

The estimation of turbulent boundary layer power-law velocity and q profiles from shape factor and momentum thickness

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Dr S. Prince	—	Cranfield University, Cranfield
Dr S. Shaw*	—	Airbus Deutschland GMBH, Germany
Mr I. Whitehouse	—	Independent
Mr P.W.C. Wong	—	Aircraft Research Association Limited, Bedford.

* Corresponding Member

The technical work in the assessment of the available information and the construction and subsequent development of the Data Item and overall responsibility for the work in this subject area was

Mr K.C. Hackett	—	Senior Engineer
Dr D.R. Philpott	—	Head of Transonic Aerodynamics Group.

THE ESTIMATION OF TURBULENT BOUNDARY LAYER POWER-LAW VELOCITY AND q PROFILES FROM SHAPE FACTOR AND MOMENTUM THICKNESS

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THE ESTIMATION OF TURBULENT BOUNDARY LAYER POWER-LAW VELOCITY AND q PROFILES FROM SHAPE FACTOR AND MOMENTUM THICKNESS

1. NOTATION

C_F	mean skin friction coefficient on a flat plate
c	aerofoil chord or local chord of wing
c_0	wing centre-line chord
H	boundary-layer shape parameter $\left(H = \frac{\delta^*}{\theta}\right)$
$\bar{H}$	transformed boundary-layer shape parameter $\left(\bar{H} = \frac{1}{\theta} \int_0^\infty \frac{\rho}{\rho_e} \left(1 - \frac{u}{U_e}\right) dy\right)$ (Note: for incompressible boundary layers $H = \bar{H}$)
l	reference length
M	local Mach number within boundary layer
M_e	local Mach number at edge of boundary layer
M_∞	freestream Mach number
n	index for boundary layer power-law velocity profile
p	local static pressure within boundary layer
p_e	static pressure at edge of boundary layer
q	$q = \frac{1}{2} \rho u^2$
q_e	$q_e = \frac{1}{2} \rho_e U_e^2$
Δq_{tot}	(Equation (4.1))
Re	Reynolds number based on l and freestream conditions
Re_1	unit Reynolds number within boundary layer $\left(Re_1 = \frac{u}{\nu}\right)$
Re_{1e}	unit Reynolds number at edge of boundary layer $\left(Re_x = \frac{U_e}{\nu}\right)$
Re_x	Reynolds number based on U_e and x $\left(Re_x = \frac{x U_e}{\nu}\right)$
Re_θ	Reynolds number based on U_e and θ $\left(Re_\theta = \frac{\theta U_e}{\nu}\right)$
r	temperature recovery factor $r = \frac{T_{aw} - T_e}{T_{se} - T_e}$ taken as 0.89)
S	reference temperature (Sutherland's law- Appendix A)
T	local temperature within boundary layer
T_{aw}	adiabatic wall temperature

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T_e	local temperature at edge of boundary layer
T_r	wall recovery temperature
T_{ref}	reference temperatures used in Sutherland's law and power-law approximation (Appendix A)
T_s	local stagnation temperature within boundary layer
T_{se}	local stagnation temperature at edge of boundary layer
T_w	wall temperature
U_e	local velocity at edge of boundary layer
u	local velocity within boundary layer
x	distance from virtual origin of turbulent boundary layer on a flat plate or distance from leading edge of aerofoil or wing
y	distance normal to surface
γ	ratio of specific heats
α	wing incidence in VFP calculation
δ	boundary-layer thickness based on $u/U_e = 0.995$ or $u/U_e = 1.0$ for power-law profile
δ^*	boundary-layer displacement thickness $\left(\delta^* = \int_0^\delta \left(1 - \frac{\rho}{\rho_e} \frac{u}{U_e} \right) dy \right)$
δ_i^*	boundary-layer displacement thickness (incompressible flow) $\left(\delta_i^* = \int_0^\delta \left(1 - \frac{u}{U_e} \right) dy \right)$
θ	boundary-layer momentum thickness $\left(\theta = \int_0^\delta \frac{\rho}{\rho_e} \frac{u}{U_e} \left(1 - \frac{u}{U_e} \right) dy \right)$
θ_i	boundary-layer momentum thickness (incompressible flow) $\left(\theta_i = \int_0^\delta \frac{u}{U_e} \left(1 - \frac{u}{U_e} \right) dy \right)$
μ	coefficient of viscosity
μ_{ref}	reference coefficients of viscosity used in Sutherland's law and power-law approximation (Appendix A)
ρ	density within boundary layer
ρ_e	density at edge of boundary layer
ν	kinematic viscosity
Other subscripts	
exp	quantity evaluated for experimental boundary-layer velocity profile
pow	quantity evaluated for power-law boundary-layer velocity profile

2. INTRODUCTION

The drag of an excrescence lying entirely (or largely) within the boundary layer is currently evaluated using data derived from tests on excrescences immersed in flat-plate boundary layers with zero external pressure gradient. When an excrescence is situated in a boundary-layer in which the velocity and density profiles significantly depart from this ideal, its drag is modified due to the changed value of q to which it is exposed.

