

Crack resistance curves

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CRACK RESISTANCE CURVES

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CRACK RESISTANCE CURVES

1. INTRODUCTION

This Data Item provides crack resistance curve data and a means for estimating the growth of a crack during the application of a given load or for estimating the combination of load and given crack length that will cause fast fracture. Crack resistance curves apply to the macro-growth of a crack during the application of a single load and their principal use is to estimate the static strength of a cracked thin sheet structure. They do not relate to crack growth under cyclic load where growth per load cycle is on the micro scale. However, the most common cause of a crack in a structure is fatigue and crack resistance curves are used to determine the length of a crack propagated by fatigue that is sufficient to cause fast fracture during a single load application. The use of this Data Item is illustrated by two worked examples.

2. NOTATION

Both SI and British units are shown but any coherent system of units may be used.

a	actual value of crack length, half-length or depth	m	in
a'	equivalent value of a	m	in
a_0	initial crack length, half-length or depth	m	in
b	dimension	m	in
f_p	0.2 per cent proof stress of material in tension	N/m ²	lbf/in ²
K	stress intensity factor defined by $K = \sigma \sqrt{\pi a} Y$	N/m ^{3/2}	lbf/in ^{3/2}
K_{crit}	critical value of K at start of fast fracture (in Mode I)	N/m ^{3/2}	lbf/in ^{3/2}
K_{Ic}	plane strain fracture toughness in Mode I	N/m ^{3/2}	lbf/in ^{3/2}
K_R	value of K , based on a' and σ , at which crack is just stable (see Section 3)	N/m ^{3/2}	lbf/in ^{3/2}
r_p	size of plastic zone*	m	in
t	thickness	m	in
w	panel width	m	in
Y	geometric factor in stress intensity factor		
Δa	$a - a_0$	m	in
$\Delta a'$	$a' - a_0$	m	in
σ	nominal gross stress in component or test specimen	N/m ²	lbf/in ²

* For a more detailed definition of r_p , see Section 3.1.1 of ESDU 80036, 'Introduction to the use of linear elastic fracture mechanics in estimating fatigue crack growth rates and residual strength of components'.

2.1 Additional notation for test summaries

E	modulus of elasticity	N/m ²	lbf/in ²
e	elongation (per cent) after failure in tension		
f_t	tensile strength	N/m ²	lbf/in ²

2.2 Suffices

1, 2, 3, ...	indicates specific values of a or σ
F	indicates a value of a or σ associated with fast fracture
s	value applied to structure

2.3 Conversion factors

$$1.0 \text{ MN/m}^{3/2} = 0.910\,047 \times 10^3 \text{ lbf/in}^{3/2} = 0.910\,047 \text{ ksi in}^{1/2}$$

$$1.0 \text{ MN/m}^{3/2} = 1.0 \text{ MPa m}^{1/2} = 31.6228 \text{ N/mm}^{3/2} \text{ (that is } 10^{3/2} \text{ N/mm}^{3/2}\text{)}$$

3. FUNDAMENTALS OF CRACK RESISTANCE CURVES

For a given environment, the resistance to Mode I growth* of a crack in a particular metallic material is governed by the combination of the length of the existing crack and the material thickness. In very thick materials, where conditions of plane strain apply, the growth of a crack under the single application of a steadily increasing tensile load is negligible until a critical load is reached. At this point, fast fracture begins, that is the crack starts to grow and accelerate rapidly to failure without any further increase in load. This critical condition is described by the fracture toughness, K_{Ic} , see Section 5.1 of ESDU 80036. In thinner materials subject to plane stress, or when the conditions are between plane stress and plane strain, cracks behave differently. This is illustrated by the case of a cracked sheet subject to a rising load. Initially when a sufficiently large load is applied, the crack grows. If, while the load is held constant, the cracked material continues to sustain the applied load, the crack is termed 'stable'. When a higher load is applied, this process is repeated. Eventually a load is reached such that, during its application, crack growth accelerates rapidly to give fast fracture. In practice, this fundamental behaviour can be further complicated by changes in stiffness that can occur within the structure (or test specimen) as a crack grows.

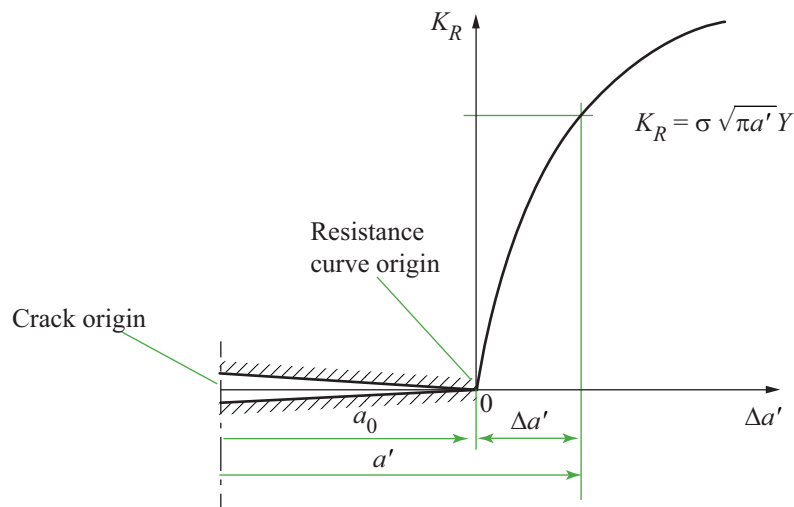
The behaviour described for conditions other than plane strain is primarily attributable to the large region of material surrounding the front of the crack tip that yields under load. This region is larger than that existing under comparable conditions of plane strain (see Section 3.2) and is referred to as the plastic zone. When a crack grows under the applied load it extends into the plastic zone and, in doing so, it further increases the size of the plastic zone. Growth therefore requires an increase in the applied load, during the application of which some of the work will lead to fracture, some grows the plastic zone and some is stored as elastic energy in the sheet. The crack stops growing when a new equilibrium is reached.

