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Fatigue crack propagation rates and threshold stress intensity factors in high strength low alloy steel plate, bar and forgings

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THE PREPARATION OF THIS DATA ITEM

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FATIGUE CRACK PROPAGATION RATES AND THRESHOLD STRESS INTENSITY FACTORS IN HIGH STRENGTH LOW ALLOY STEEL PLATE, BAR AND FORGINGS

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FATIGUE CRACK PROPAGATION RATES AND THRESHOLD STRESS INTENSITY FACTORS IN HIGH STRENGTH LOW ALLOY STEEL PLATE, BAR AND FORGINGS

1. NOTATION

A	position of maximum stress		
$2a$	crack length	m	in
a_c	critical crack half-length at failure	m	in
a_1	effective crack half-length	m	in
$2b$	plate width	m	in
C	coefficient for linear interpolation of crack growth rate curve defined by $da/dN = C(\Delta K)^n$	$\text{m} \left(\frac{\text{m}}{\text{N}} \right)^n$	$\text{in} \left(\frac{\text{in}}{\text{lbf}} \right)^n$
c	crack depth	m	in
f_p	0.2 proof stress of material	N/m ²	lbf/in ²
f_t	tensile strength of material	N/m ²	lbf/in ²
K	stress intensity factor	N/m ^{3/2}	lbf/in ^{3/2}
K_c	critical value of stress intensity factor	N/m ^{3/2}	lbf/in ^{3/2}
K_{cl}	stress intensity factor value at crack closure	N/m ^{3/2}	lbf/in ^{3/2}
ΔK	stress intensity factor range	N/m ^{3/2}	lbf/in ^{3/2}
ΔK_{eff}	effective stress intensity factor range defined as $K_{max} - K_{cl}$	N/m ^{3/2}	lbf/in ^{3/2}
K_{Ic}	plane strain fracture toughness in Mode I	N/m ^{3/2}	lbf/in ^{3/2}
ΔK_p	stress intensity factor range corrected for plasticity	N/m ^{3/2}	lbf/in ^{3/2}
ΔK_{TH}	threshold value of stress intensity factor range	N/m ^{3/2}	lbf/in ^{3/2}
ΔK_0	$S(\pi a)^{1/2}$	N/m ^{3/2}	lbf/in ^{3/2}
N	number of load cycles		
n	characteristic slope of crack growth rate curve		
M	moment	N m	lbf in

R	stress ratio, S_{min}/S_{max}		
r_p	extent of plastic zone size, or plasticity correction term	m	in
S	stress based upon gross cross-sectional area of specimen	N/m ²	lbf/in ²
α	geometrical factor in the calculation of stress intensity factor range		
α_1	geometrical factor in the calculation of stress intensity factor range, corrected for plasticity		
σ_y	yield (or 0.2 per cent proof) stress of material	N/m ²	lbf/in ²
<i>Suffixes</i>			
max, min	indicate maximum and minimum values of stress or stress intensity factor in the loading cycle		
f, i	indicate final and initial crack length		

1.1 Conversion Factor

Both SI and British units are shown in the notation. The data are presented in terms of SI units and the following conversion factor for British units applies.

$$1 \text{ MN/m}^{3/2} = 910.047 \text{ lbf/in}^{3/2}$$

Note also that,

$$1.0 \text{ MN/m}^{3/2} = 1.0 \text{ MPa m}^{1/2} = 31.6228 \text{ N/mm}^{3/2} \text{ (that is, } 10^{3/2} \text{ N/mm}^{3/2} \text{)}$$

2. NOTES

2.1 General

This Item presents fatigue crack propagation rate data in the range 10^{-4} to 10^{-11} m/cycle, (4×10^{-3} to 4×10^{-10} in/cycle) and also threshold stress intensity factor ranges, ΔK_{TH} , in terms of linear elastic fracture mechanics, as described in ESDU 80036*. Data are presented on the rate of growth of fatigue cracks in high strength ($f_t > 850 \text{ MN/m}^2$ ($123 \times 10^3 \text{ lbf/in}^2$)) low alloy steel plate, bar and forgings. A low alloy steel is here defined as a steel where the contribution (by weight) of any particular alloying element is less than 5 per cent. The chemical composition of the steels used is given in Table 7.1.

* ESDU 80036, 'Introduction to the use of linear elastic fracture mechanics in estimating fatigue crack growth rates and residual strength of components'.

2.2 Presentation of Data

Figures 1 to 22 present curves of crack propagation rate, da/dN , plotted against the stress intensity factor range, ΔK , for each of the material designations. Curves have been drawn through the data for the various values of stress ratio, R . The tables containing the mechanical properties and crack growth data for each figure appear on the pages facing the relevant figures.

Figures 23 to 25 present the effect of the change of R on the crack growth rate for three ranges of 0.2 per cent proof stress. Figure 26 summarises the curves and the effect that changing R and the different 0.2 per cent proof stresses have on the crack growth rate.

