice

The method of this Item has been derived using thin plate data for rectangular, square, and circular orifice geometries. In addition to these geometries the method should also be applicable to elliptical orifices and to other similar geometries.

In this Item l is the maximum dimension of the orifice aligned with the direction of the external flow and w is the maximum dimension perpendicular to the external flow. These dimensions are shown in Sketches 1.1 and 1.2 for rectangular and elliptical orifice geometries, respectively. For the purposes of the method the leak orifice geometry is defined by the ratio w/l and the leak orifice area, A_L .

The leak orifice area is

$$A_L = wl, \quad (3.1)$$

for a rectangular orifice and

$$A_L = \frac{\pi}{4}wl, \quad (3.2)$$

for an elliptical orifice.

3.2 Plenum

The plenum is a chamber within the vehicle structure which collects leakage inflows and distributes leakage outflows. It is assumed that the flow in the plenum is stationary relative to the plate before exiting via the leak orifice in the plate.

4. LEAK FLOWFIELD

It is useful to divide the leak flowfield into an external flow and a leakage flow. The external and leakage flows are described in Sections 4.1 and 4.2, respectively.

4.1 External Flow

The external flow follows the surface contours of the air vehicle and its direction might therefore be different from the direction of the freestream, see Sketch 1.5. Sketch 1.3 shows the external flow over the surface of a flat plate with a leak discharging into that flow.

Section 4.1.1 defines the leak station. The external flow conditions at the leak station when the leak is sealed are usually known. The mass flow rate in the external stream is defined in Section 4.1.2.

4.1.1 Leak station

Sketch 1.4 shows the external flow for the case when the leak has been sealed. The leak station is shown in this sketch and is located at the centre of the leak orifice (see Sketches 1.1 and 1.2).

4.1.2 Mass flow rate in external stream, $\dot{m}_e$

The external stream is defined here as the flow at the edge of the boundary layer at the leak station when the leak is sealed, see Sketch 1.4. In Sketch 1.4 ρ_e is the stream density, V_e is the flow velocity and M_e is the Mach number at the edge of the boundary layer at the leak station when the leak is sealed. The external mass flow rate, $\dot{m}_e$, is conveniently defined as the mass flow rate of the external stream through

a streamtube with the same cross-sectional area as the leak orifice, so that

$$\dot{m}_e = \rho_e A_L V_e . \quad (4.1)$$

The boundary-layer thickness, δ , is the height of the boundary layer at $V = 0.99V_e$, where V is the local velocity in the boundary layer, see Sketch 1.4. The unit Reynolds number, R_1 , is also based on the flow conditions at the edge of the boundary layer at the leak station when the leak is sealed.

4.2 Leakage Flow

The leakage flow is the flow in the streamtube passing through the leak orifice. The flow is dependent primarily on the pressure difference between the plenum and the external flow adjacent to the leak orifice.

A cross-sectional view of a leak orifice with a leakage flow discharging into an external stream is shown in Sketch 1.3. The leak orifice exit plane is denoted by (L) and the leak mass flow rate, $\dot{m}_L$, is the mass flow rate through the leak orifice.

5. METHOD

The leak drag coefficient, C_{DLA} , based on an air-vehicle reference area, S_{ref} , and the freestream kinetic pressure, is

$$C_{DLA} = \frac{D_L}{\frac{1}{2}\rho_\infty V_\infty^2 S_{ref}} , \quad (5.1)$$

where D_L is the drag due to the individual leak, measured in the freestream direction and ρ_∞ and V_∞ are the density and velocity of the freestream. The drag due to the individual leak, D_L , is the difference between the drag of the air vehicle with the leak and the drag with the leak sealed.

The drag due to the individual leak, D_L , is greatly dependent on the external flow conditions in the vicinity of the leak. It is therefore useful to express C_{DLA} as

$$C_{DLA} = \left[\frac{\rho_e V_e^2 A_L}{\rho_\infty V_\infty^2 S_{ref}} \right] C_{DLe} , \quad (5.2)$$

where the leak drag coefficient, C_{DLe} , based on the conditions in the external stream, is

$$C_{DLe} = \frac{D_L}{\frac{1}{2}\rho_e V_e^2 A_L} . \quad (5.3)$$

The leak drag coefficient, C_{DLe} , can also be expressed as two components

$$C_{DLe} = C_{DL,E} + C_{DL,b} , \quad (5.4)$$

where $C_{DL,E}$ is the drag coefficient due to leakage efflux momentum at the leak exit plane (L) (see Sketch 1.3) and $C_{DL,b}$ is the drag coefficient due to interference of the leakage flow with the boundary layer. The drag coefficients due to leakage efflux momentum and leakage flow interference with the boundary layer are dealt with in Sections 5.1 and 5.2, respectively. Calculation flow diagrams aiding the use of the method are presented in Section 5.3.

