

Force limited random vibration testing

The computation of the semi-empirical constant C^2 for test article and supporting structure

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FORCE LIMITED RANDOM VIBRATION TESTING: THE COMPUTATION OF THE SEMI-EMPIRICAL CONSTANT FOR TEST ARTICLE AND SUPPORTING STRUCTURE

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FORCE LIMITED RANDOM VIBRATION TESTING: THE COMPUTATION OF THE SEMI-EMPIRICAL CONSTANT C^2 FOR TEST ARTICLE AND SUPPORTING STRUCTURE

1. NOTATION

		Units
A	maximal amplitude acceleration	m/s^2
C^2	force limiting vibration constant	
E	Young's modulus, modulus of elasticity	Pa
f	frequency	Hz
f_i	natural frequency	Hz
f_o	natural frequency	Hz
F	force	N
G	shear modulus, modulus of rigidity	Pa
$H(f)$	frequency transfer function	
j	$\sqrt{-1}$	
k_{ij}	spring stiffness (CSMA)	N/m
L	length	m
M_l	apparent mass	kg
M_o	rigid body mass	kg
M	residual mass	kg
m_2	mass of load	kg
m_{ij}	modal effective mass (CSMA)	kg
m_{il}	modal effective mass	kg
m_{rl}	residual mass	kg
Q	amplification factor	
$\ddot{U}$	base acceleration for two degree of freedom system	m/s^2
$\ddot{U}^*$	base acceleration for single degree of freedom system	m/s^2
W_{AA}	acceleration spectral density	g^2/Hz

W_{FF}	force spectral density	N^2/Hz
x	coordinate	
y	coordinate	
$\ddot{x}$	acceleration	m/s^2
ζ_i	modal viscous damping	
ν	Poisson's ratio	
μ	mass ratio	
ξ	natural coordinate	
η	natural coordinate	
ω_i	natural frequency	rad/s
ω_{ij}	natural frequency (CSMA)	rad/s

Abbreviations

BEA	boundary element analysis
CNES	Centre National d'Étude Spatiales (National Centre for Space Studies)
CSA	Canadian Space Agency
CTDFS	complex two degree of freedom system
CSMA	coupled systems modal analysis
ECSS	European Cooperation for Space Standardization
ESA/ESTEC	European Space Agency/European Space Research and Technology Centre
FEA	finite element analysis
FLVT	force limited vibration loading
FRF	frequency response function
JAXA	Japanese Aerospace Exploration Agency
NASA/JPL	National Aeronautical and Space Administration/Jet Propulsion Laboratory
PSD	power spectral density
SDOF	single degree of freedom
SEA	statistical energy analysis
SEM	semi-empirical method

SPL	sound pressure level
SRS	shock response spectra
STDFS	simple two degree of freedom system
TDFS	two degree of freedom system

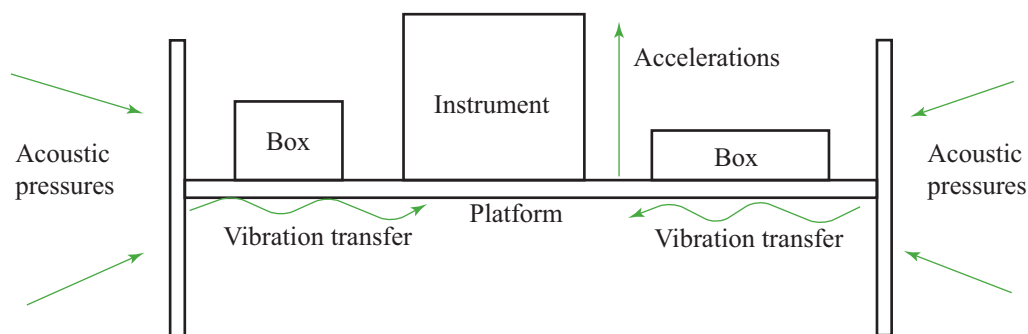
2. INTRODUCTION

2.1 General

During the early phase of the launch (lift-off) a spacecraft will be exposed to severe random acoustic loads. Acoustic loads are the cause of structure-borne random mechanical vibrations inside the spacecraft. Before launch the spacecraft system is tested to be sure it will withstand (survive) the acoustic loads. Additionally, a validated mathematical model has to be developed with which the dynamical structural responses can be calculated.