2.3 Test Conditions

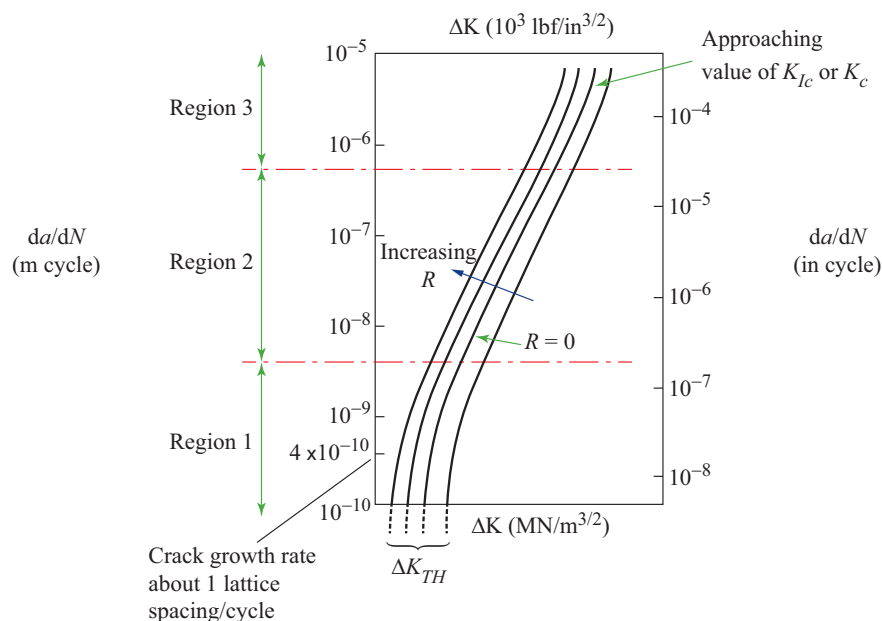
The data are derived from constant amplitude fatigue tests carried out on specimens of various standard types. The thicknesses of the specimens are given, where known, in the tables that accompany Figures 1 to 22. The tests were conducted under laboratory conditions (unless otherwise stated), at room temperature, and at loading frequencies in the range 0.1 to 175 Hz, with the exception of the tests reported in Derivation 26 which were conducted at 375 Hz.

The majority of tests were carried out within the framework of linear elastic fracture mechanics, implying that the stress across the remaining ligament of the specimen did not exceed the yield strength of the material and that the plastic zone size is small compared with the crack length (see Section 1 of ESDU 80036).

2.3.1 Fracture toughness testing

Some of the materials for which data are presented were also subjected to fracture toughness tests. The values of the critical stress intensity factors obtained from these tests are indicated on the figures and these values are designated K_{Ic} . The values of K_{Ic} for these and for other steels at other temperatures, and with different heat treatments, may be found in ESDU 83023.

2.4 Estimating Fatigue Crack Propagation Rates



Sketch 2.1 Typical set of crack growth rate curves

During fatigue loading, the process eventually leads to the failure of components and this process can be conveniently divided into three regions, see Sketch 2.1 and Table A1.1 (in Appendix A). In Table A1.1 the range of crack sizes and crack growth rates in each region, and the shape of the curves and the methods for analysing the data, are given. Note that although the transition from one region to the next is a continuous process, the boundaries between these regions are not precisely defined. References 38 and 40 give more detailed descriptions about these three regions. Many ESDU Data Items are available that present fatigue crack propagation rate data in forms similar to that shown in Sketch 2.1 for many materials. These crack propagation rate curves will not necessarily follow, in detail, the form shown in Sketch 2.1. Also, the value of K at failure in some materials may not be the value of K_{Ic} or K_c particularly if there has been extensive plasticity like, for example, in naturally aged aluminium alloys or in many structural steels.

Under constant amplitude fatigue loading, the crack propagation rate of a given crack depends primarily on the range of stresses in the fatigue cycle, $(S_{max} - S_{min})$, the crack half-length, a , and the stress ratio, $R = S_{min}/S_{max}$. The stress intensity factor range, ΔK , is defined by

$$\Delta K = K_{max} - K_{min}. \quad (2.1)$$

In its most simple form, ΔK may be expressed as

$$\Delta K = (S_{max} - S_{min})(\pi a)^{1/2} \alpha. \quad (2.2)$$

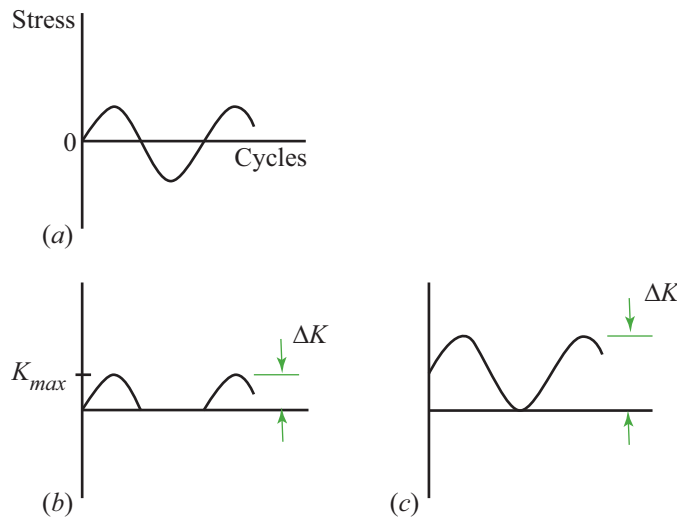
The value of K_{max} is always a function of S_{max}

$$K_{max} = S_{max}(\pi a)^{1/2} \alpha, \quad (2.3)$$

however, the effective value of K_{min} may be governed by crack closure effects.

The effect of mean stress on the fatigue crack propagation rate is important, albeit less so than the value of ΔK . Allowances for mean stress are considered in the values of R , K_{max} , or K_{min} .

Experimental fatigue crack growth data from simple specimens can be presented in terms of fatigue crack propagation rates (da/dN) versus ΔK for various values of R , see Sketch 2.1. In cases where S_{min} is compressive the crack may close during the fatigue cycle meaning that there is no clear convention for calculating ΔK . Therefore when calculating ΔK either the full range of the stress cycle must be considered, or only the tensile part of the cycle used (that is $\Delta K = K_{max} (K_{min} = 0)$), see Sketch 2.2.



Sketch 2.2 (a) Typical fatigue cycle with a negative stress ratio, R , (b) finding ΔK using tensile part of the stress cycle only, (c) finding ΔK using the full range of the stress cycle

Note that if the results of independently derived test data are used it is vital, when calculating ΔK , to use the same method that was applied to the analysis and presentation of the test data. The method of presentation of crack propagation rate data is given in the relevant ESDU Data Items*.