5.1 Leak Efflux Drag Coefficient, $C_{DL,E}$

The momentum of the leakage flow exiting the leak orifice results in a force which acts in the direction of the leak flow. The leak efflux drag is the component of this force on the air vehicle measured in the freestream direction. It may have a positive value (*i.e.* drag) or a negative value (*i.e.* a thrust) depending on the location of the leak on the air vehicle, the geometry, size and orientation of the leak orifice and the external flow conditions.

Equation (A4.3) in Appendix A gives

$$C_{DL,E} = -2 \frac{\dot{m}_L V_{La}}{\dot{m}_e V_e} \cos(\alpha_{VL} - \beta_s) - (C_p)_L \sin \beta_s, \quad (5.5)$$

where V_{La} is the average flow velocity at the leak exit plane (L) and $(C_p)_L$ is the pressure coefficient at the leak exit plane. In Equation (5.5) α_{VL} is the angle between the directions of V_{La} and V_e , measured positive in the direction shown in Sketch 1.3. The leak mass flow ratio $\dot{m}_L/\dot{m}_e$ is obtained using ESDU 16005. Also β_s is the angle between V_e and the freestream direction; measured positive in the direction shown in Sketch 1.5a for a backward-facing surface relative to the freestream and negative in the direction shown in Sketch 1.5b for a forward-facing surface.

The pressure coefficient at the leak orifice exit plane is

$$(C_p)_L = \frac{p_L - p_e}{\frac{1}{2} \rho_e V_e^2}, \quad (5.6)$$

where p_L is the static pressure at the leak orifice exit plane and p_e is the static pressure in the external flow at the edge of the boundary layer at the leak station when the leak is sealed.

Sections 5.1.1 and 5.1.2 present $C_{DL,E}$ for incompressible and compressible flow, respectively.

5.1.1 $C_{DL,E}$ for incompressible leakage flow

For an incompressible flow field Equation (5.5) in ESDU 16005 gives

$$(C_p)_L \approx 0.$$

For an incompressible flow Equation (B3.4) in Appendix B gives

$$\frac{V_{La}}{V_e} = \frac{\dot{m}_L}{\dot{m}_e}. \quad (5.7)$$

5.1.1.1 Leak orifice geometries with $w/l > 1$

Substituting Equation (5.7) and Equation (5.5) of ESDU 16005 into Equation (5.5) gives

$$C_{DL,E} \approx -2 \left(\frac{\dot{m}_L}{\dot{m}_e} \right)^2 \cos(\alpha_{VL} - \beta_s) \quad (5.8)$$

The value of α_{VL} for $w/l \geq 4$ is estimated using Figure 1, which gives the variation of α_{VL} with $\dot{m}_L/\dot{m}_e$ for various values of M_e . Linear interpolation for M_e is acceptable. The development of Figure 1 is outlined in Section E7.1 of Appendix E.

To a first approximation, the value of α_{VL} for $1 < w/l < 4$ is obtained using linear interpolation over this range, see Section 6.4 and Sketch 6.3, so that

$$\alpha_{VL} = 90^\circ - \frac{1}{3} \left[\frac{w}{l} - 1 \right] \left[90^\circ - (\alpha_{VL})_{\text{Fig1}} \right], \quad \text{for } 1 < \frac{w}{l} < 4 \quad (5.9)$$

where $(\alpha_{VL})_{\text{Fig1}}$ is the value of α_{VL} obtained from Figure 1 for $w/l \geq 4$.