In the test set-up these acoustic loads (sound pressures) are simulated as a reverberant (diffuse) acoustic sound field, both analytically and during acoustic tests in a reverberant acoustic chamber, which is more or less an approximation of the real acoustic environment under the fairing of the launch vehicle.

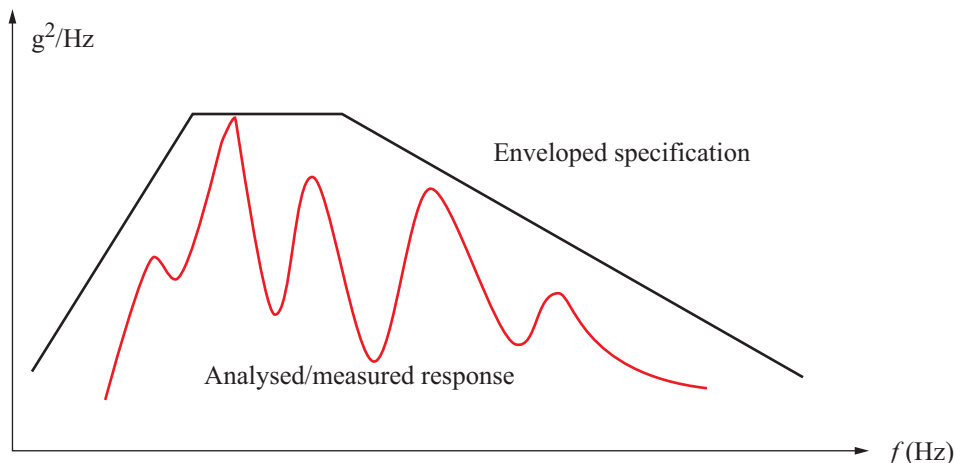
The mathematical description of a diffuse sound field and the simulation during testing in a reverberant acoustic chamber are well known and, in general, accepted in spacecraft design. Lightweight large areas of structures (solar arrays, antennae, *etc.*) are very sensitive to the quite severe acoustic loads and within the spacecraft the acoustic loads are transferred into random mechanical vibrations, the so-called structure-borne noise. This is illustrated in Sketch 2.1.



Sketch 2.1 Principle acoustical/mechanical vibration transfer

These mechanical random vibrations excite the spacecraft mounted instruments, equipment and other units by random enforced accelerations. In general, the random enforced acceleration specifications are provided to investigate analytically and by test the stiffness and strength characteristics of the subsystems. The random enforced acceleration specifications are derived from the acoustic random response characteristics estimated by fluid/structure response interaction analysis (finite element analysis (FEA), boundary element analysis (BEA) or statistical energy analysis (SEA)) or by an acoustic test in a reverberant chamber.

The dynamic response characteristics between two or more elastic structures show peaks and valleys at resonances and anti-resonances. In practice, the generation of a random enforced acceleration specification means that the peaks are enveloped to achieve a more or less smoothed random acceleration specification as shown in Sketch 2.2. The enveloped random acceleration specification is conservative at anti-resonance frequencies, where valleys in the dynamic responses will be encountered. The random vibration specification will be applied during the design and qualification process of instruments, equipment and units against the random vibration environment. **For convenience the test item (instrument, equipment, unit, etc.) is called the load and the supporting structure (panels, side walls, etc.) the source.**

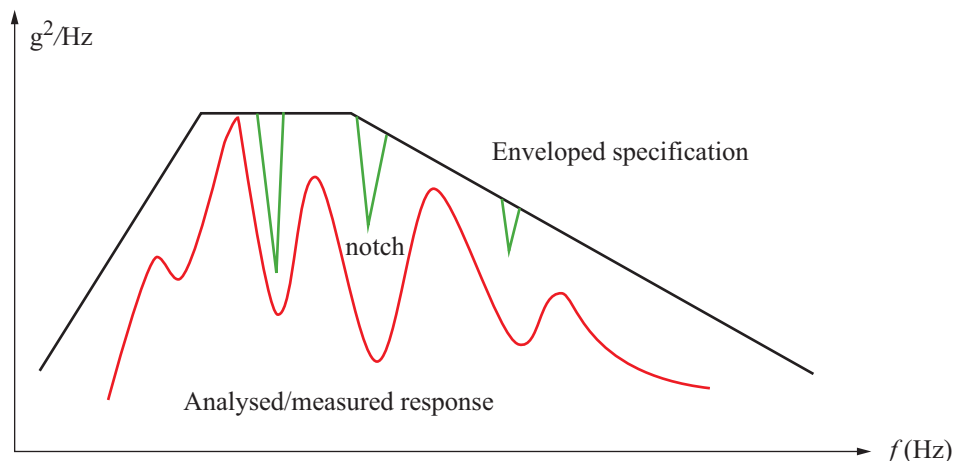


Sketch 2.2 Enveloped random vibration specification of the load

During the random vibration test the load is placed on a shaker system to apply the random acceleration specification. The impedance of the shaker head or shaker slip table is significantly greater than the structural impedance of the source. The boundary condition of the load during the vibration test is quite different from the supported load during launch.