The geometrical factor, α , in Equation (2.3) is influenced by both the specimen geometry and the loading conditions and its effect on stress intensity factor depends on the following bullet points:

- For an elliptical planform crack in the centre of a plate which is infinite in both the thickness and the in plane extent, when the governing stress is normal to the planform then the stress intensity factor varies around the crack perimeter. It is greatest at the ends of the minor axes and lowest at the ends of the major axes. For all cases, the geometrical factor is less than for a two-dimensional crack of the same dimension. In the limit where the crack is a circular planform (penny shaped), the stress intensity factor is uniform round the perimeter and is approximately 0.65 (Reference 42). When the crack depth, c , is small relative to the major axis, a , it becomes effectively a two-dimensional crack with an factor of unity.
- In an infinite solid there are stresses parallel to the plane of the internal crack. When a crack breaks the surface of a semi-infinite solid the effect is to release these stresses by creating a free surface making the crack semi elliptical. The stress intensity factor on the surface increases by up to 11 per cent. This is sometimes called the 'edge' or 'free surface' factor.

* ESDU 91015, 'Fatigue crack propagation rates of experimental aluminium lithium sheet and plate alloys'.
 ESDU 90028, 'Effect of environment on fatigue crack propagation rate in titanium alloy sheet and plate'.
 ESDU 89051, 'Fatigue crack propagation rates for titanium alloy sheets'.
 ESDU 84003, 'Fatigue crack propagation rates and threshold stress intensity factors in high alloy and corrosion resistant (stainless) steel'.
 ESDU 83007, 'Fatigue crack propagation rates and threshold stress intensity factor ranges for aluminium alloy sheet'.
 ESDU 82015, 'Fatigue crack propagation rates and threshold stress intensity factor ranges for titanium alloy plate, bar and forgings'.
 ESDU 81031, 'Fatigue crack propagation rates and threshold stress intensity factor ranges for aluminium alloy plate, extruded bar and forgings'.
 ESDU 81012, 'Fatigue threshold stress intensity factors and slow crack propagation rates in low and medium strength low alloy steel'.

- A further effect on the geometrical factor is the relationship between the depth of the crack and the finite thickness of the component, in the direction of the crack (c/b). Even a two dimensional crack will experience an increase in α factor if the width of the cracked component is less than infinity. This applies to an internal crack in a two dimensional finite solid or an edge crack breaking a free surface (for this later case, the α factor is increased by a further 11 per cent)
- The bending constraint in single edge-cracks also affects the geometrical factor. The starting point for finite width cases is a wide plate with symmetrical edge cracks. If this geometry is split down the middle, the single edge-crack will experience greater stress intensity as the load is now unsymmetrical to the cracked section and this will apply some bending that is not present in the symmetrical case. This effect increases the α factor when the value of c/b increases.

The plasticity correction term that is applied to account for the effect of plastic flow at the crack tip is smaller under cyclic loading compared to its value for monotonic loading as the effective yield strength is doubled and the full range of the stress cycle contains both tensile and compressive behaviour. The extent of the plastic zone, r_p , for conditions of plane stress and monotonic loading for a particular value of K can be estimated using:

$$r_p = \frac{1}{2\pi} \left(\frac{K}{\sigma_y} \right)^2 \text{ or } r_p = \frac{a}{2} \left(\frac{S}{\sigma_y} \right)^2 \alpha^2. \quad (2.4)$$

For conditions of plane strain and monotonic loading, the shape of the plastic zone is more complex but is small and is taken to be one third of the plane stress plastic zone size:

$$r_p = \frac{1}{6\pi} \left(\frac{K}{\sigma_y} \right)^2. \quad (2.5)$$

To determine the value of r_p under cyclic loading conditions, K is replaced by ΔK and the yield stress value, σ_y , is replaced by $2\sigma_y$. Neglecting the possibility of crack closure the condition for plane stress becomes

$$r_p = \frac{1}{8\pi} \left(\frac{\Delta K}{\sigma_y} \right)^2, \quad (2.6)$$

and for conditions of plane strain:

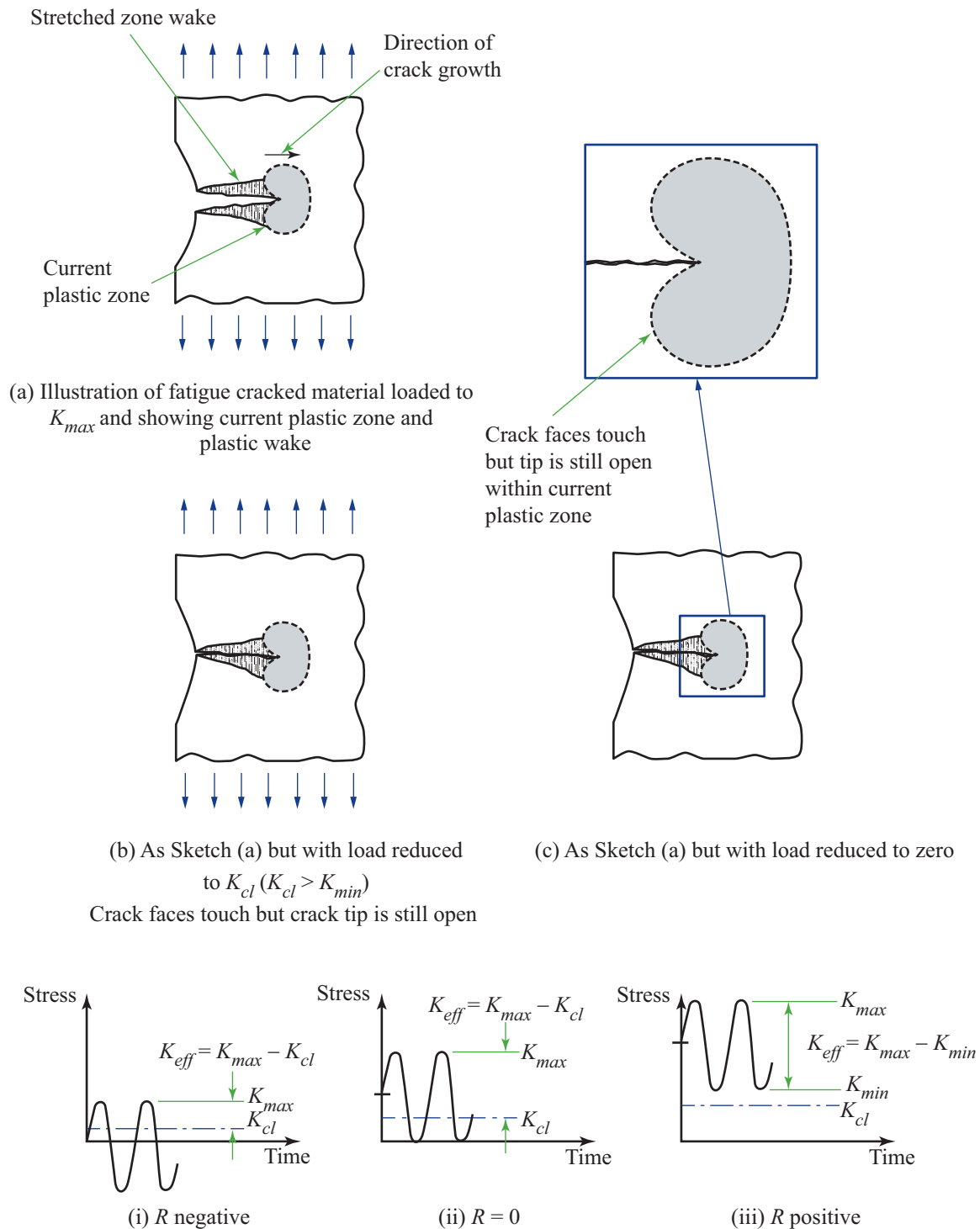
$$r_p = \frac{1}{24\pi} \left(\frac{\Delta K}{\sigma_y} \right)^2. \quad (2.7)$$

The appropriate value of r_p is added to the crack half-length, a , to obtain the effective crack half length, a_1 , for cyclic loading situations so that Equation (2.2) can now be expressed as

$$\Delta K_p = (S_{max} - S_{min})(\pi a_1)^{1/2} \alpha_1. \quad (2.8)$$

Note that α_1 is normally only corrected once for plasticity, that is without iteration.

In situations where an overload produces larger plastic zones than those due to cyclic loading, subsequent crack growth is slowed temporarily and this phenomenon, caused primarily by compressive residual stresses, is known as retardation. Fatigue cracks also experience crack tip shielding when the effective range of crack tip stress intensity, ΔK_{eff} , is not the same as the nominal range, $\Delta K = K_{max} - K_{min}$. This is because of the fracture surface contact that forms in the wake of the advancing fatigue crack due to, for example, plasticity-induced crack closure, see Sketch 2.3.



Sketch 2.3 Crack tip shielding and schematic of premature crack closure (based on ESDU 80036)

For a wide variety of materials, the fatigue crack propagation behaviour is represented graphically by curves similar to those shown in Sketch 2.1 which can be described, in general terms, by (Reference 36):

$$\frac{da}{dN} = f(R, \Delta K). \quad (2.9)$$

For crack growth rates in Region 2, and over small ranges of ΔK , Equation (2.9) can be given as (see References 36 and 37):

$$\frac{da}{dN} = C(\Delta K)^n, \quad (2.10)$$

where C and n are the intercept and slope of the crack propagation rate curve, respectively, and they depend on the material properties and the mean stress, and they remain approximately constant over small ranges of da/dN and ΔK within Region 2. The effect on the crack growth rate of different values of R is accounted for by changes to C and n within Region 2. Note that Equation (2.10) does not adequately describe the curves in Regions 1 and 3 therefore the use of this equation to predict crack growth life in these regions is severely limited.

In Region 3, at high values of ΔK , the slope of the crack propagation rate curve increases so that $n \rightarrow \infty$. For some materials the magnitude of ΔK at high crack growth rates approaches a limiting value of ΔK such that:

$$\frac{\Delta K}{(1 - R)} = K_{max} \rightarrow K_{Ic} \text{ or } K_c. \quad (2.11)$$

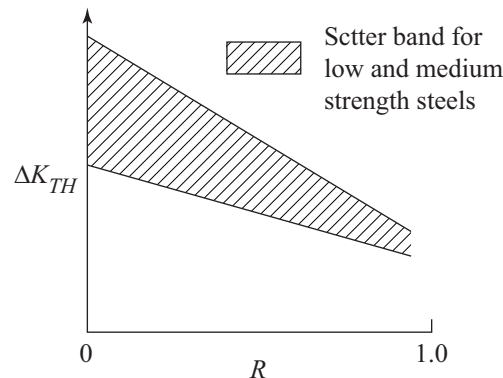
There have been many attempts made to derive laws to describe the complete crack propagation curves (see Reference 41). One example used in aerospace applications is Equation (2.12) from Reference 39 which takes into account the region of the curves as they near the value of K_c at high crack growth rates. However this equation does not describe adequately the low crack growth rate regime.

$$\frac{da}{dN} = \frac{C(\Delta K)^n}{(1 - R)(K_c - \Delta K)}. \quad (2.12)$$

Integration of Equations (2.10) and (2.12) by analytical or graphical methods can be used to estimate the crack growth life of a structure or component. The term "crack growth life" in this case is simply the number of load cycles required to make a crack grow from an initial size (or length), a_0 , until it reaches an unacceptable length, a_c , or an unsuitable crack growth rate is reached, or if failure of the component occurs (use whichever of these is the selected failure criterion). Note that in some cases it may be necessary to check whether or not the fatigue cracked component is able to withstand a separate static design loading.