If $\beta_s \approx 0$, Equation (5.8) simplifies to

$$C_{DL,E} \approx -2 \left(\frac{\dot{m}_L}{\dot{m}_e} \right)^2 \cos \alpha_{VL} \quad (5.10)$$

5.1.1.2 Leak orifice geometries with $w/l \leq 1$

Test data from Derivations 1 and 2 indicate that $\alpha_{VL} \approx 90^\circ$ for flush leak orifice geometries where $w/l \leq 1$ (Sketches 1.1a and 1.2a), see Sketch 6.2 and Section 6.4.

Substituting $\alpha_{VL} \approx 90^\circ$ into Equation (5.8) and simplifying gives

$$C_{DL,E} \approx -2 \left(\frac{\dot{m}_L}{\dot{m}_e} \right)^2 \sin \beta_s, \quad (5.11)$$

for flush leak orifice geometries where $w/l \leq 1$.

If $\beta_s \approx 0$, Equation (5.11) simplifies to

$$C_{DL,E} \approx 0 \quad (5.12)$$

5.1.2 $C_{DL,E}$ for compressible leakage flow

When sonic conditions are reached at the leak orifice the flow is said to be choked. Section 5.1.2.1 calculates a value of a critical leak mass flow ratio in order to determine whether the leakage flow is choked or unchoked. Sections 5.1.2.2 and 5.1.2.3 calculate values of $C_{DL,E}$ for choked and unchoked leakage flows, respectively. The total temperatures in the external flow and leakage flow are assumed to be equal.

5.1.2.1 Critical leak mass flow ratio, $\frac{(\dot{m}_L^*)_{crit}}{\dot{m}_e}$

The critical leak mass flow ratio, $(\dot{m}_L^*)_{crit} / \dot{m}_e$, is the value of $\dot{m}_L / \dot{m}_e$ at which the leakage flow becomes choked. A simplifying assumption is made that when the flow is choked the area of sonic flow is equal to A_L and it occurs at the leak orifice exit plane (L)[†]. Equation (C5.1) in Appendix C gives

$$\frac{(\dot{m}_L^*)_{crit}}{\dot{m}_e} \approx \frac{1}{M_e} \sqrt{\frac{1+\gamma}{2+(\gamma-1)M_e^2}}, \quad (5.13)$$

where γ is the ratio of specific heat capacities, taken as 1.4 for air.

For $\dot{m}_L / \dot{m}_e \geq (\dot{m}_L^*)_{crit} / \dot{m}_e$ the leakage flow is choked and $C_{DL,E}^*$ is obtained using the method in Section 5.1.2.2. For $\dot{m}_L / \dot{m}_e < (\dot{m}_L^*)_{crit} / \dot{m}_e$ the leakage flow is unchoked and $C_{DL,E}$ is obtained using the method in Section 5.1.2.3.

5.1.2.2 Choked flow

For a choked flow Equation (5.5) becomes

$$C_{DL,E}^* = -2 \frac{\dot{m}_L^* V_{La}^*}{\dot{m}_e V_e} \cos(\alpha_{VL}^* - \beta_s) - (C_p^*)_L \sin \beta_s, \quad (5.14)$$

where $\dot{m}_L^*$ is the mass flow rate and V_{La}^* is the average flow velocity at the leak orifice exit plane (L) for a choked leakage flow, respectively. Also in Equation (5.14) α_{VL}^* is the value of α_{VL} and $(C_p^*)_L$ is the value of $(C_p)_L$ for a choked leakage flow, respectively. For a choked flow, sonic conditions are assumed to exist at the leak orifice exit plane so that the leakage flow at (L) is not affected by the external flow. Therefore $\alpha_{VL}^* = 90^\circ$ for a leak orifice opening normal to the external flow (Sketches 1.1 and 1.2), so that Equation (5.14) becomes

$$C_{DL,E}^* = -2 \frac{\dot{m}_L^* V_{La}^*}{\dot{m}_e V_e} \sin \beta_s - (C_p^*)_L \sin \beta_s. \quad (5.15)$$

[†] Section 3.1 of ESDU 82009 (Reference 6) describes the development of choking for a plane circular orifice. Sketch 3.1 of ESDU 82009 (in the Internal Flow series) shows that for a choked flow the sonic surface is curved, not planar.

From Equation (5.6), for a choked flow

$$(C_p^*)_L = \frac{p_L^* - p_e}{\frac{1}{2}\rho_e V_e^2}, \quad (5.16)$$

where p_L^* is the value of p_L when the leakage flow is choked.