* See ESDU 80036, Section 3, for the definition of Mode I growth.

3.1 The Crack Resistance Curve Test

The crack resistance curve represents the effects of all the phenomena described in Section 3. To deal with the effects of plasticity, the crack resistance curve represents growth, not of an actual crack, but of a crack with the same behaviour when treated by linear elastic fracture mechanics (LEFM). This is referred to as the 'equivalent crack' of length a' , which is longer than the actual crack to account for the size of the plastic zone.

The value of a' is usually determined by tests. For the chosen specimen type, a relationship between the crack length and the compliance across the crack under conditions of LEFM is required. This is obtained by applying very low loads to the test specimen for a range of crack lengths. At each crack length, a gauge is placed across the crack to measure the crack opening displacement and knowing the applied load allows the calculation of the associated compliance. This relationship holds for all specimens that have the same proportions, irrespective of their size, and it can be applied to any homogenous ductile metallic materials provided that a correction is made to account for differences in the modulus of elasticity and the thickness. The resistance curve is then obtained from a specimen of identical proportions. When this specimen is loaded in increments causing the crack to tear, for each instance of stability, the crack opening displacement is measured using an identically sited gauge and the load. At each point of crack stability, the value of a' is found from the established linear elastic compliance/crack length relationship at the identical value of compliance. From the load applied to the specimen and a' the value of K_R is calculated and the crack resistance curve constructed by plotting K_R versus a' . It should be noted that the K_R curve may vary for the different specimen types (such as wedge loaded specimens or centre-cracked tensions specimens and note that care should be taken when comparing results from different specimen), or modes of loading.



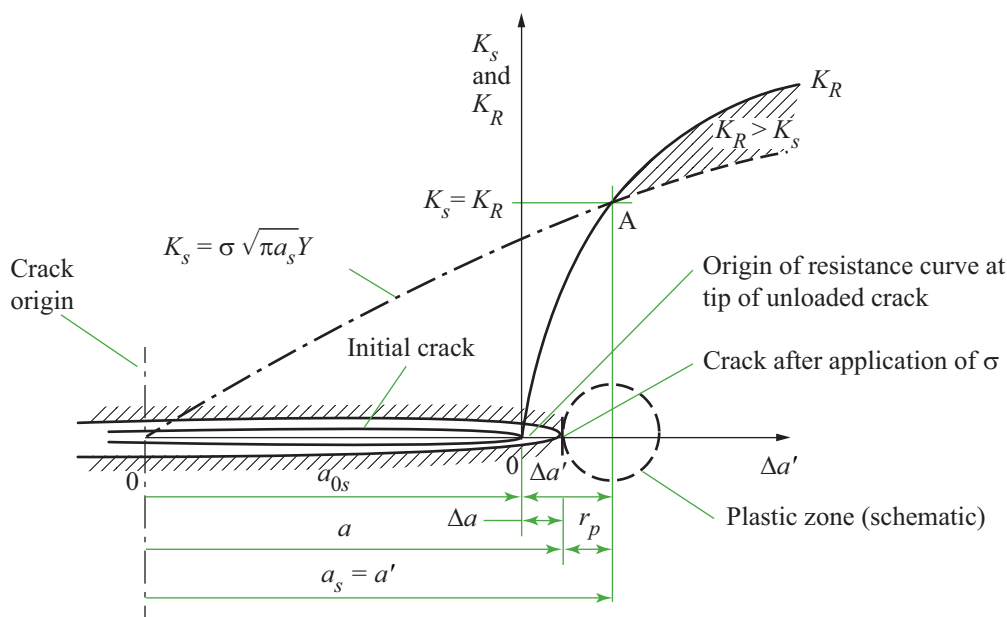
Sketch 3.1 Typical crack resistance curve

The resulting crack resistance curve does not vary with the length of the initial crack, a_0 , because the crack resistance curve is related to the work done forming new fracture surfaces and changing the plastic zone size. As a result of the independence of K_R from a_0 , the general form of a crack resistance curve is K_R versus $\Delta a'$, where $\Delta a'$ is found by subtracting a_0 from a' , see Sketch 3.1. Thus, all the effects of plasticity are included in the crack resistance curve and estimations of the load at which fast fracture occurs can be made using LEFM.

Crack resistance curves are affected by the test temperature. This effect is non-linear and it is not possible to state a general rule about the effect the test temperature has. The temperature at which each set of crack resistance curve data was found is given with the relevant test summary.

There is evidence that the rate of loading will influence the crack resistance curve. For this reason, the test loading rate, where known, is also included in the relevant test summary. During some types of loading procedures, the load is applied in increments. After the application of each increment, time is allowed for the crack to grow before measurements are taken. For most cracks, growth stops within a few seconds of holding the load constant; this period is referred to as crack stabilisation. Crack resistance curves obtained from tests in which the crack is allowed to stabilise give the lowest value of K for a particular value of $\Delta a'$ and therefore offer the most conservative value of either the critical crack length or the stress, depending on how the crack resistance curve data is used.

Sketch 3.2 presents a simple application of a crack resistance curve and shows the relationship between r_p , $\Delta a'$ and a' . The sketch relates to a case in which a stable crack has developed and fast fracture has not occurred. For the given crack a graph of K_s against a_s is plotted. The resistance curve is then added, using identical scales for a_s and $\Delta a'$, with its origin sited at the crack tip. The sketch shows that $K_s > K_R$ until a point A is reached at which point $K_s = K_R$. The crack has grown from a_0 in a stable manner and at point A the crack is stable at a length of $a_0 + \Delta a$ and an effective length of $a_0 + \Delta a'$. Beyond this point K_R exceeds K_s indicating that the resistance to crack growth exceeds the driving force. Note that, at point A, $a_s = a + r_p$.