Flaws or crack-like defects in components propagate under fatigue loading if the stress intensity factor is above the threshold stress intensity factor, ΔK_{TH} , for that material. If ΔK is held below ΔK_{TH} then an effectively infinite fatigue life is attained despite the presence of cracks and flaws. In practice this can lead to very low design stresses even for quite small macro-cracks but the value of ΔK_{TH} is of interest for designing lightly stressed components that may be subject to large numbers of service loadings. In cases where components are crack free, or contain flaws or cracks of a size defined by Region 1 and 2, and are subjected to normal design stresses then the crack initiation stage can last for many loading cycles and the overall fatigue life includes both the initiation and the propagation stage.

Sketch 2.4 shows how ΔK_{TH} varies with R for typical steels. The value of ΔK_{TH} is sensitive to variations in the stress ratio, R , and also to the stress history, corrosive environments and the mechanical and microstructural properties of the material. These effects may be due to crack closure. As an example, the stress ratio becomes important at slow crack propagation rates for low and medium strength steels that usually show little sensitivity to any changes in R within the normal crack propagation rate regime (Region 2).

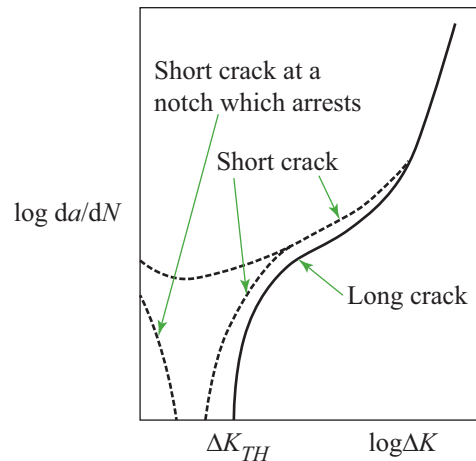


Sketch 2.4 Practical variation of ΔK_{TH} with the stress ratio, R

When using linear elastic fracture mechanics to analyse the propagation of cracks of different lengths there are significant differences in behaviour between long cracks and short cracks. A short crack is one that is either microstructurally small (its length is close to that of the dominant microstructural dimension or the grain size) and/or physically small (and it is not substantially larger than the plastic zone at the crack tip). Crack tip shielding, and in particular the effect of premature crack closure, is a foremost consideration to explain the differences in crack propagation behaviour between long and short cracks.

For long cracks, any crack closure before S_{min} is reached may shield the crack tip from the full fatigue cycle meaning that the effective stress intensity factor range, ΔK_{eff} , is smaller than the nominal stress intensity factor range, ΔK .

On the other hand whilst cracks remain within the initial plastic zone boundary, the wake of plastically deformed material may not fully develop, see Sketch 2.3, hence short cracks are unlikely to experience premature crack closure. As a short crack grows, any plastic wake effects begin to develop more fully and the value of K_{cl} becomes larger resulting in a decrease in ΔK_{eff} . This reduction may negate, or even outweigh, the natural increase in ΔK_{eff} due to increasing the crack length. This effect is illustrated in Sketch 2.5. In the extreme case of a microstructurally small crack developing at a notch, there may be a region of non-propagation. ESDU 80036 gives further details on the short crack effect.



Sketch 2.5 Comparison of the crack propagation rates of typical long and short cracks and for short cracks growing from a notch which arrests.

All curves have a common value of stress ratio, R

In Region 2, the analytical integration of Equations (2.10) and (2.12) over several discrete values of da/dN and ΔK followed by the summation of the values of N can be used to estimate the fatigue crack growth life. This integration procedure may only be carried out analytically if the value of n is an integer greater than 2 and the geometrical factor, α , is in the form of a polynomial, $f(a)$. For example, the integration of Equation (2.10) for the case of an infinite width plate ($\alpha = 1.0$) where $n \neq 2$ gives:

$$\left[N \right]_{N_i}^{N_f} = \frac{1}{C \left[(S_{max} - S_{min}) \pi^{1/2} \right]^n} \left[\frac{a^{[1 - (n/2)]}}{1 - (n/2)} \right]_{a_i}^{a_f} \quad (2.13)$$

Other solutions to the integral for finite width components may be found in References 44 and 45. If Equations (2.10) and (2.12) cannot be integrated analytically, graphical solutions using step by step summations of N over discrete ranges of da/dN and ΔK can be useful. In yet more complex cases numerical integration procedures, such as a Runge Kutta method, are suitable (see Reference 45, for example).

If the fatigue loading is not of constant amplitude then the analysis of the loading spectrum can be carried out using the methods described in ESDU 95006.

2.5 Determination of Fatigue Threshold

Certain derivations quote threshold values of stress intensity factor range, ΔK_{TH} (that is the value of ΔK below which fatigue cracks will not grow). A common definition of ΔK_{TH} is the maximum value of ΔK at which no detectable crack growth occurs in 10^7 cycles. The values of ΔK_{TH} are considered to be valid if the crack growth rate curves are vertical, or very nearly vertical, and since a crack extension of less than 10^{-4} m (4×10^{-3} in) would remain undetected by conventional crack length monitoring equipment, testing at 10^7 cycles implies a growth rate of 10^{-11} m/cycle (4×10^{-10} in/cycle). This corresponds to an average growth rate of about one quarter of a lattice spacing per cycle.