If $\beta_s \approx 0$, Equation (5.15) simplifies to

$$\boxed{C_{DL,E}^* \approx 0}, \quad (5.17)$$

for a leak orifice opening normal to the external flow (Sketches 1.1 and 1.2).

For choked leakage flows, values of velocity ratio, V_{La}^*/V_e , and leak exit pressure coefficient, $(C_p^*)_L$, are calculated as follows.

Velocity ratio, $\frac{V_{La}^*}{V_e}$

For a choked flow, the value of V_{La}^* is constant for a given leak. Equation (B3.14) in Appendix B gives

$$\boxed{\frac{V_{La}^*}{V_e} = \frac{1}{M_e} \sqrt{\frac{2 + (\gamma - 1)M_e^2}{1 + \gamma}}}. \quad (5.18)$$

Leak exit pressure coefficient, $(C_p^*)_L$

For a choked flow, Equation (D4.3) in Appendix D gives

$$\boxed{(C_p^*)_L = \left(\frac{\dot{m}_L^*}{\dot{m}_e} \right) \frac{2}{\gamma M_e} \sqrt{\frac{2 + (\gamma - 1)M_e^2}{1 + \gamma}} - \frac{2}{\gamma M_e^2}}. \quad (5.19)$$

The equation shows that the value of $(C_p^*)_L$ increases with increasing $\dot{m}_L^* / \dot{m}_e$. The leak exit pressure coefficient, $(C_p^*)_L \approx 0$ when $\dot{m}_L^* / \dot{m}_e = (\dot{m}_L^*)_{crit} / \dot{m}_e$ from Equation (5.13).

5.1.2.3 Unchoked flow

For unchoked leakage flows, the external flow can affect the leakage flow exit angle for certain leak orifice geometries. An estimated value of leakage flow exit angle, α_{VL} , is obtained as part of the method. Values of $C_{DL,E}$ for leak orifice geometries with $w/l > 1$ and $w/l \leq 1$ are calculated as follows.

Leak orifice geometries with $w/l > 1$

For an unchoked leakage flow, Equation (5.5) in ESDU 16005 gives

$$(C_p)_L \approx 0.$$

Substituting Equation (5.5) of ESDU 16005 into Equation (5.5) gives

$$C_{DL,E} = -2 \frac{\dot{m}_L V_{La}}{\dot{m}_e V_e} \cos(\alpha_{VL} - \beta_s). \quad (5.20)$$

The value of α_{VL} for $w/l \geq 4$ is estimated using Figure 1, which gives the variation of α_{VL} with $\dot{m}_L/\dot{m}_e$ for various values of M_e . Linear interpolation for M_e is acceptable. The development of Figure 1 is outlined in Section E7.1 of Appendix E.

To a first approximation, the value of α_{VL} for $1 < w/l < 4$ is obtained using linear interpolation over this range, see Section 6.4 and Sketch 6.3, and Equation (5.9) gives

$$\alpha_{VL} = 90^\circ - \frac{1}{3} \left[\frac{w}{l} - 1 \right] \left[90^\circ - (\alpha_{VL})_{\text{Fig1}} \right], \quad \text{for } 1 < \frac{w}{l} < 4$$

where $(\alpha_{VL})_{\text{Fig1}}$ is the value of α_{VL} obtained from Figure 1 for $w/l \geq 4$.

For an unchoked flow the velocity ratio, V_{La}/V_e , is dependent on the leak mass flow ratio, $\dot{m}_L/\dot{m}_e$, and the Mach number of the external flow, M_e .

Equation (B3.23) in Appendix B gives

$$\frac{V_{La}}{V_e} = \frac{-1 + \sqrt{1 + 4 \left(\frac{\dot{m}_L}{\dot{m}_e} \right)^2 \left(\frac{\gamma-1}{2} \right) M_e^2 \left[1 + \left(\frac{\gamma-1}{2} \right) M_e^2 \right]}}{2 \frac{\dot{m}_L}{\dot{m}_e} \left(\frac{\gamma-1}{2} \right) M_e^2}. \quad (5.21)$$

If $\beta_s \approx 0$, Equation (5.20) simplifies to

$$C_{DL,E} = -2 \frac{\dot{m}_L V_{La}}{\dot{m}_e V_e} \cos \alpha_{VL}, \quad (5.22)$$

in which V_{La}/V_e is given by Equation (5.21).