Sketch 3.2 Application of a crack resistance curve

3.2 The Plastic Zone Size

The use of effective crack length means that it is important to ensure that the plastic zone size is not influenced by any boundary other than that of the crack itself. Thus, when a crack, in either a test specimen or a component, approaches a boundary that begins to influence r_p , the resistance curve or its application ceases to be valid. Wheeler, in his report for GARTEur, for example, indicates that the net section stress should not exceed $0.9f_p$ (see Reference 13). Similarly, in very short cracks in components where the plastic zone is influenced by a plastic zone at the other end of the crack, or by the boundary from which the crack was initiated, the resistance curve approach does not apply. For conditions of plane stress, an estimate of r_p is given by

$$r_p = (K/f_p)^2 / 2\pi, \quad (3.1)$$

and for conditions of plane strain, r_p is about one third of the value given by Equation (3.1). For conditions intermediate between plane stress and plane strain, a simple method of estimating r_p is not available. Plane stress may be assumed to apply if

$$r_p/t \geq 1/2, \quad (3.2)$$

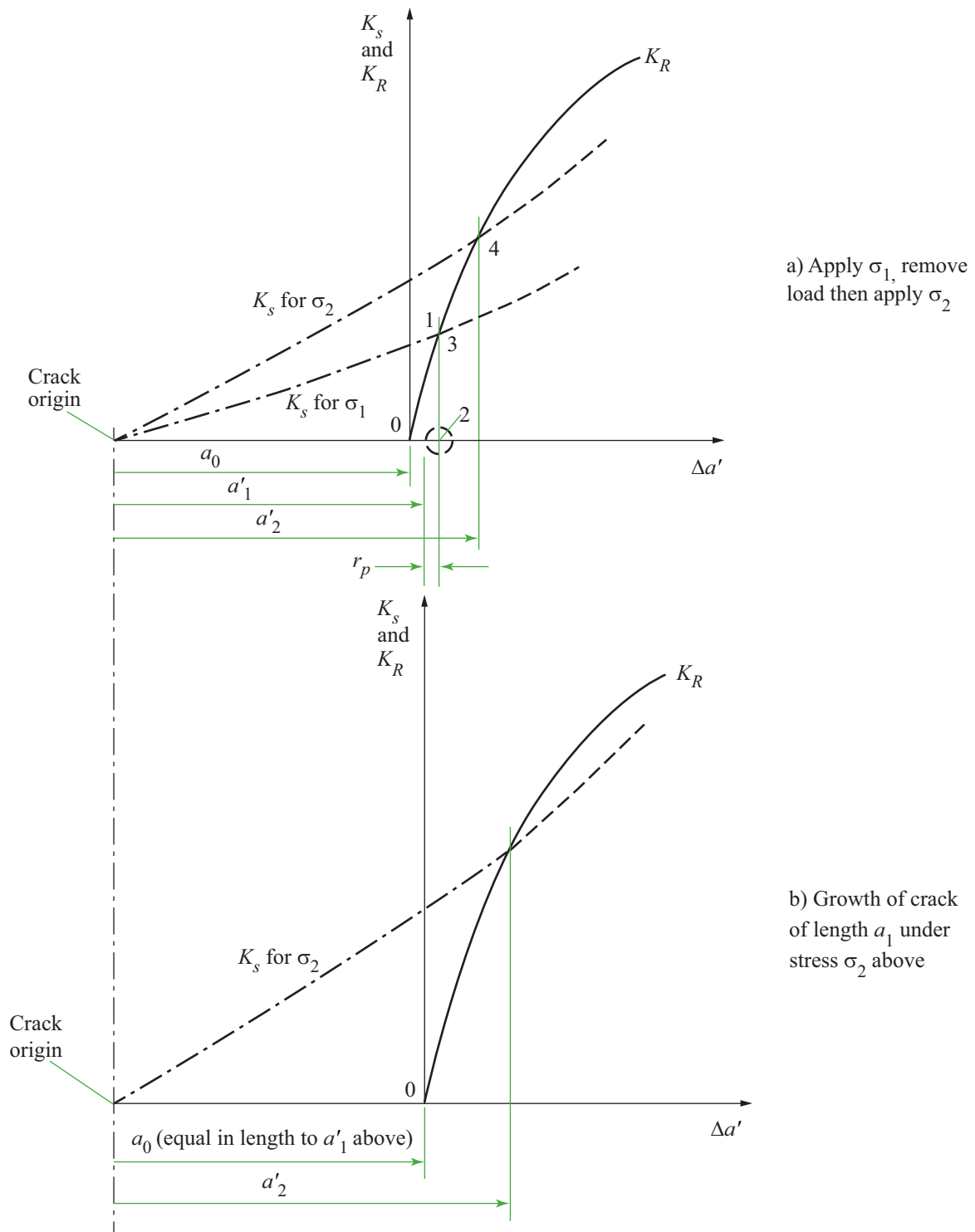
and plane strain applies if

$$r_p/t \geq 1/50, \quad (3.3)$$

3.3 Repeated Loading

Resistance curves are strictly valid only for a steadily increasing load. The origin of the resistance curve is only relocated to the tip of the physical crack length if previous growth was not by tearing. Application of a load that causes the crack to grow by tearing, but that is not enough to cause fast fracture, leaves a lengthened crack. A second application of the same load after complete or partial load removal does not cause further tearing. This is illustrated in Sketch 3.3 which shows the growth of a crack from a_0 to a'_1 under the application of stress σ_1 . Following the removal of that stress and the subsequent application of a larger stress, σ_2 , the crack grows to a'_2 . The numbers on Sketch 3.3 show the sequence of the positions associated with these stages. If, in the same circumstances, the initial crack length was equal to a_1 , as shown on Sketch 3.3, then the resistance curve would have its origin at the physical crack tip as shown on that sketch. Under the application of σ_2 only, this results in a longer equivalent crack, as a comparison between part (a) and (b) of Sketch 3.3 shows. The reason why the resistance curve was not relocated at the tip of the physical crack of length a'_1 in Sketch 3.3 for the application of stress σ_2 was that the plastic zone from σ_1 still existed. Thus applying σ_2 would not extend the crack in this case until σ_1 is exceeded.