A frequently used method to determine very low rates of crack propagation near the threshold is the load reduction (or load shedding) technique. This technique involves reducing the load, usually in increments of less than 10 per cent, between sets of measured crack extension and load cycles, until no further crack extension can be detected in 10^7 cycles. In the tests at each cyclic load level and before crack measurements are taken, the crack is grown until it is several times longer (usually 3 to 5 times) than the maximum plastic zone size produced at the previous load level. This procedure is designed to reduce any effect of stress history but it cannot be continued for cracks which have ceased to propagate. At this point in the test, however, the plastic zone size is considered negligible.

3. DISCUSSION OF DATA

3.1 Method of Analysis of Test Data

Data from the derivations were in the form of curves of da/dN against ΔK , as defined by Equation (2.2). The values of α in Equation (2.2) from the derivations are values appropriate to the loading conditions and geometry of the specimens tested. Allowances for the effects of crack tip plasticity on the value of ΔK were not made except for from the data obtained from Derivations 16 and 20.

3.2 Effect of Fracture Toughness

The steels with low values of fracture toughness ($< 50 \text{ MN/m}^{3/2}$ ($46 \times 10^3 \text{ lbf/in}^{3/2}$)) tended to have higher crack propagation rates for given values of ΔK and also had steeper crack growth rate curves compared to those with medium to large values of fracture toughness. The steepness of the crack propagation rate curve is indicated by the value of n as used in Equation (2.10).

3.3 Effect of Mean Stress

The effect of mean stress on the fatigue crack growth is presented in terms of the stress ratio R . Note that in some cases the value of R used was not held constant during the test or its value was not stated in the derivation. The effect of R on the fatigue crack propagation rate is more pronounced at low values of stress intensity near to ΔK_{TH} and at high values of stress intensity near to K_{Ic} . The summary curves given in Figures 23 to 25 show that for higher values of stress ratio there is an increase in crack propagation rate, da/dN , for given values of ΔK .

3.4 Variation with Material Proof Stress

Figure 26 shows that a larger value of 0.2 per cent proof stress, f_p , increases the crack propagation rate for a given value of ΔK . However, this effect is less pronounced for smaller values of stress ratio, R , or lower values of crack propagation rate da/dN . This is the opposite effect to that presented in ESDU 81011* for lower strength low alloy steels where $f_t < 850 \text{ MN/m}^2$ ($123 \times 10^3 \text{ lbf/in}^2$).

* ESDU 81011, "Fatigue crack propagation in low and medium strength low alloy steel plate, bar and forgings".

3.5 Effect of Environment and Frequency

It is generally thought that crack propagation tests carried out at low frequency loading (< 10 Hz) in a corrosive environment will produce different crack growth rates compared to tests conducted in air at room temperature. However, the effect of different frequencies of the loading cycle on crack propagation rate is not considered here.

4. CALCULATION OF CRACK GROWTH LIFE

4.1 Use of Curves

As described in ESDU 80036 the crack growth rate data can be used to estimate the total crack growth for a given number of cycles, or vice versa, using integration. In the absence of any suitable equation to cover the full range of da/dN on the figures in this Item, the complete curves available from the data are presented. Any values read from these curves may be used in computer programs which predict crack growth life; note that the accuracy of such predictions depend on the number of values provided. Alternatively when the slope of a straight portion of the curve, or a mean straight line representing a portion of a curve, is represented by the characteristic value n , then Equation (2.10) is applicable (see Reference 36). This simple law can be analytically integrated to obtain the total crack growth life or it can be inverted to calculate the number of cycles needed for a crack to grow from one length to another. The value of ΔK described by Equation (2.2) includes the geometrical correction factor for stress intensity, α , which may be treated as a constant (usually around the value of 1) over a short interval of crack growth.

The value of n is calculated from any two co-ordinates, such as $((da/dN)_1, (\Delta K)_1)$ and $((da/dN)_2, (\Delta K)_2)$, using the following equation:

$$n = \frac{\log \left[\frac{(da/dN)_2}{(da/dN)_1} \right]}{\log \left[\frac{(\Delta K)_2}{(\Delta K)_1} \right]}. \quad (4.1)$$

The value of C is calculated by inserting n , together with one set of co-ordinate values, into Equation (2.10). For convenience, suggested values of n and C , obtained from the mean crack propagation rate curves for various ranges of da/dN , are given in the tables accompanying Figures 1 to 22.

To determine the fatigue crack growth rate using the straight line law in Equation (2.10) in preference to carrying out a step-by-step integration procedure, the values of da/dN and ΔK used to calculate n and C should correspond to points along the straight line approximation rather than directly from fatigue crack propagation rate curves.

5. DERIVATION AND REFERENCES

5.1 Derivation

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5.2 References

The references given are recommended sources of information supplementary to that in this Item.

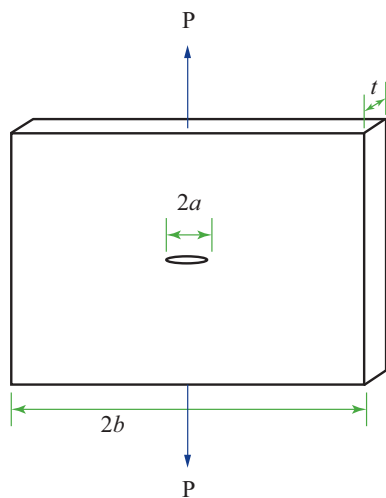
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6. EXAMPLES

6.1 Example 1

A structural steel plate 300 mm wide by 6 mm thick is manufactured from BS 970 817 M40 (EN 24) steel and has been found to have a central crack through its thickness, see Sketch 6.1. When first observed, the crack half-length is 5 mm. It is required to know the number of load cycles that can be applied before the maximum crack half-length reaches 25 mm (1 in). The tensile fatigue loading on the plate ranges from 0 to 120 MN/m² (0 to 174 × 10³ lbf/in²), the 0.2 per cent proof stress of the material is 1200 MN/m² (174 × 10³ lbf/in²) and the plane strain fracture toughness is 95 MN/m^{3/2} (86 × 10³ lbf/in^{3/2}).