Leak orifice geometries with $w/l \leq 1$

Section 5.1.1.2 indicates that $\alpha_{VL} \approx 90^\circ$ for flush leak orifice geometries where $w/l \leq 1$.

Substituting $\alpha_{VL} \approx 90^\circ$ into Equation (5.20) and simplifying gives

$$C_{DL,E} = -2 \frac{\dot{m}_L V_{La}}{\dot{m}_e V_e} \sin \beta_s, \quad (5.23)$$

where the velocity ratio, V_{La}/V_e , is obtained using Equation (5.21).

If $\beta_s \approx 0$, Equation (5.23) further simplifies to

$$C_{DL,E} \approx 0. \quad (5.24)$$

5.2 Boundary-layer Interference Drag Coefficient, $C_{DL,b}$

The interference effects of the emerging leakage flow on the external flow boundary layer are expected to influence drag. The emerging leakage flow will cause the boundary layer to separate upstream and downstream of the leak orifice. The associated momentum loss in the separated flow results in a drag increase. The boundary-layer interference drag coefficient, $C_{DL,b}$, accounts for this drag component.

The test data from Derivation 1 include a component due to the interference of the leakage flow with the boundary layer, see Section 6.1.1. These data indicate that the value of $C_{DL,b}$ is dependent on the geometry and orientation of the leak orifice, as well as the leak mass flow rate.

Section 5.2.1 shows that $C_{DL,b}$ is small enough to be neglected for leak orifice geometries with $w/l \leq 1$. Section 5.2.2 provides an approximate equation giving $C_{DL,b}$ for leak orifice geometries with $w/l > 1$.

5.2.1 Leak orifice geometries with $w/l \leq 1$

The test data from Derivation 1 presented in Sketch 6.2 indicate that

$$C_{DL,b} \approx 0, \quad (5.25)$$

for those leak orifice geometries where $w/l \leq 1$.

It is thought that for orifice geometries with $w/l \leq 1$ the external stream divides and travels around the sides of the emerging leakage flow. This would account for the negligible value of $C_{DL,b}$ since the region of separated external flow is likely to be small in this case.

The test data from Derivation 2 do not include the component due to boundary layer interference, see Section 6.1.2. However, the circular orifice data from this Derivation presented in Sketch 6.2 show a negligible change in skin friction on the balance plate. It is therefore surmised that the region of separated flow is small for orifice geometries with $w/l = 1$.

5.2.2 Leak orifice geometries with $w/l > 1$

An analysis of test data from Derivation 1 presented in Sketch 6.1 covering a range of leak orifice geometries, together with predicted $C_{DL,E}$ data, gives

