



ESDU 17011

Issued January 2018
Supersedes ESDU 66016

Power spectral density and bandwidth correction of spectrum level



**ROYAL
AERONAUTICAL
SOCIETY**

Endorsed by
The Royal Aeronautical Society

ESDU DATA ITEMS

Data Items provide validated information in engineering design and analysis for use by, or under the supervision of, professionally qualified engineers. The data are founded on an evaluation of all the relevant information, both published and unpublished, and are invariably supported by original work of ESDU staff engineers or consultants. The whole process is subject to independent review for which crucial support is provided by industrial companies, government research laboratories, universities and others from around the world through the participation of some of their leading experts on ESDU Technical Committees. This process ensures that the results of much valuable work (theoretical, experimental and operational), which may not be widely available or in a readily usable form, can be communicated concisely and accurately to the engineering community.

We are constantly striving to develop new work and review data already issued. Any comments arising out of your use of our data, or any suggestions for new topics or information that might lead to improvements, will help us to provide a better service.

THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item which supersedes ESDU 66016, was monitored and guided by the Vibration and Acoustic Fatigue Committee. This Committee first met in 1966 and now has the following membership:

Chairman	
Prof. R.D. Adams	— University of Bristol
Members	
Dr N.S. Ferguson	— University of Southampton
Mr E. Green	— Rolls-Royce plc, Derby
Dr P. Green*	— BAE Systems, Preston, Lancashire
Dr A. Grillenbeck*	— IABG Space Test Center, Germany
Mr A.R. Harris*	— Northrop Grumman, USA
Dr D.M.A. Millar*	— Bombardier Aerospace Short Brothers plc, Belfast
Mr C.D. Munger*	— The Boeing Company, USA
Dr S. Patsias	— Rolls-Royce plc, Derby
Dr J. Santiago Prowald*	— European Space Agency (ESTEC), The Netherlands
Mr E. Smyth	— Bombardier Aerospace Short Brothers plc, Belfast
Prof. R.G. White	— University of Southampton (Member Emeritus)
Dr. Ir. J.J. Wijker	— University of Twente, Enschede, The Netherlands.

* Corresponding Member

The construction and subsequent development of the Data Item were undertaken by

Dr J. Anderson	— Principal Engineer
Dr M. Bratos-Anderson	— Consultant

The person with overall responsibility for the work in this subject area is Dr C.B. Chinoy, Head of the Aircraft Noise and Structural Dynamics Group.

POWER SPECTRAL DENSITY AND BANDWIDTH CORRECTION OF SPECTRUM LEVEL

CONTENTS

	Page
1. NOTATION	1
2. INTRODUCTION	2
3. POWER SPECTRAL DENSITY FUNCTION	2
3.1 Stress in Panel Subjected to Random Acoustic Loading	2
3.2 Definition of Power Spectral Density Function	3
3.3 Band Pressure Level Measurements	4
3.4 Bandwidth Correction	4
3.5 Example – Estimation of PSDF	5
3.5.1 Spectrum level estimation in one-third octave bands	6
3.5.2 Spectrum level estimation in octave bands	7
3.5.3 PSDF in one-third octave and octave bands	8
3.6 Limitations of Method	10
4. REFERENCES	10

POWER SPECTRAL DENSITY AND BANDWIDTH CORRECTION OF SPECTRUM LEVEL

1. NOTATION

f	frequency	Hz
f_c	centre frequency	Hz
f_0	fundamental frequency of panel (Equation (3.1))	Hz
G_p	power spectral density function	Pa ² /Hz
K_δ	damping ratio correction factor (defined by Equation (3.5))	
$L_{p,\Delta f}$	band sound pressure level	dB
$L_{p,1}$	spectrum level	dB
$\overline{p_{f_c,1}^2}$	mean-squared sound pressure within bandwidth of 1 Hz and of centre frequency f_c	Pa ²
$\overline{p_{f_c,\Delta f}^2}$	mean-squared sound pressure within bandwidth of Δf and of centre frequency f_c	Pa ²
p_{rms}	root mean-squared sound pressure	Pa
$\overline{p^2}$	mean-squared sound pressure	Pa ²
S_{rms}	r.m.s. stress of stiffened panel	Pa
S_0	stress ratio	
Δf	frequency bandwidth	Hz
Δf_{oct}	octave frequency bandwidth	Hz
$\Delta f_{1/3oct}$	one-third octave frequency bandwidth	Hz
ΔL	bandwidth correction	dB
δ	equivalent viscous damping ratio	