At approximately the natural frequencies of the load (at low modal damping) a large increase of the dynamic mass of the load will occur and the response at the interface of the load/source combination will decrease. This is the same as the principle of the reduction of responses using auxiliary mass spring systems (absorbing vibration (Den Hartog, 1985)). At resonance the source with a normal structural impedance will not be able to put enough energy in the load to maintain the vibration levels, but the shaker is certainly able to put the required energy into the load. Following the random vibration specification the shaker system will over-test the load considerably. That means the random response acceleration and random force response are higher than may be expected during the launch event.

To avoid over-testing of the subsystem during the random vibration test the specified random enforced acceleration vibration specification will be adapted. That means that at the resonant frequency (frequencies) of the load the input spectrum will be reduced or notched. In general, the adapted (notched) random vibration acceleration spectrum is defined based on either on limiting it to allowable responses (response limited vibration test) or on an allowable force spectrum (force limited vibration test). The force limits are defined at the interface between the load and the shaker. An example of a notched random vibration spectrum is shown in Sketch 2.3. In general, the notches will occur at the natural frequencies of the load when the interface force are beyond the specified force limits in fact, at the anti-resonances in the load/source combination. Many methods for defining the notched spectra are discussed in the literature review in this Item.



Sketch 2.3 Modified input acceleration of random vibration specification of the load

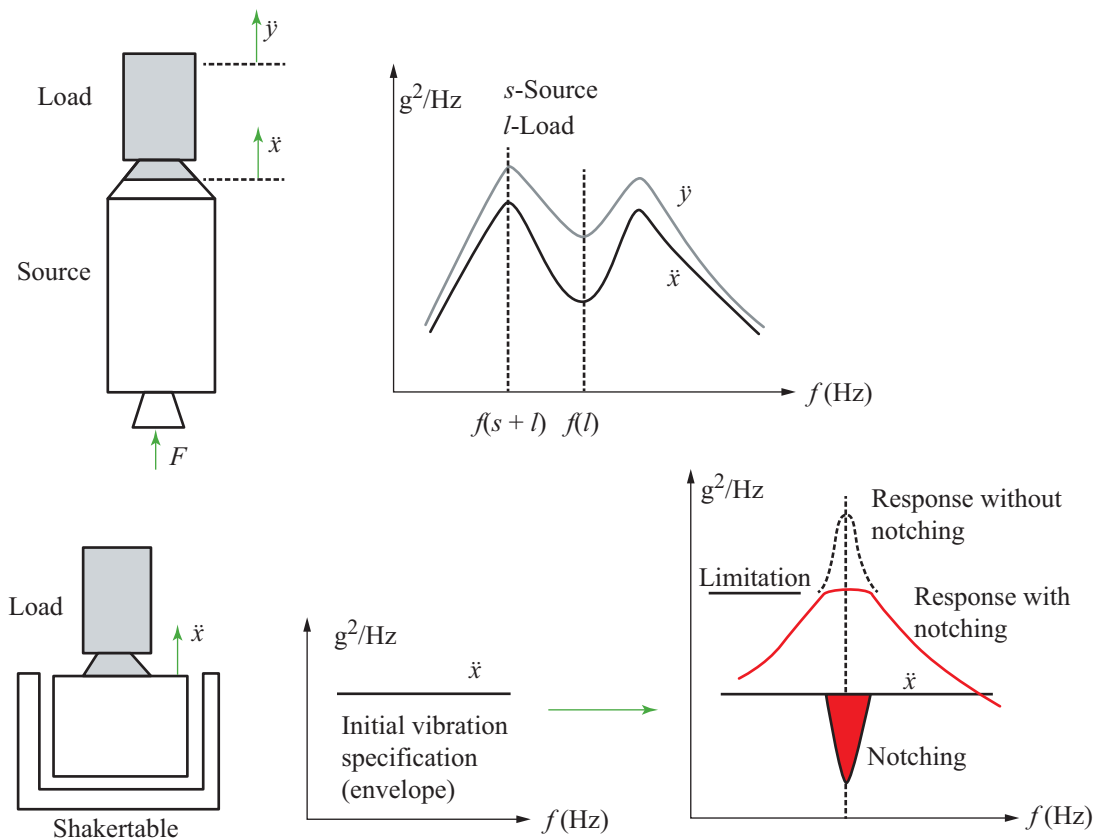
3. RANDOM VIBRATION TESTS, MODIFICATION OF INPUT SPECTRUM (NOTCHING)