Flow solvers, such as the 2D aerofoil method, VGK¹, and the wing-body Viscous Full-Potential method, VFP², are used, in excrescence drag estimation, to evaluate local flow conditions on a wing. These codes output local boundary-layer parameters, including displacement thickness, momentum thickness, skin friction and boundary-layer shape factor. However, no information is generated regarding velocity and density profiles within the boundary layer and so departures of local q from a flat-plate boundary-layer profile, with zero pressure gradient, cannot be evaluated directly.

This Data Item describes a simple process that can be used to calculate a turbulent boundary-layer velocity profile, which can be adequately represented by a simple power-law relationship, using only momentum thickness and shape factor. This velocity profile is then used to derive the q distribution within the boundary layer. Examples are given to illustrate the use of the method to determine the mean q acting on an excrescence.

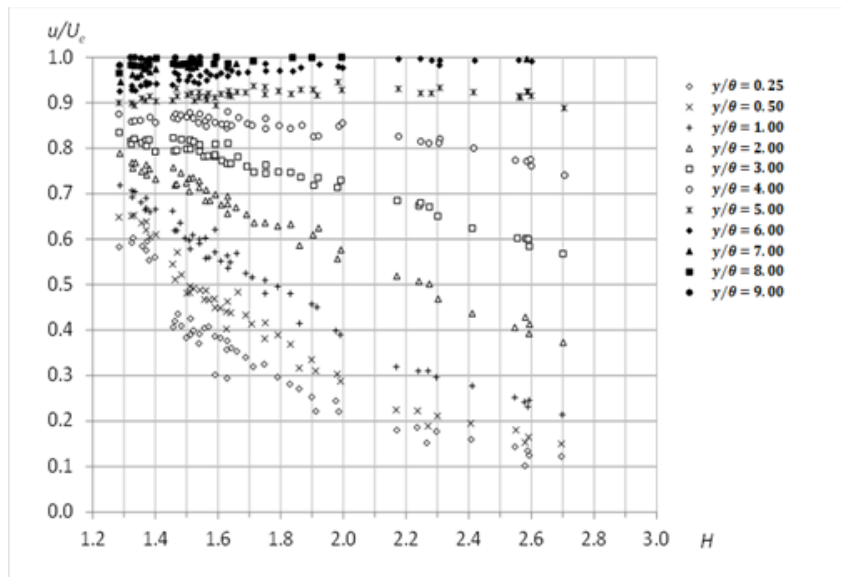
3. METHOD

3.1 Determination of Velocity Profile from $\bar{H}$ and θ

3.1.1 Incompressible flow

In Reference 3 an analysis is described of an extensive amount of data for turbulent boundary layers, at low Mach number, over wings and bodies of various shapes. These data were taken from the literature available at that time including tests in the NACA two-dimensional low-turbulence tunnel. One of the main conclusions of this analysis is that, for all the turbulent boundary-layer profiles examined (Sketch 3.1), u/U_e can be expressed, with reasonable accuracy, as:

$$\frac{u}{U_e} = f\left(H, \frac{y}{\theta_i}\right). \quad (3.1)$$



Sketch 3.1 Reduction of turbulent boundary-layer profiles from Reference 3

If a power-law velocity profile:

$$\frac{y}{\delta} = \left(\frac{u}{U_e} \right)^{\frac{1}{n}} \quad (3.2)$$

is assumed δ^* and θ can be calculated directly in terms of n from Equation (3.2) giving, for an incompressible boundary layer:

$$\frac{\delta^*}{\delta} = \frac{1}{n+1} \quad (3.3)$$

$$\frac{\theta_i}{\delta} = \frac{n}{(n+1)(n+2)} \quad (3.4)$$

$$H = \frac{2+n}{n} \quad (3.5)$$

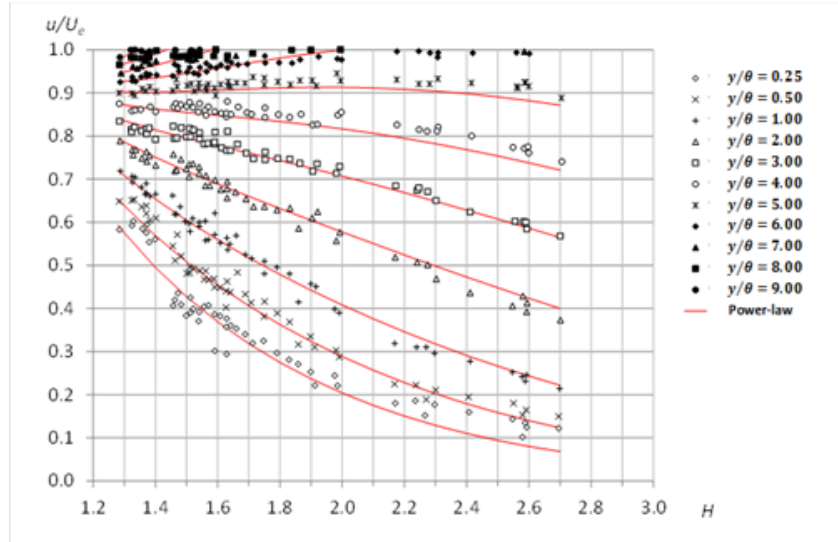
From Equation (3.5)

$$n = \frac{2}{H-1} \quad (3.6)$$

and from Equations (3.2) and (3.4):

$$\frac{u}{U_e} = \left(\frac{n}{(n+1)(n+2)\theta_i} y \right)^{\frac{1}{n}} \quad (3.7)$$

Sketch 3.2 shows a set of curves, corresponding to the data of Sketch 3.1, constructed by calculating n for a given H using Equation (3.6) and the value of u/U_e at each y/θ_i using Equation (3.7).



Sketch 3.2 Power-law velocity profiles compared with the data of Reference 1

Sketch 3.2 shows that the assumed power-law profiles correspond reasonably well with the experimental data of Reference 3 and that, within the range of these data, Equations (3.6) and (3.7) can be used to reconstruct the velocity profile if H and θ_i are known.

3.1.2 Compressible flow

For compressible flows, H can be transformed into an equivalent incompressible shape parameter, $\bar{H}$. In Reference 4 this relationship is given as:

$$\bar{H} = \frac{(H+1)}{\left(1 + r \frac{\gamma-1}{2} M_e^2\right)} - 1. \quad (3.8)$$

$\bar{H}$, is output directly by VGK and VFP but, where other data sources are used, it can be calculated from Equation (3.8), taking r as 0.85.

At low speed, H and $\bar{H}$ are equal and correlations obtained at low speed can be generalised to include compressible flow cases simply by using the transformed parameter, $\bar{H}$, instead of H and θ instead of θ_i .

Equation (3.1) thus becomes:

$$\frac{u}{U_e} = f\left(\bar{H}, \frac{y}{\theta}\right). \quad (3.9)$$

The function, $f\left(\bar{H}, \frac{y}{\theta}\right)$, is identical to that given in Equation (3.1) and approximated by the power-law curves in Sketch 3.2, except that H is replaced with $\bar{H}$ and θ_i with θ .

Thus, for the power-law approximation:

$$n = \frac{2}{\bar{H} - 1} \quad (3.10)$$

and (from Equation (3.7)) the velocity profile is given by:

$$\frac{u}{U_e} = \left(\frac{n}{(n+1)(n+2)\theta} y \right)^{\frac{1}{n}}. \quad (3.11)$$

The momentum thickness, θ , for the compressible boundary layer is also output directly by VGK and VFP.

3.2 Determination of q Distribution

The q distribution is approximately related to the velocity distribution (see Appendix A) by:

$$\frac{q}{q_e} = \frac{\frac{1}{2}\rho u^2}{\frac{1}{2}\rho_e U_e^2} = \frac{\frac{u^2}{U_e^2}}{1 + \frac{r(\gamma-1)}{2} M_e^2 \left[1 - \frac{u^2}{U_e^2} \right]}. \quad (3.12)$$

Once the velocity profile in the compressible boundary layer is determined using the method of Section 3.1, Equation (3.12) is used to calculate q/q_e at different values of y .