Most cracks are initiated and grow by fatigue under alternating loads. As a consequence, it is usually of importance to determine the crack length, a_{0F} , at which, during a single load application, fast fracture from the existing crack occurs. This assumes that no previous tearing has occurred or, if it has, that the crack has grown by fatigue (under loads much less than the highest load) beyond the plastic zone associated with the tearing. The value of r_p used with the maximum load to determine a_{0F} should be that given by Equation (3.1) for plane stress since fast fracture is dependent on the full r_p . Note that the maximum load could be higher for the load case that is used for residual strength checks, compared to the load used for fatigue case. (This should not be confused with the plastic zone that governs fatigue crack propagation which is controlled by the region subjected to reversed plastic flow only and in consequence is smaller than that given by Equation (3.1)).



Sketch 3.3

3.4 Synopsis of Properties of Crack Resistance Curves

From the previous sections, the following synopsis of the properties of crack resistance curves has been compiled.

- (a) Crack resistance curves depend on the material, its thickness and the test temperature. It follows that, for each material, with a specific heat treatment and grain orientation, sets of curves are required for each thickness and test temperature.
- (b) Crack resistance curves obtained may depend on the method of testing, and the type of specimen used. For further details see Section 4.1.
- (c) Crack resistance curves are independent of the initial crack length and should be sited at the crack tip, a_0 . The crack resistance curve may only be moved to a new (longer) a_0 if the intervening crack growth has occurred without tearing via fatigue fracture or stress corrosion cracking (or whatever the method of propagating the crack in a non-tearing manner).
- (d) Crack resistance curves should have some dependence on the rate of loading.
- (e) The fast fracture condition can be determined from the appropriate crack resistance curve using LEFM.
- (f) Crack resistance curves cover, in principle, the complete region of Mode I growth from plane strain through to plane stress conditions.
- (g) All crack resistance curves have a positive slope from the origin since a plastic zone begins to evolve and grow as soon as a load is applied.
- (h) Crack resistance curves only apply when the plastic zone is not significantly influenced by boundaries other than those of the crack itself, see Section 3.2.

4. USE OF CRACK RESISTANCE CURVES

Crack resistance curves are often used for two types of calculation:

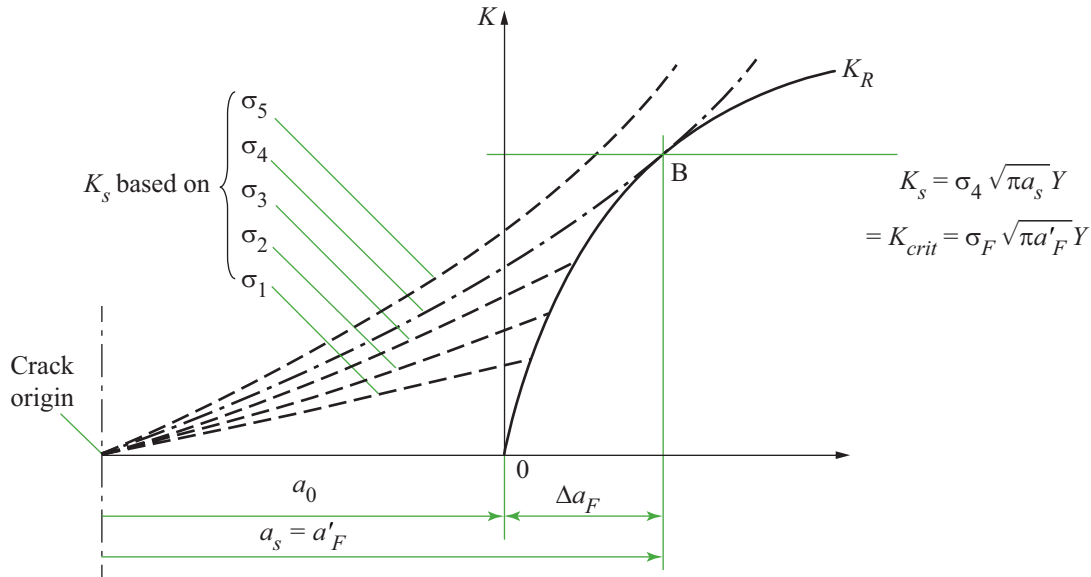
Type I Given a crack of known length, determine the stress that causes fast fracture.

Type II Given a stress field, find the length of the initial crack that will grow and cause fast fracture.