3.1 Introduction

In this section an overview of a number of methods of defining modifications in the random vibration specifications are mentioned and discussed. This modification or reduction in the vibration levels is needed to prevent over-testing of the test item on a shaker or shaker table.

The dynamic configuration of the test item (spacecraft, instrument, equipment) on the shaker table is totally different from the flight configuration, spacecraft launch vehicle, instrument, spacecraft, radiator, instrument, *etc.* In the flight configuration the test item will act like a vibration absorber, which is not the case when placed on a shaker table with a very high impedance. When the test item acts like a vibration absorber the vibration in the interfaces will be reduced, which is not the case during the dynamic test. This was already noticed by Blake where he stated: “The neglect of the effects of impedance generally leads to tests of real equipment which are far more severe than any field conditions” (Blake, 1956).

The need to limit the levels during the vibration tests arises when the specifications for the load are derived from an envelope covering the mechanical load environment generated by a source excited by forces F (see Sketch 3.1). In this case, the specified acceleration levels $\ddot{x}$ are based on the dynamic responses of the combined source and load structure ignoring the minima (valleys) related to the natural frequencies (resonances) of the load as illustrated in Sketch 3.1. Reduction of the test random vibration levels (notching) is called the “response limiting vibration” and a reduction of the random interface forces is indicated by “force limiting”.



Sketch 3.1 Principle of notching (Gerald *et al*, 1999)

Notching is the reduction of acceleration input levels around natural (resonant) frequencies, to avoid over-testing. There are two forms of notching

- Primary notching
- Secondary notching

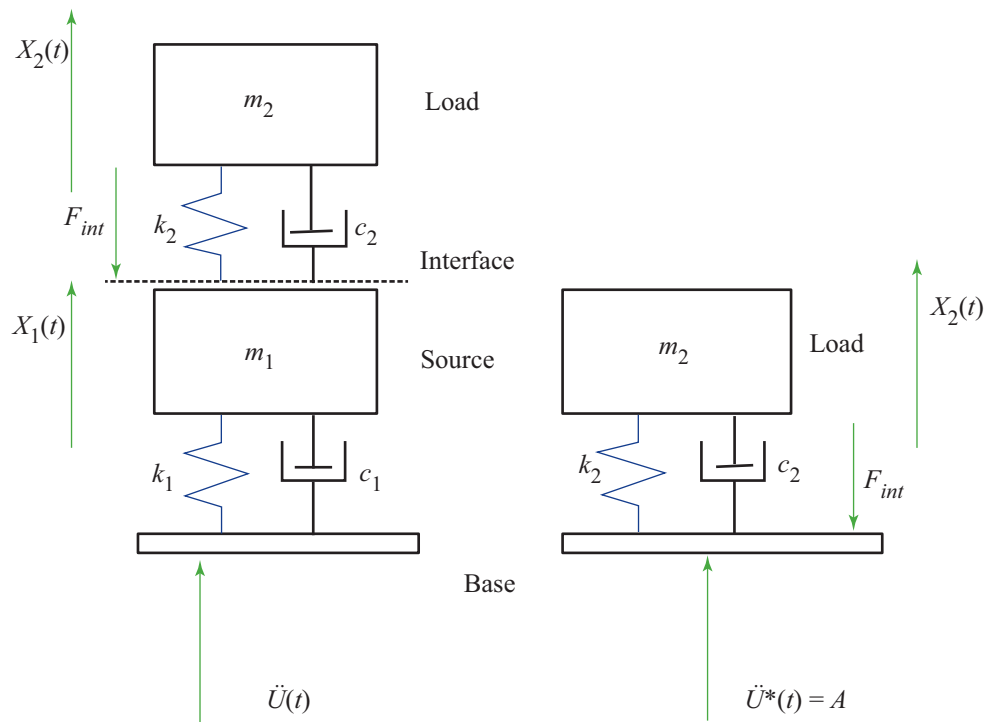
The primary notching is performed to limit the load interface forces to the specified values. This is basically the same as limiting the equivalent accelerations at the centre of gravity of the load. The secondary notching is performed to limit local accelerations with a view to protecting instruments and equipment. In general, instruments and equipment have been qualified to a certain vibration level, which has to be respected.