Dimensions	Plate width	$2b$	300 mm	(11.8 in)
	Plate thickness	t	6 mm	(0.24 in)
	Crack length	$2a$	10 mm	(0.4 in)
Steel properties	0.2 per cent proof stress	f_p	1200 MN/m ²	(174 × 10 ³ lbf/in ²)
	Plane strain fracture toughness	K_{Ic}	95 MN/m ^{3/2}	(86 × 10 ³ lbf/in ^{3/2})

Sketch 6.1

The number of load cycles to determine the maximum acceptable crack half-length is calculated by integrating Equation (2.10) using values of n and C . Since no actual data are available for the values of mean stress ratio ($R = 0$) and the 0.2 per cent proof stress in this example, a conservative approach is to use the closest available data from Figure 5, namely $R = 0.17$ and $f_p = 1275$ MN/m² (185 × 10³ lbf/in²). Considering geometric effects means that α is:

$$\alpha = (1 - (a/b)^{1.8})^{-0.54} \quad (6.1)$$

Equation (6.1) was obtained from Reference 43 and is a "closed form" solution which may be readily integrated and lies within 2 per cent of the more generally quoted polynomial solutions for a central crack in a finite plate under tensile loading. No allowance for plasticity is made in this example.

Using Equation (2.2), the initial and final values of ΔK are:

$$\begin{aligned} \Delta K_i &= (120(\pi \times 0.005))^{1/2} (1 - (0.005/0.15)^{1.8})^{-0.54}, \\ &= 15.1 \text{ MN/m}^{3/2} \text{ (13.7} \times 10^3 \text{ lbf/in}^{3/2}\text{)}. \\ \Delta K_f &= (120(\pi \times 0.025))^{1/2} (1 - (0.025/0.15)^{1.8})^{-0.54}, \\ &= 34.4 \text{ MN/m}^{3/2} \text{ (31.3} \times 10^3 \text{ lbf/in}^{3/2}\text{)} \end{aligned}$$

From the table accompanying Figure 5, the values of C and n in the range 10^{-8} to 10^{-7} m/cycle (4.0×10^{-7} to 4.0×10^{-6} in/cycle) are $4.4 \times 10^{-12} \text{ m} \left(\frac{\text{m}^{3/2}}{\text{N}} \right)^n \left(1.9 \times 10^{-10} \text{ in} \left(\frac{\text{in}^{3/2}}{10^3 \text{ lbf}} \right)^n \right)$ and 3.0 respectively.

Equation (2.2) is substituted into Equation (2.10) to give

$$\frac{da}{dN} = C \left((S_{max} - S_{min}) (\pi a)^{1/2} \alpha \right)^n. \quad (6.2)$$

This equation can be re-written as

$$\frac{da}{dN} = C Q (a^{1/2} \alpha)^n, \quad (6.3)$$

where $Q = \left((S_{max} - S_{min}) \pi^{1/2} \right)^n. \quad (6.4)$

So that in this case $Q = (120 \pi^{1/2})^{3.0} = 9.622 \times 10^6 (\text{MN/m}^2)^{3.0}$.

For the general case, Equation (6.3) is expressed as an integral with limits a_i and a_f and N_i and N_f

$$\frac{1}{CQ} \int_{a_i}^{a_f} \left(\frac{a^{-1/2}}{\alpha} \right) da = \int_{N_i}^{N_f} dN. \quad (6.5)$$

Substituting α from Equation (6.1) in to Equation (6.5) gives

$$\frac{1}{CQ} \int_{a_i}^{a_f} \left(a^{-1/2} (1 - (a/b)^{1.8})^{0.54} \right)^n da = \frac{1}{CQ} \int_{a_i}^{a_f} \left(\frac{(1 - (a/b)^{1.8})^{0.54n}}{a^{n/2}} \right) da = \int_{N_i}^{N_f} dN. \quad (6.6)$$

Noting that the right-hand side of Equation (6.6) becomes ΔN (because the origin of the line given by Equation (2.10) is unknown and therefore individual values of N have no meaning) and then expanding the left-hand side of Equation (6.6), it is possible to use Maclaurin's series before integrating term by term for three terms to give the solution:

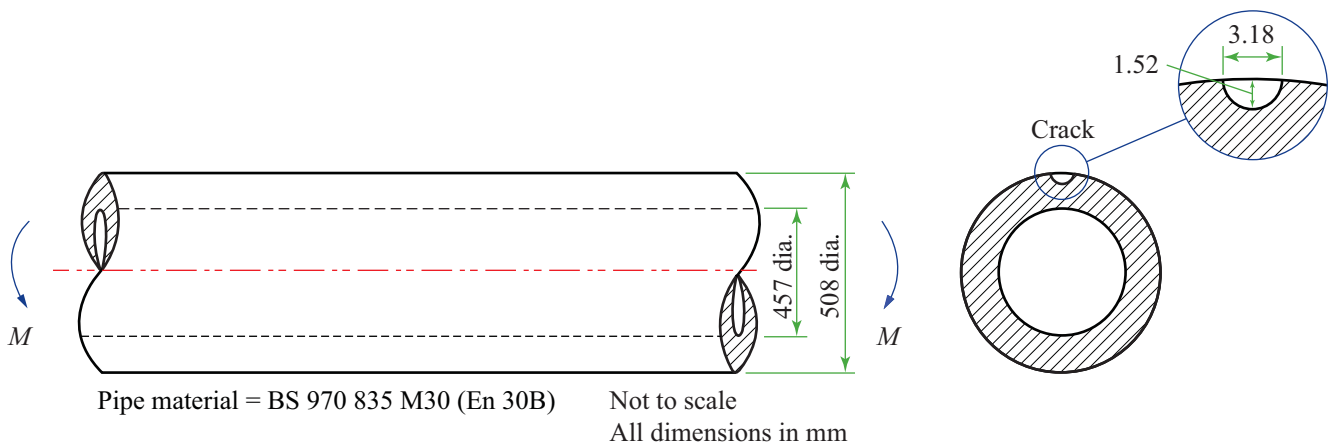
$$\begin{aligned} \frac{b^{(1-n/2)}}{1.8CQ} \left[\left(\frac{a}{b} \right)^{(1-n/2)} \left[\frac{1}{\left(\frac{2-n}{3.6} \right)} - 0.54n \left(\frac{a}{b} \right)^{1.8} \left[\frac{1}{1 + \left(\frac{2-n}{3.6} \right)} - \left(\frac{(0.54n-1) \left(\frac{a}{b} \right)^{1.8}}{2} \right) \right] \right. \right. \\ \left. \left. \left[\frac{1}{2 + \left(\frac{2-n}{3.6} \right)} - \left(\frac{(0.54n-1) \left(\frac{a}{b} \right)^{1.8}}{2} \right) \right] \right] \right] \right]_{a_i}^{a_f} = \Delta N. \quad (6.7) \end{aligned}$$