3.3 Boundary Layer for Flat Plate in Zero Pressure Gradient

If direct boundary-layer data are not available, the equivalent incompressible shape factor, $\bar{H}$, can be estimated using the empirical relationship between $\bar{H}$ and Re_x given graphically in Fig. 5 of Reference 5. This relationship is very closely represented by:

$$\bar{H} = -0.0032(\log_{10} Re_x)^3 + 0.0826(\log_{10} Re_x)^2 - 0.7488(\log_{10} Re_x) + 3.5957. \quad (3.13)$$

The momentum thickness of the compressible boundary layer can be determined from Fig 4 of Reference 5, which relates Re_θ to Re_x for an adiabatic wall over a Mach range from zero to 5. Alternatively the program associated with Ref. 5 may be used. This calculates C_F when Re_x and M_e (*i.e.* the freestream Mach number for a flat plate) are input.

4. APPLICABILITY AND ACCURACY

4.1 Evaluation Method

The power-law method (Section 3) has been previously evaluated⁶ against published experimental boundary-layer velocity profiles obtained:

- (a) on flat plates, in zero pressure gradient over a range of subsonic and supersonic Mach numbers (Reference 7),
- (b) at low speed, with pressure gradient sufficient to cause separation (Reference 8)

and

- (c) as streamwise profiles measured on a wing at a freestream Mach number of 0.843 (Reference 9).

In Reference 6 several alternative methods are also evaluated, including a similar method using polynomial fits to the lines of constant y/θ in Sketch 3.1 and a number of published empirical and semi-empirical methods (References 7, 10 and 11).

The power-law profile method allows velocity and q profiles to be easily constructed from given values of $\bar{H}$ and θ (or δ^*), gives a good fit to the experimental data of Reference 3 (Sketch 3.2) and generally to the velocity profiles used for comparison in Reference 6. It is, therefore, chosen for its simplicity and acceptable accuracy in determining the distribution of q within the boundary layer for the purpose of drag estimation (Section 1).

In this Data Item the power-law profiles are evaluated against the experimental data by comparing:

- velocity profiles $\left(\frac{y}{\theta} \text{ vs. } \frac{u}{U_e}\right)$,
- profiles showing the local deficit in $q \left(\frac{y}{\theta} \text{ vs. } 1 - \frac{q}{q_e}\right)$, where $\frac{q}{q_e}$ is obtained directly from the velocity profile using Equation (3.12),
- profiles showing the accumulated deficit in $q \left(\frac{y}{\theta} \text{ vs. } \int_0^{y/\theta} \left(1 - \frac{q}{q_e}\right) d\frac{y}{\theta}\right)$,

A complete set of these profiles is given in Appendix B.

Because the same value of θ applies to an experimental profile and its derived power-law equivalent, y/θ is taken as the ordinate for all profiles.

However, the values of δ for an experimental profile and its power-law equivalent will not, in general, be equal and so comparison is also made of Δq_{tot} , which is defined as:

$$\Delta q_{tot} = \left[\frac{\left[\int_0^{\delta/\theta} \left(1 - \frac{q}{q_e}\right) d\frac{y}{\theta} \right]_{\text{exp}} - \left[\int_0^{\delta/\theta} \left(1 - \frac{q}{q_e}\right) d\frac{y}{\theta} \right]_{\text{pow}}}{\left[\int_0^{\delta/\theta} \left(1 - \frac{q}{q_e}\right) d\frac{y}{\theta} \right]_{\text{exp}}} \right] \quad (4.1)$$

and reflects the magnitude of the difference in the total deficit in q for an experimental profile and its derived power-law equivalent, when integrated across the entire boundary layer.