Abbreviations

DFT	discrete Fourier transform
FFT	fast Fourier transform
ISO	International Standards Organization
PSDF	power spectral density function

2. INTRODUCTION

In the spectral analysis of a signal, which is a function of time, the power spectral density function (PSDF) is a fundamental concept (Bendat and Piersol, 2010). In this Item the signal is considered in terms of sound pressure, but the same principles apply to other quantities, such as velocity or the displacement of a vibrating surface. The PSDF is not usually measured directly. Instead, the sound pressure signal is digitised to form a time series that is transformed into the frequency domain by means of the fast Fourier transform (FFT) algorithm (Cooley and Tukey, 1965; Marple, 1987; Shin and Hammond, 2008), which is a computationally efficient version of the discrete Fourier transform (DFT). The result of the FFT is a series of Fourier components in the frequency domain. With suitable selection of windowing and scaling, the distribution of the squared moduli of the Fourier components represents a power spectrum in the form of a series of lines in the frequency domain. Several spectral lines can lie within a particular band, such as, for example, a one-third octave band, hence summation of the values of the squared moduli for the lines within the band is necessary to provide the sound pressure level in the band.

In many engineering problems there is a necessity to analyse data, for example, in the form of band pressure levels or spectrum levels in the frequency domain. In such cases with the assumption of a continuous sound spectrum without prominent pure tones, the PSDF can be estimated from the band pressure levels, see Equations (3.4), (3.7) and (3.9). When the analysed signal is a combination of two signals, for example a sinusoidal signal and a random signal, the power spectrum can be treated as the sum of the power spectrum of the sine wave and the power spectrum of the random sound (Bendat and Piersol, 2010).

The second part of the Item (Sections 3.4 and 3.5) refers to the bandwidth correction, which is associated with the conversion of the band sound pressure level to the spectrum level. The conversion only has meaning when the power spectrum is continuous and fairly uniform within the measurement band (Beranek, 1971). The restriction implies that the considered sound signal should not contain prominent pure tone components. Usually, sound pressure levels are converted to spectrum levels in order to compare the results obtained with different filters.

3. POWER SPECTRAL DENSITY FUNCTION

The power spectral density function is involved in the estimation of the stress in panels which are subjected to intense random acoustic loading (ESDU, 1972). Random jet noise usually fulfils the restrictions imposed on the sound spectra, described in Section 2.

3.1 Stress in Panel Subjected to Random Acoustic Loading

The r.m.s. stress S_{rms} for a stiffened panel subjected to random acoustic loading on one side is given approximately by the expression

$$S_{rms} = \left[\frac{\pi}{4\delta} f_0 G_p(f_0) \right]^{1/2} K_\delta S_0, \quad (3.1)$$

where $G_p(f_0)$ is the power spectral density function of the random acoustic loading at a frequency f_0 , which is usually the fundamental frequency of the panel with all edges fixed (ESDU, 1972). As explained in the ESDU Item, δ is the equivalent viscous damping ratio (ESDU, 2009), K_δ is a damping ratio correction factor for different viscous damping ratios and S_0 is a stress ratio. In many cases of acoustic fatigue the acoustic loading is the noise from a jet engine, which is either estimated or measured in, for example, one-third octave bands. In order to estimate the power spectral density the bandwidth of the one-third octave filters, or the bandwidth of any other types of filter used, must be known.