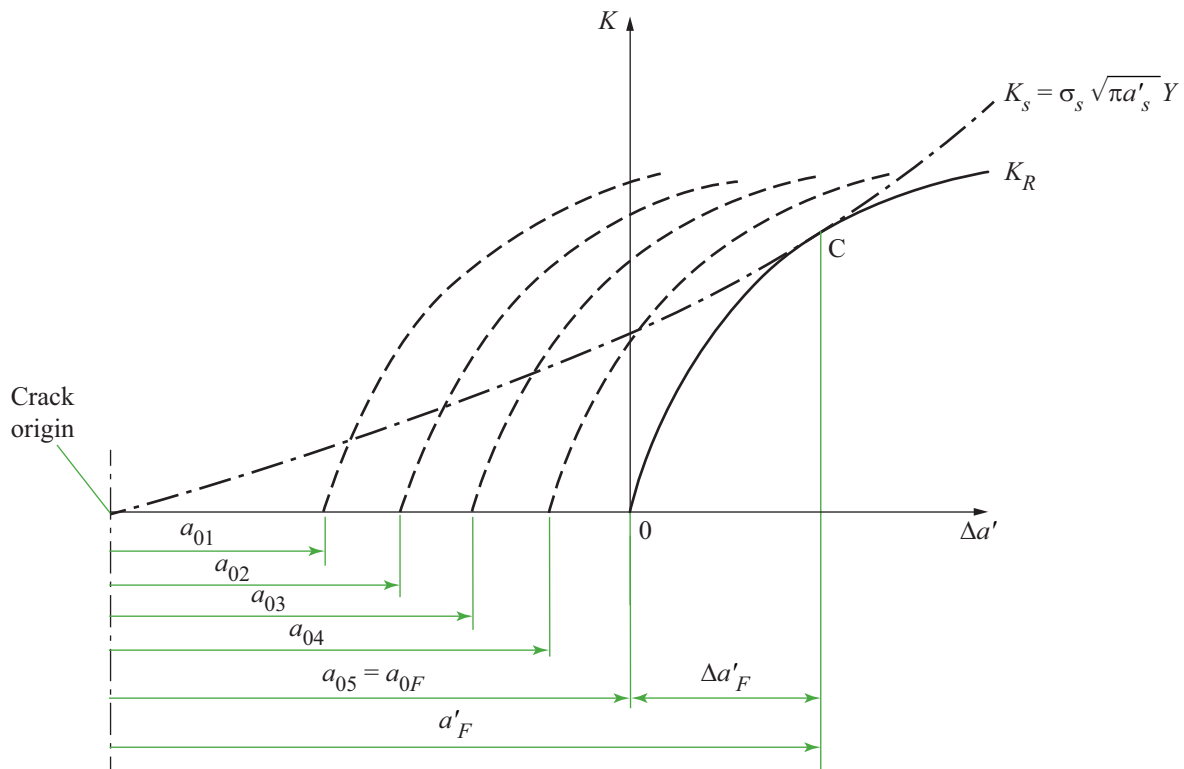
Sketch 4.1 illustrates a Type I calculation by plotting a series of K_s versus a_s curves with increasing values of σ until a curve is found that is tangential to the crack resistance curve. In Sketch 4.1, the K_s curve based on σ_4 has a point of tangency with the crack resistance curve at point B. Thus, the K_s curve based on σ_4 (and all curves with a higher value of σ) has $K_s \geq K_R$ at all times and fast fracture will occur at this stress – also termed σ_{sF} .

Sketch 4.2 shows as Type II calculation to find the minimum crack length that causes fast fracture under a given applied stress. This is found by moving the crack resistance curve in the direction of crack growth (that is, larger values of a_0) until a point of tangency, C, is found between the K_s and K_R curves. This yields the initial crack length that will cause fast fracture, a_{0F} , under the given applied stress.

The actual crack length at the start of fast fracture, a_F can be estimated by subtracting an estimate of the plastic zone size, r_p , from a'_F . This is practical if conditions of plane stress apply (see Equation (3.2)) since for this case, an estimate of r_p can be found from Equation (3.1). For conditions between the case for plane stress and plane strain, a simple method for estimating r_p is not available. Since r_p is a maximum under conditions of plane stress it follows that a conservative estimate of the smallest value of a_F may be found by assuming plane stress and using Equation (3.1) to estimate r_p .



Sketch 4.1 Type I calculation. Determination of stress that will cause fast fracture



Sketch 4.2 Type II calculation. Determination of initial crack length that will lead to fast fracture

4.1 Limitations on the Use of Crack Resistance Curves

The methods of testing and the choice of specimens limit the applicability of crack resistance curves. Some of those limitations have been noted in Section 3 but they are explained more fully in the following sections. Notes on avoiding the errors that can result from these limitations are also given.

4.1.1 The initial slope of the crack resistance curve

When using a crack resistance curve to predict the future behaviour of a structure containing a crack, extreme care should be taken when using the initial part of the curve since this region corresponds to conditions of very limited stable crack growth. This is likely to be an important issue for shorter cracks or for cracked structures in which the K_s curve rises steeply; in such cases, the shape of the initial part of the curve can be critical.

When sufficiently large load is applied to a test specimen, the size of the plastic zone ahead of the crack starts to grow so that $\Delta a'$ increases. Sometimes the first recorded point on the resistance curve is after the first tear has occurred and thus the form of the initial part of the curve is not available. Also before any tearing occurs, the length of $\Delta a'$ will increase with the load along a curve governed by the compliance/crack length relationship (see Section 3). Over this region $a_0 = a_s$ and a general form of the curve is not available at the time of writing. For both these regions, extreme care should be taken when using any curve that has been interpolated in the initial region.

4.1.2 Shape of a crack resistance curve at large value of $\Delta a'$

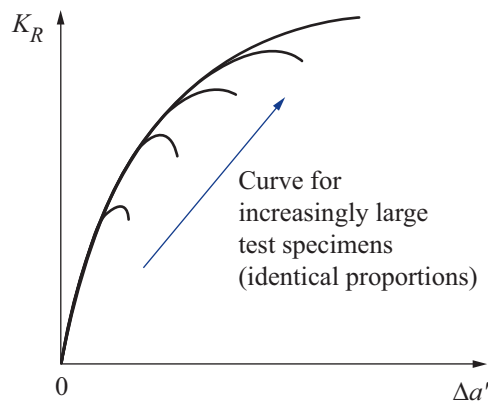
In practice, crack resistance curves are usually terminated when the boundaries of the specimen start to significantly influence the size of the plastic zone. Most standard test specimens and procedures are defined to ensure that the curve is terminated at this point (see the relevant references in Section 5). This limitation also applies to the interaction between the plastic zone and the boundaries of the component. Thus, even if a valid crack resistance curve is available it cannot be applied to the component if the net section is small and is influencing r_p .