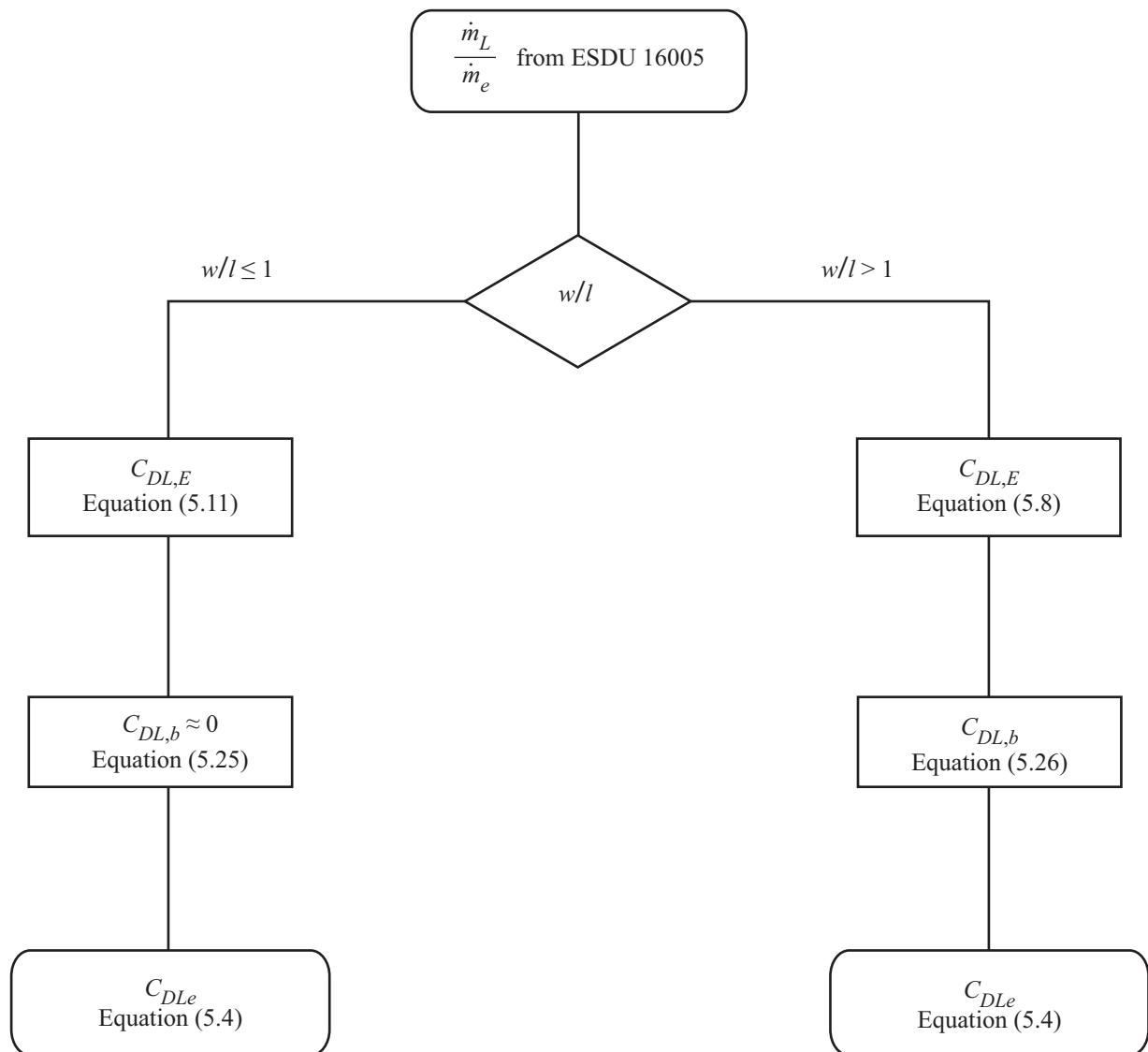
$$C_{DL,b} \approx k_{w/l} \frac{\dot{m}_L}{\dot{m}_e}, \quad (5.26)$$

where $k_{w/l}$ is a factor accounting for the leak orifice geometry. Figure 2 presents an approximate curve giving the variation of $k_{w/l}$ with w/l for leak orifices in thin plates opening normal to the external flow (Sketches 1.1 and 1.2). The effect of compressibility is not thought to affect the interaction of the leakage flow with the boundary layer, see Section 6.6.2. Figure 2 is therefore taken to be applicable for both incompressible and compressible leakage flows. The leak mass flow ratio $\dot{m}_L/\dot{m}_e$ is obtained using ESDU 16005.