Substituting the values $a_i = 0.005$ m, $a_f = 0.025$ m, $a_f = 0.025$ m, $b = 0.15$ m and the previously determined values of C , n and Q in to Equation (6.7), the number of load cycles to reach the maximum acceptable crack half-length is

$$\Delta N = 369\,000 \text{ cycles.}$$

6.2 Example 2

During an inspection of a pipe run, a surface crack was discovered, see Sketch 6.2. Further ultrasonic investigation revealed the crack to be approximately semi-circular in shape, having a width, $2a$, of 3.18 mm (0.125 in) and a depth, c , of 1.52 mm (0.06 in). It is required to know if the crack is likely to propagate under the specific condition of constant amplitude loading which is applying an alternating axial stress of ± 41.4 MN/m² (± 6000 lbf/in²) at the crack position.



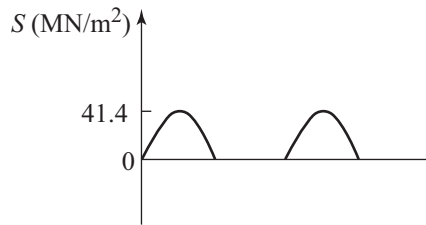
Sketch 6.2 Details of crack and pipe

The solution to this problem requires establishing if the combination of the existing crack and the loading cycle is sufficient to create a value of ΔK greater than the threshold ΔK of the pipe material. In practice, the required complete crack growth rate curve may not always be available therefore assumptions are often made such as those shown in this example.

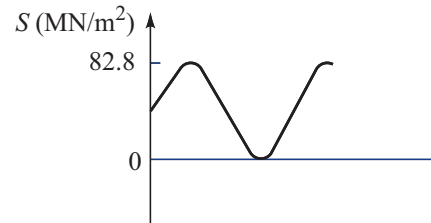
The curve of da/dN against ΔK for EN 30B steel is given in Figure 7 but as no threshold data is available from this figure, the appropriate summary curve for steels having similar 0.2 per cent proof stresses can be consulted. In the table accompanying Figure 7 for EN 30B steel, f_p is 1351 MN/m² (196×10^3 lbf/in²) therefore the summary curve in Figure 25 can be used.

For this example, $R = \frac{S_{min}}{S_{max}} = \frac{-41.4}{41.4} = -1$.

Figure 25 does not provide data for $R = -1$ however data for $R = 0$ are available from which the value of ΔK_{TH} is 4.01 MN/m^{3/2} (126.8 N/mm^{3/2} or 3650 lbf/in^{3/2}). To compare the present loading cycle with the data for $R = 0$, one assumption is to consider only the tensile part of the loading cycle and to neglect the compressive part (because during compression the crack may be closed and no crack growth occurs). This loading cycle is illustrated in Sketch 6.3 but note that this assumption will lead to errors since crack closure effects are not taken into account (see Section 2.4). A different assumption is to consider the whole loading cycle when calculating ΔK , see Sketch 6.4, but this may lead to an over-conservative result. In this example, the value of ΔK was estimated using Sketch 6.4.

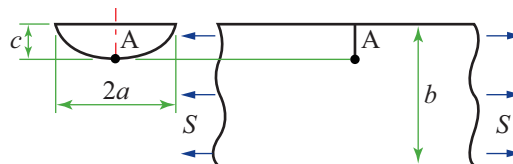


Sketch 6.3



Sketch 6.4

The value of α can now be estimated. Figure 186 of Reference 42 gives values of K_I/K_0 , where $K_0 = S(\pi c)^{1/2}$, for a semi-elliptical edge crack in a slab subjected to a uniform uniaxial tensile stress (Sketch 6.5).



Sketch 6.5

To make use of these curves in the solution of the present problem it is necessary to assume that the curvature effect of the pipe on is small. This is considered to be valid, in this case, since the crack depth is small in relation to the pipe outside diameter. Also the stress intensity factor will be greatest at the ends of the minor axis of the crack (see Section 2.4). The crack depth, c , in this example is just less than the crack half length, a , therefore position A is the critical location where the crack will first grow. The crack tip on the breaking surface will experience an edge effect which is thought to increase the stress intensity factor slightly but note that this has not been taken into account in this example.

$$\Delta K = (S_{max} - S_{min})(\pi c)^{1/2} \alpha,$$

and the critical value of c representing threshold can be established thus,

$$\Delta K_{TH} = 126.8 = [41.4 - (-41.4)](\pi c)^{1/2} \alpha.$$

$$\text{Hence } (c)^{1/2} \alpha = 0.8674 \text{ mm}^{1/2}.$$

In order to determine the likelihood of crack propagation relative to a threshold value, various values of c are considered in Table 6.1.