In this Item the emphasis is on notching applied to random enforced acceleration during random vibration testing and no distinction between primary and secondary notching needs to be made. Sinusoidal vibration testing is, in general, in a frequency range between 5 and 150 Hz. Also over-testing problems may occur and notching will be applied. The nature of acoustic and shocks test is such that adaptation of sound pressure levels (SPL) and shock response spectra (SRS) is not possible (Wijker, 2008).

3.2 Why Force and Response Limiting

The need for notching will be explained with a very simple example. The total system is represented by a two degree of freedom system (TDFS). The supporting structure (source) in the TDFS is represented by mass m_1 , damper c_1 and stiffness k_1 . The test item (load) of interest is mounted on top of the supporting structure and is represented by mass m_2 , damper c_2 and stiffness k_2 . The load is also shown as a simple

one degree of free system (SDOF). Both systems are shown in Sketch 3.2. The source represents the supporting structure of the load. The properties of the TDFS and SDOF are shown in Table 3.1. The TDFS will be excited by a constant amplitude enforced sinusoidal acceleration $\ddot{U}(\omega)$ m/s². The SDOF will be excited by a constant amplitude enforced sinusoidal acceleration $\ddot{U}^*(\omega) = \max\{\ddot{X}_1(\omega)\} = A$ m/s². The modal superposition method is used to solve numerically the sinusoidal acceleration responses $\ddot{X}_1$ and $\ddot{X}_2$ of the TDFS system when sinusoidally excited at the base $\ddot{U}(\omega) = 10$ m/s².