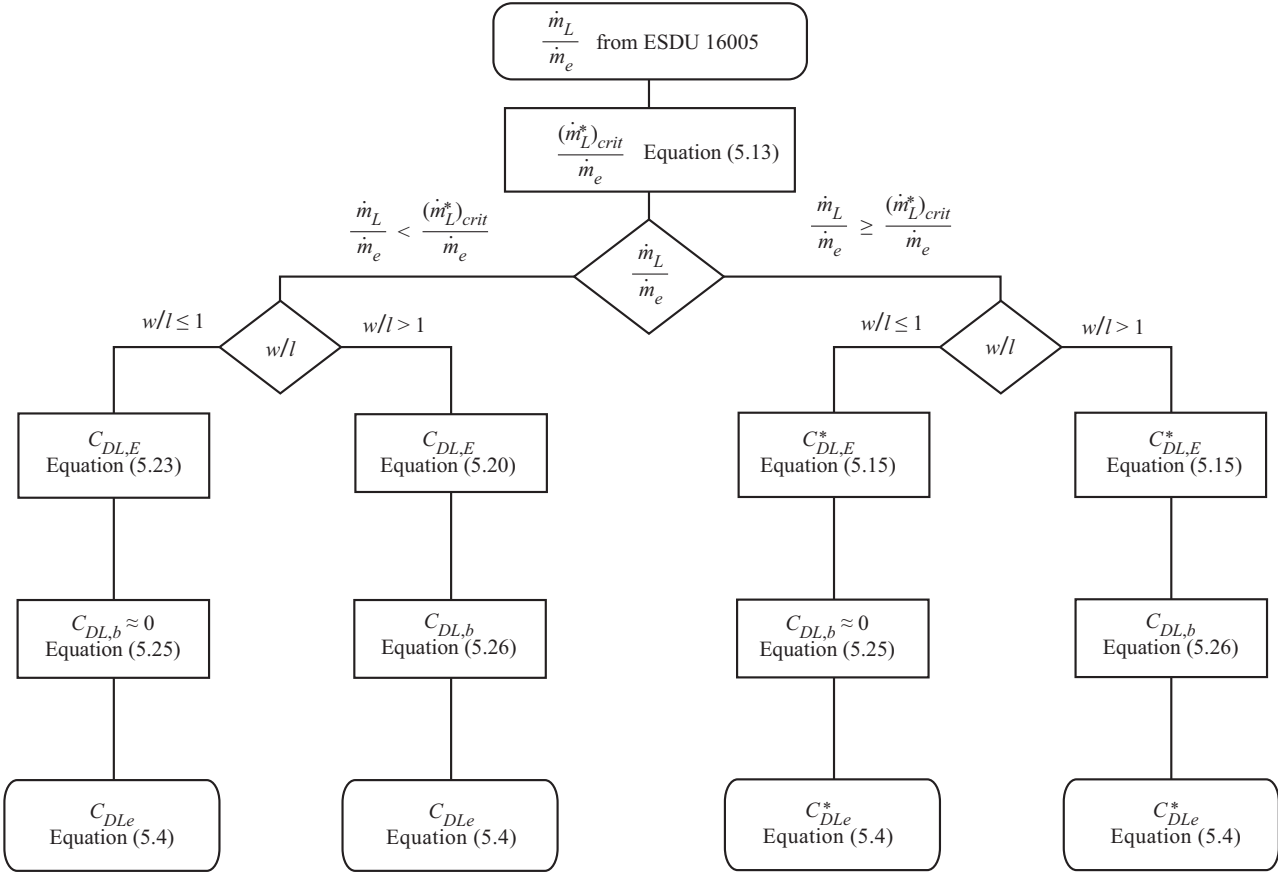
The test data in Sketch 6.1 indicate extensive regions of separated flow exist for most of the orifice geometries presented. This is expected since when there is leakage, attached flow can only exist near the ends of those orifice geometries with large values of w/l .

5.3 Calculation Flow Diagrams

The methods in Sections 5.1 and 5.2 are dependent on whether the leak orifice geometry has $w/l \leq 1$ or $w/l > 1$. In addition, the method in Section 5.1 also depends on whether the leakage flow is choked or unchoked. The flow diagrams in Sketches 5.1 and 5.2 give an aid to the use of the method for incompressible flow and compressible flow, respectively. The method requires a value of $\dot{m}_L/\dot{m}_e$ to be obtained from ESDU 16005.



Sketch 5.1 Incompressible flow method



Sketch 5.2 Compressible flow method

6. ACCURACY AND APPLICABILITY

Sketches 6.1 and 6.2 present leak drag coefficient test data obtained from low-speed wind tunnel tests (Derivations 1 and 2). Since the external flows are low speed the leakage flows are taken to be incompressible. Sketch 6.4 indicates that most of the leak drag coefficient test data are predicted within ± 0.1 for rectangular orifices and ± 0.05 for circular and annular orifices. Section 6.1 provides information on the wind-tunnel tests in Derivations 1 and 2. Sections 6.2 and 6.3 present test data for leak orifice geometries with $w/l \geq 3.98$ and $w/l \leq 1$, respectively. Sections 6.4 and 6.5 discuss the leakage flow exit angle and boundary-layer drag factor due to leakage flow, respectively. Compressibility effects are discussed in Section 6.6. This Item has the same parameter ranges of applicability as ESDU 16005, see Section 6.7.