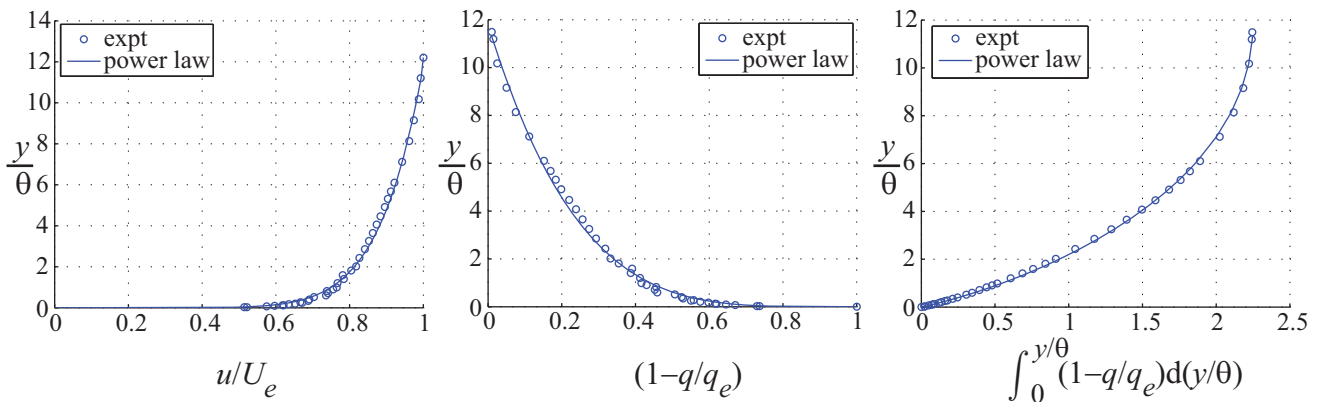
4.2 Evaluation of Results

The method was evaluated against several experimental profiles. These profiles were obtained from flat-plate tests both with zero pressure gradient over a Mach number range of approximately $0.2 \leq M_e \leq 1.6$ with $\bar{H} \approx 1.24$ (Reference 7), and at low Mach number ($M_e \approx 0.14$) with an adverse external pressure gradient giving transformed shape factors in the range $1.35 \leq \bar{H} \leq 2.85$ (Reference 8). Additional data sets were obtained from wind-tunnel measurements of streamwise boundary-layer profiles on a Harrier-like wing (Reference 9), tested at a freestream Mach number of 0.83. A complete set of these comparisons is given in Appendix B.

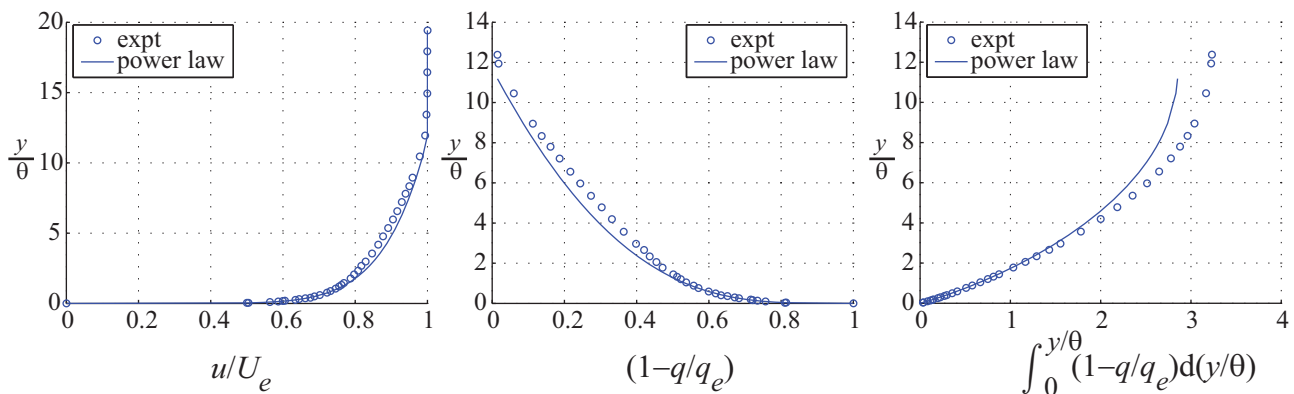
Generally agreement is very good in subsonic cases and, even at relatively high Mach numbers ($M_e = 1.597$ on a flat plate and $M_e = 1.4706$ on a wing surface), gives useful guidance to the anticipated mean value of q at different locations in the boundary layer.

4.2.1 Boundary layer on a flat plate with varying Mach number (zero pressure gradient)

With Mach number variation and zero pressure gradient on a flat plate, the mean value of $\Delta q_{tot} = 3.8\%$ (for all cases shown in Appendix B Section B1) and the maximum value (11.7%) occurs at the highest Mach number, $M_e = 1.597$. Sketches 4.1a and 4.1b (taken from Appendix B) illustrates best- and worst-fit cases.



Sketch 4.1a Effect of M_e for profiles measured on a flat plate with zero pressure gradient
 $M_e = 0.1999$ $H = 1.2492$ $\bar{H} = 1.2334$ (Appendix B: Case B1 ii)



Sketch 4.1b Effect of M_e for profiles measured on a flat plate with zero pressure gradient
 $M_e = 1.5970$ $H = 2.2528$ $\bar{H} = 1.2372$ (Appendix B: Case B1 iv)