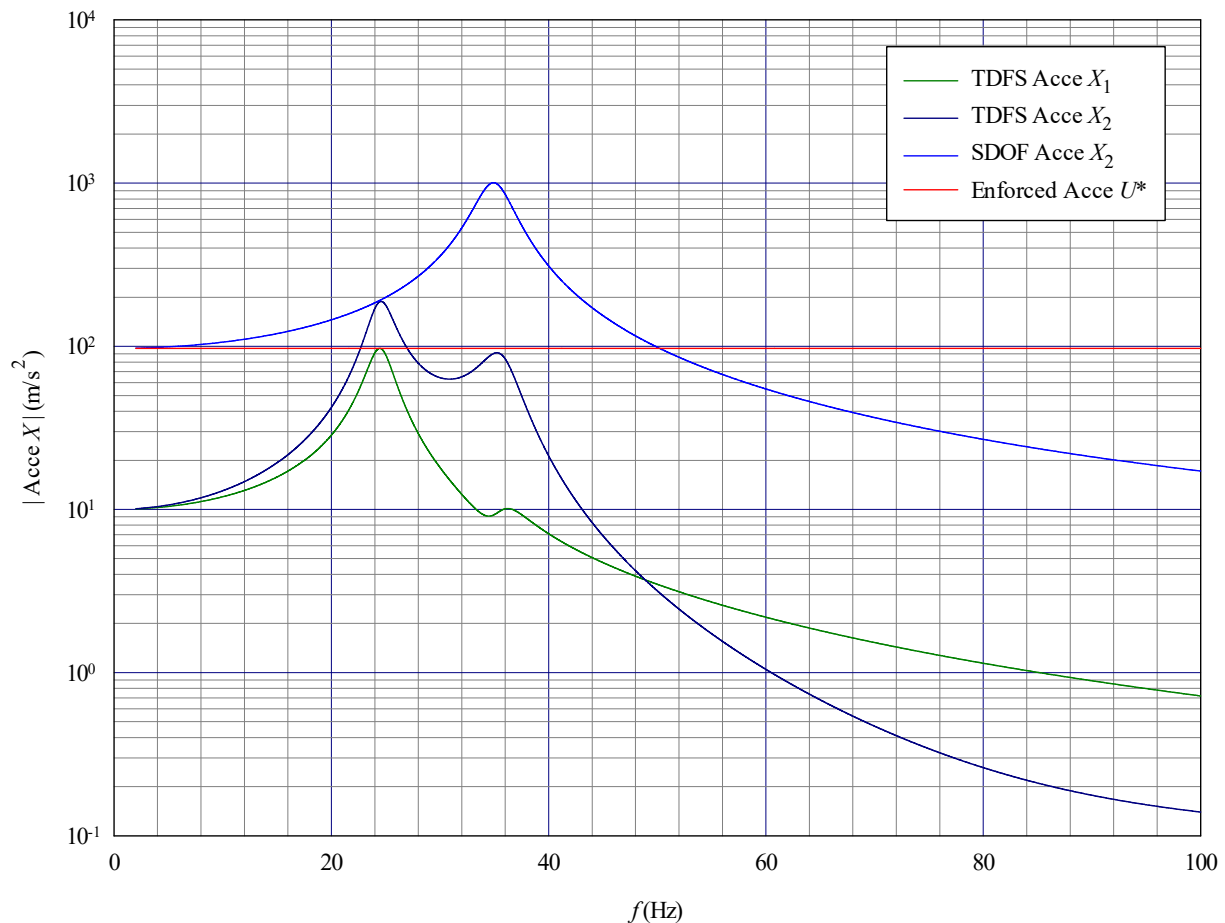
Sketch 3.2 Base-excited TDFS and SDOF

TABLE 3.1 Properties TDFS and SDOF

Item	Natural frequency (rad/s)	Damping Ns/m	Amplification factor
Source Load	$\omega_1 = \sqrt{k_1/m_1}$ $\omega_2 = \sqrt{k_2/m_2}$	$c_1 = 2\zeta_1\sqrt{k_1m_1}$ $c_2 = 2\zeta_2\sqrt{k_2m_2}$	$Q_1 = 1/2\zeta_1$ $Q_2 = 1/2\zeta_2$
Mass ratio	$\mu = m_2/m_1$		
Frequency ratio	$\phi = \omega_2/\omega_1 = f_2/f_1$		
Frequency ratio	$r = \omega/\omega_1 = f/f_1$; ω, f excitation frequency		
Frequency ratio	$\bar{r} = \omega/\omega_2 = f/f_2 = r/\phi$; ω, f excitation frequency		

The following values for the parameters are used: $m_1 = 500$ kg, $m_2 = 10$ kg, $f_1 = 25$ Hz and $f_2 = 35$ Hz. The modal damping ratio is $\zeta = 0.05$ for both modes. The acceleration responses are shown in Sketch 3.3. The natural frequencies of the undamped system are 24.5 Hz and 35.7 Hz and the associated modal effective

masses are 501.37 kg and 8.63 kg, respectively. The first mode will be excited very well and the second mode slightly. This can be seen in Sketch 3.3. The maximum acceleration for $\ddot{X}_1 = 97.2 \text{ m/s}^2$ and for $\ddot{X}_2 = 188.6 \text{ m/s}^2$.