Sketch 4.3 illustrates crack resistance curves found by testing proportionally larger specimens of the same material and thickness. It shows that the curves are coincident until the boundaries of the specimen begin to influence the size and position of the plastic zone. The termination of a curve is therefore a consequence of the choice of specimen size and is not a real material effect.

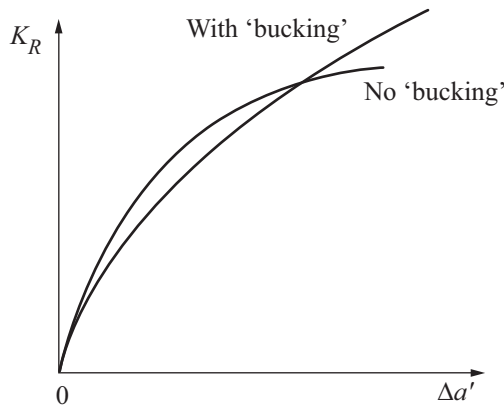
In the event that a crack resistance curve has been terminated prior to the required $\Delta a'$ value, either for the reasons described above or simply because the test was terminated prematurely, it is advisable to re-test to determine the form of the full curve. For preliminary calculations the data may be extrapolated but any results obtained in this way must be treated with extreme caution.

4.1.3 Buckling of crack components

In a thin sheet containing a large crack, part of the sheet near to the crack may be forced into lateral compression and deformed out-of-plane. This is often termed 'buckling'. It can be prevented in tests by providing suitable restraints to maintain the specimen in its original plane. Where 'buckling' has been allowed to occur, it modifies the crack resistance curve as shown in Sketch 4.4. 'Buckling' can occur in a real structure.



Sketch 4.3 Effect of specimen on K_R curve



Sketch 4.4 Effect of 'buckling'

4.1.4 Variations in the measured values of K_R

For each of the K_R curves given in Appendix B, the accompanying test summary gives details, where available, of the materials, test specimens and testing method used. The section in the test summaries that is headed 'Variations in measured values of K_R ' relates only to tests described in that summary. It should be noted that, where the test data for a particular alloy have been obtained from tests on a single batch of material, the variations should not be taken to indicate scatter across different batches of the same material. A summary of the data in Appendix B is given in Table 4.1.

4.1.5 Determining crack resistance curves for wide panel specimens

Crack resistance curves are usually determined for specimens of standard proportions but various sizes as recommended in the appropriate testing standard. However, as explained in Section 4.1.2, the size of the specimen affects the extent of the curve that can be obtained. Thus, for ductile materials with a large value of toughness, it is not possible to use the data obtained from small specimens to predict the fracture instability behaviour in large structures.

To provide data for the prediction of failure in large structures, crack resistance curves must be developed using large specimens. However, the size of the specimen that can be tested will depend on the size of the test apparatus. For example, the testing standard for a centre-cracked tension (CCT) specimen requires the length of the specimen to be more than 1.5 times the width to avoid adverse stress gradients in the specimen. Thus, test specimens up to 2 m wide would have to have a length of 3 m, requiring a crosshead spacing on the test machine of approximately 4 m. Reducing the specimen length to width ratio allows testing to be carried out without the need for greater crosshead spacing therefore reducing the amount of test material required too.

However, it is necessary to calibrate experimentally the normalised compliance specific to the nonstandard specimens used. A valid test carried out when the geometric correction factor (and hence the stress intensity factor, K) and the compliance against normalised crack length calibration are found for specific length to width ratios less than 1.5 (for CCT specimens).

It has been shown that for CCT specimens valid tests can be carried out for nonstandard specimen geometries with a range of length to width ratios between 2 and 0.5. Furthermore, it has been shown that crack resistance curve tests carried out on a range of panel sizes showed good agreement (Reference 15).

4.2 Use of Resistance curves Provided in Appendix B

The two major uses of resistance curves described in Section 4 require either the repetitive calculation and plotting of K_s versus a' curves for the component or the repeated plotting of the resistance curve on the same axes as those of the single curve. Both of these processes are tedious and for the vast majority of cases can be avoided by using the resistance curves presented in Appendix B.

To determine a_{0F} under a given applied stress, the K_s versus a' curve for the given stress and a function is plotted with the appropriate resistance curve positioned so that there is a point of tangency between the two curves. The value of $\Delta a'_F$ and a_{0F} can then be read from the abscissae, see Example 1 for a presentation of how to use this method. Similarly, to determine the value of σ_F for a given crack, the resistance curve is plotted and a curve of K_s/σ versus $\Delta a'_F$ is plotted using the same scale for a'_F as used in $\Delta a'$ on the matching resistance curve. From the definition of K_s ,