6.1 Wind-tunnel Test Data

Leak drag test data were determined by subtracting the drag force on the wind-tunnel balance with the leak sealed from the drag force with leakage flow. Sections 6.1.1 and 6.1.2 present an overview of the wind-tunnel tests and the forces measured in Derivations 1 and 2, respectively.

6.1.1 Derivation 1

The Derivation 1 test data were obtained by placing leak orifices at various positions in the surface of a $1/5^{\text{th}}$ scale model of a Hawker Hurricane fighter aeroplane, which was connected to a drag balance. The fuselage and cockpit were hollowed out and the model was made airtight. The internal pressure was controlled by a compressed air system connected to the inside of the model and each orifice geometry could be tested separately. The measured force from the drag balance represents the leak efflux drag plus the drag changes due to the interference of the leakage flow with the boundary layer. The various test cases provided in Derivation 1 are summarised in Tables 6.1a to 6.1c.

6.1.2 Derivation 2

The Derivation 2 test data were obtained using a floating-element drag balance set into the wind-tunnel wall. The measured force represents the leak efflux drag plus any changes in skin friction on the balance plate due to the presence of the leak less the skin friction corresponding to the area of the plate removed by the leak orifice, but did not include the effect of drag changes induced upstream and downstream of the plate. The test conditions for the circular orifice geometry in Derivation 2 are summarised in Table 6.2.

6.2 Leak Drag Coefficient for Orifice Geometries with $w/l \geq 3.98$

Sketch 6.1 presents test data obtained from Derivation 1. The sketch shows the variation of C_{DLe} with leak mass flow ratio for various rectangular orifice geometries where $3.98 \leq w/l \leq 60$. Test data for two annular orifice geometries with $d/s = 7.81$ and 15.63 are also included in the sketch, where d is the outer diameter and s is the annular gap, see the inset sketch in Sketch 6.1. For annular orifices the external flow is likely to affect the leakage flow in a similar way to rectangular orifices with $w/l > 1$. Therefore, for the two annular orifice geometries identified in Sketch 6.1 the method of this Item was used with values of d/s instead of w/l , and taking A_L as the area of the annulus. For the flow fields considered in Derivation 1, the range $1.91 \leq \delta/l \leq 9.6$ indicates that the leak streamtube is wholly immersed in the external flow boundary layer, see Section 6.2.2 of ESDU 16005.

Sketch 6.1 shows that the leakage flow provides a small thrust component (*i.e.* $-C_{DLe}$). This indicates that there is a component of leakage flow momentum exiting the leak orifice in the direction of the external flow. Therefore, for those leak orifice geometries represented in Sketch 6.1 the effect of the external flow is to turn the leakage flow streamtube at the leak orifice exit. For orifice geometries with $w/l \geq 3.98$ Sections 6.2.1, 6.2.2 and 6.2.3 discuss C_{DLe} , $C_{DL,E}$ and $C_{DL,b}$, respectively.

6.2.1 Leak drag coefficient, C_{DLe}

The continuous curve in Sketch 6.1 shows the predicted variation of C_{DLe} with leak mass flow ratio. The predicted values of C_{DLe} were obtained via Equations (5.4), (5.10) and (5.26) for leak orifice geometries with $w/l \geq 8$. A comparison of this curve with the test data indicates that the method of this Item provides a reasonable prediction of C_{DLe} .

6.2.2 Leak efflux drag coefficient, $C_{DL,E}$

For the locations of the various leak orifices considered in Derivation 1, β_s is small enough to be neglected so that $C_{DL,E}$ is predicted using Equation (5.10), with values of α_{VL} obtained from Figure 1. The chain-dashed curve in Sketch 6.1 shows the variation of $C_{DL,E}$ with leak mass flow ratio. A comparison of this curve with the test data indicates that the variation of α_{VL} with mass flow ratio in Figure 1 is applicable to leak orifices in thin plates opening normal to the external flow (Sketches 1.1 and 1.2).

6.2.3 Boundary-layer interference drag coefficient, $C_{DL,b}$

For those leak orifice geometries with $w/l > 1$ the leakage flow exiting the orifice has a disruptive effect on the external flow, see Section 5.2.2. A comparison of the predicted $C_{DL,E}$ and C_{DLe} curves indicates that $C_{DL,b}$ is a significant component of C_{DLe} for those orifice geometries with $w/l \geq 8$.