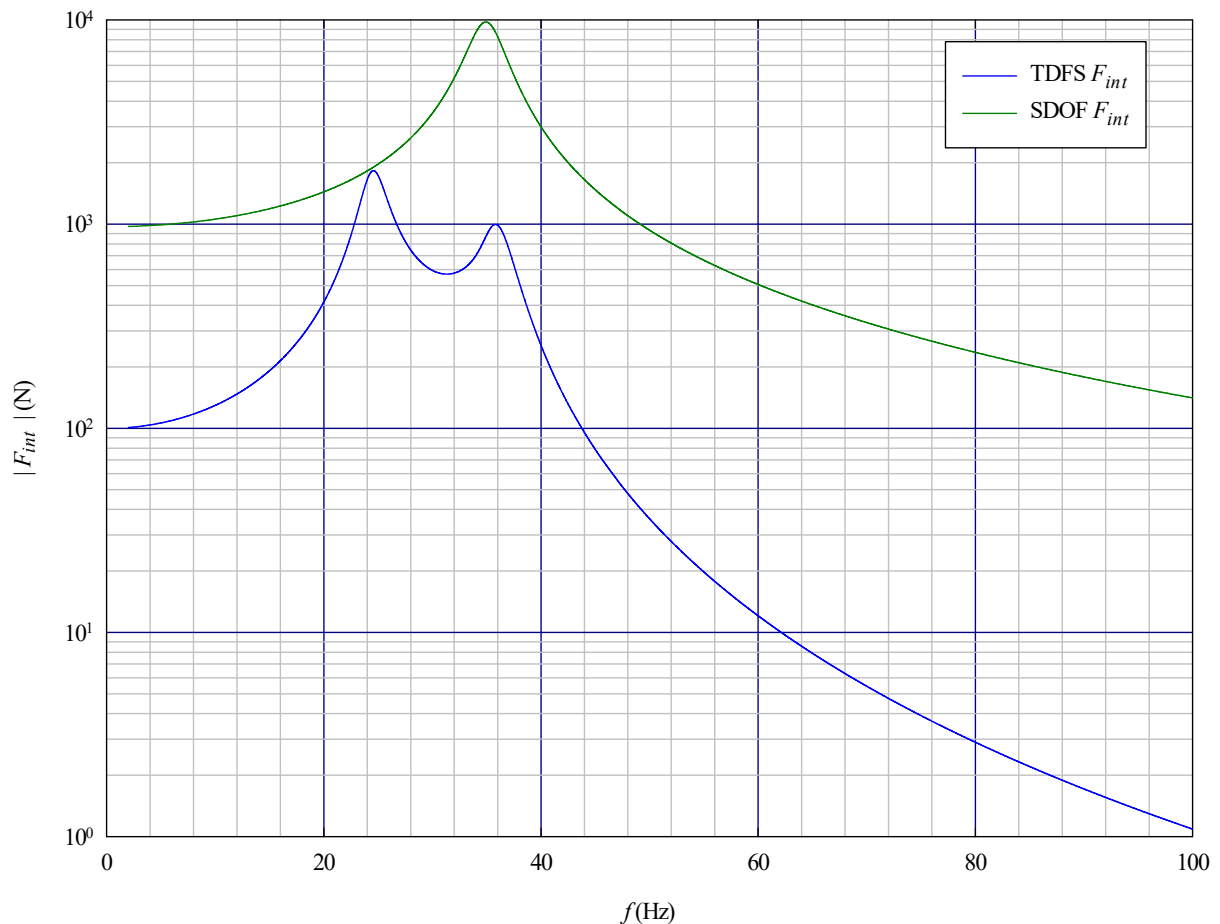


Sketch 3.3 Sinusoidal accelerations TDFS and SDOF system

The SDOF system, the load, will be now excited at the base by a conservative constant amplitude sinusoidal enforced acceleration $\ddot{U}^* = \max \ddot{X}_1 = A = 97.2 \text{ m/s}^2$. No adaptation of $\ddot{X}_1$ has been made for the second natural frequency at 35.7 Hz. In fact, the load is placed on a shaker with an infinite impedance. The maximum acceleration response of the load is $\ddot{X}_2 = 1005.8 \text{ m/s}^2$, however, at 35 Hz and not 24.5 Hz. In fact, the load is over-tested by a factor $1005.8/188.6 = 5.33$.

The interface forces in the TDFS and SDOF system as indicated in Sketch 3.2 and plotted in Sketch 3.4. The maximum amplitude of the interface force in the TDFS system $F_{int} = 1829 \text{ N}$ and the maximum amplitude of the interface force in the SDOF system is $F_{int} = 9782 \text{ N}$. The over testing by interface force is a factor 5.35.

The high over-testing factors for both the enforced accelerations and interface forces, during vibration testing of the load by an enveloping enforced acceleration spectrum, makes it necessary to adapt the vibration test spectrum in order to prevent over-testing of the load during the vibration test.



Sketch 3.4 Interface forces TDFS and SDOF system

3.3 Previous Work

In an early paper Blake described the need to reduce or notch the specified base random acceleration specification at the natural frequency of the load (test item) (Blake, 1956). On the shaker table with a very high impedance the test item cannot act as a vibration absorber, as in the case where the test item had been coupled on to another structural system.

In general the following methods are discussed in reports and papers from institutes, working groups, conferences and journals.

- *Response limiting methods.* The random vibration specification is modified (notched) in such a manner that the responses will not be beyond a specified acceleration level (this may be primary or secondary notching). This is illustrated in Sketch 3.1. One method for defining the modified random test levels is discussed:
 - Approximate Computation of Notch Spectrum (Curtis and Boykin, 1975).
- *Quasi Static Loads.* The following methods are discussed:
 - Mass participation factors (Wijker, 2018)