$$K_s/\sigma = Y\sqrt{\pi a'_F}. \quad (4.1)$$

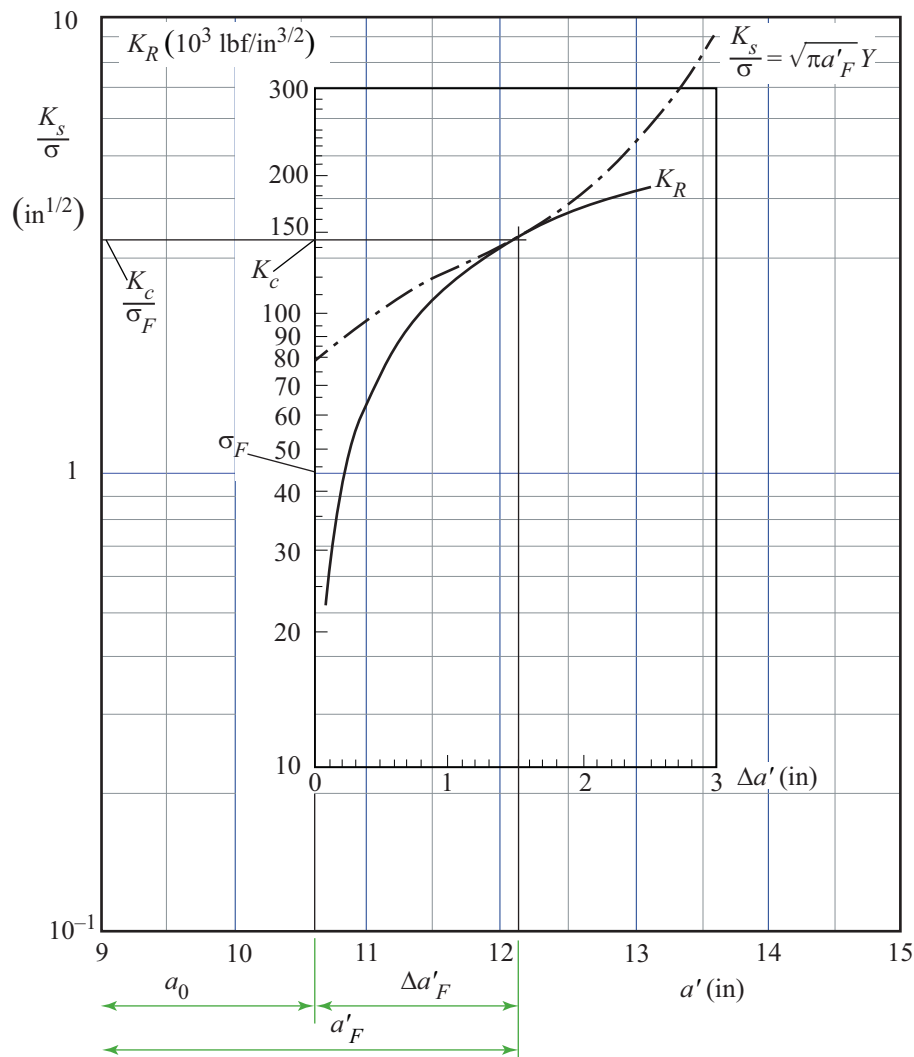
Since a crack of length a_{0F} exists it is only necessary to plot the K_s/σ curve for the region $a'_F \geq a_{0F}$. The resistance curve is moved relative to the K_s/σ plot, with the ordinate at the a_0 value, until the point of tangency and hence the fast fracture point is located. This is shown in Sketch 4.5. The tangency yields K_c and $\Delta a'_F$, from the axes of the resistance curve, and a'_F but, more importantly, also yields σ_F directly. This is because $K_s = K_c$ so that:

$$K_c/(K_s/\sigma_F) = \sigma_F. \quad (4.2)$$

Therefore,

$$\log K_c - \log (K_s/\sigma_F) = \log \sigma_F, \quad (4.3)$$

and since K_s/σ_F and K_R can be plotted using the same axis, it follows that σ_F can be read directly from the K_R scale at the point where it intersects the $K_s/\sigma_F = 1.0$ value (because $\log K_s/\sigma_F = 0$ when $K_s/\sigma_F = 1$ and $\log K_c = \log \sigma_F$ or $K_c = \sigma_F$). The units of σ_F will be the stress units that would give the correct value of K_s/σ_F and $K_c = K_s = K_R$.



Sketch 4.5

There are circumstances where K_s is not linearly dependent on σ . In such instances the presentation employing logarithmic plotting of K_s/σ_F cannot be used. The linear plots can be used, but in complex cases where α and σ are obtained by extensive computation, it is likely to be more efficient to store the resistance curve coordinates in a computer and append a subroutine to determine σ_F and a_{0F} .

TABLE 4.1 Specifications for which resistance curves are provided in Appendix B

Table number	Derivation	Specification or designation	Heat Treatment	Form	Previously	Superseded by – check these	Equivalent specification or designation
Aluminium alloys							
B1.1	1	BS L70	T3	Sheet and strip	DTD 803	BS 1156	2014-T3
B1.2	1	BS L71*	T6	Sheet and strip	-	BS L104 BS L157	2014-T6 pr EN 2089
B1.3	5	BS L72	T3	Sheet and strip (clad)	DTD 601	BS L163 BS L164	2014-T3 (clad)
B1.4	4	BS L73	T6	Sheet and strip (clad)	DTD 546	BS L165	2014-T6 (clad)
B1.5	1	BS L88	T6	Sheet and strip (clad)	DTD 687	-	7075-T6 (clad) pr En 2092
B1.6	4	BS L93	T651	Plate	DTD 5020	-	pr EN 2125
B1.7	4	BS L97	T351	Plate	DTD 5090	-	2024-T351 pr En 2422
B1.8	4	BS L104†	T6	Sheet and strip	BS L71	BS L157	2014-T6
B1.9	6	ALCOA 2024	T3	Sheet (clad)	-	-	2024-T3 BS L109 pr En 2090
B1.10	8	2024	T3	Sheet (unclad)	-	-	-
B1.11	7	2024	T3	Sheet (unclad)	-	-	-
B1.12	7	2024	T3	Sheet (unclad)	-	-	-
B1.13	7	2091	T84	Sheet	-	-	-
B1.14	7	6013	T6	Sheet	-	-	-
B1.15	5	AMS4085	T761	Clad sheet	-	-	7475-T761
B1.16	4	WERKSTOFF 13314	T7351	Plate	-	-	7475-T7351
B1.17	8	8090	T81	Sheet	-	-	-
B1.18	7	8090	T81	Sheet	-	-	-
Steels							
B2.1	4	TA 10	-	-	-	-	-
B2.2	3	4130	-	Strip	-	-	AISI and SAE4130, AMS 6351 and UNS G41300
Titanium alloys							
B3.1	2	414Ti	-	Sheet			

* Resistance curves also presented for BS L104 which has superseded BS L71.

† Resistance curves also presented for BS L71 which has preceded BS L104.

5. DERIVATION AND REFERENCES

5.1 Derivation

This section lists selected sources that have assisted in the preparation of the Data Item.

1. BRADSHAW, F. J.
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2. HAYER, R. H.
McCABE, D. E. Plane stress fracture toughness using a crack-line-loaded specimen. *Engineering Fracture Mechanics*, Vol. 4, pp. 393-412, 1972.
3. SULLIVAN, A. M.
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STOOP, J. Comparison of *R*-curves determined from different specimen types. In *Fracture Toughness Evaluation by R-Curve Methods*. American Society for Testing and Materials, ASTM STP 572, pp. 85-104, 1973.
4. SHARP, R. P. Assessment of the crack growth resistance curve technique for residual strength prediction. Stage 1: Determination of *R*-curve data. *British Aerospace plc*, Warton Division, Report No. SOR(P)155, January 1985.
5. NOWAK, S. The effect of temperature and loading rate on *R*-curves using the CLWL technique. *British Aerospace plc*, UK, Report No. BAe-MSM-R-GEN-0544, July 1985.
6. de KONNING, A. U. Applicability of the effective crack length model to slow stable crack growth in centrally cracked sheet specimens. *National Aerospace Laboratory NLR*, The Netherlands, Memorandum SC-85-045 U, October 1985.
7. HART, W. G. J.
SCHRA, L. Residual strength of damage tolerant aluminium-lithium sheet materials. *NLR Technical Report*, NLR TP 92439 U, Amsterdam, the Netherlands, 1992.
8. SUTTON, G. R.
THOMAS, C. E. The measurement of fracture toughness of thin sheet aluminium-lithium 8090-T81, 2090-T8X and unclad 2024-T3. *UK DRA Technical Report*, TR 94008, Farnborough, UK, 1994.

5.2 References

The references given are recommended sources of information supplementary to that in this Data Item.

9. ASTM Fracture toughness evaluation by *R*-curve methods. ASTM STP 527, 1973.
10. CREAGER, M. A note on the use of a simple technique for failure prediction using resistance curves. ASTM STP 527, pp. 105-112, April 1973.
11. ROOKE, D. P.
CARTWRIGHT, D. J. Compendium of stress intensity factors. *HMSO*, London, UK, September 1974.

12. McCABE, D. E.
LANDES, J. D. Elasticplastic *R*curves. *Transactions of the American Society of Mechanical Engineers, Journal of Materials and Technology*, Vol. 100, pp. 258-265, July 1978.
13. WHEELER, C. The effect of strain rate on the failure of aluminium alloys. *UK RAE Report TR 79149*, December 1979.
14. WHEELER, C.
et al. Recommendations for measurement of *R*-curves using centrecracked panels. *Journal of Strain Analysis*, Vol. 17, No. 4, 1982. Also UK RAE Report TR 81142 and GARTEur/TP-04, November 1981.
15. EWALDS, H. L.
WANHILL, R. J. H. Fracture Mechanics. Published by Edward Arnold Ltd, London, 1984.
16. SUTTON, G. R.
WHEELER, C.
THOMAS, C. E.
WILSON, R. N. Determination of *R*-curves for short, wide centre cracked panels of aluminium alloy sheet. *Fatigue and Fracture of Engineering Materials and Structures*, Vol. 14, No. 8, pp. 823-832, 1991.
17. ASTM Standard practice for *R*-curve determination. ASTM E561-86, 1986. (Note that this Standard has been superseded by 'Standard test method for K_R curve determination'. ASTM E561-19, 2019.)

6. EXAMPLES

6.1 Example 1

Sketch 6.1 shows one side of a rectangular I-section frame that is part of the structure of an aircraft wheel well. The frame is found to contain a 1.5 in long central fatigue crack initiated by impact damage. Under flight conditions the frame is under tension and must sustain an extreme load of 34 000 lbf. It is required to find the length of fatigue crack that will cause fast fracture. The frame is manufactured from BS L88 aluminium alloy.

The values of α for this configuration can be found on Sketch 6.2 (derived from Figure 123, Reference 11). The cross-sectional area of each end flange is 0.384 in² giving a flange to web area ratio of 0.5 and Sketch 6.2 is appropriate to this value. The panel is manufactured from materials to BS L88 specification. A curve of K_s against α under the applied load is required. For $\alpha = a_0 = 0.75$ in this is calculated as follows. From Sketch 6.2, $a = 0.665$ so that:

$$K_s = (34\,000 / (2 \times 6 \times 0.064))0.665 = 45.2 \times 10^3 \text{ lbf/in}^{3/2}.$$

Table 6.1 repeats this calculation of K_s for $0.75 \leq a \leq 4.5$ in and from this the curve of Sketch 6.3 is produced.

Sketch 6.3 shows a plot of K_s with the K_R curves superimposed such that the primary point of tangency is shown. This gives a_{0F} as 1.8 in and a'_F at 2.3 in at $K_R = 83 \times 10^3 \text{ lbf/in}^{3/2}$. The physical crack length at fast fracture is $a'_F - r_p$. Equation (3.1) is used to find the value of r_p – which requires f_p for BS L88 as 62 400 lbf/in².

Therefore,

$$r_p = 1/2\pi(83 \times 10^3 / 62\,400)^2 = 0.282 \text{ in.}$$

Hence the physical crack length is estimated as:

$$2.30 - 0.282 = 2.018 \text{ in.}$$

The net section stress is:

$$34\,000 / ((12 - 2 \times 2.018) \times 0.064 + 2 \times 0.384) = 26\,600 \text{ lbf/in}^2.$$

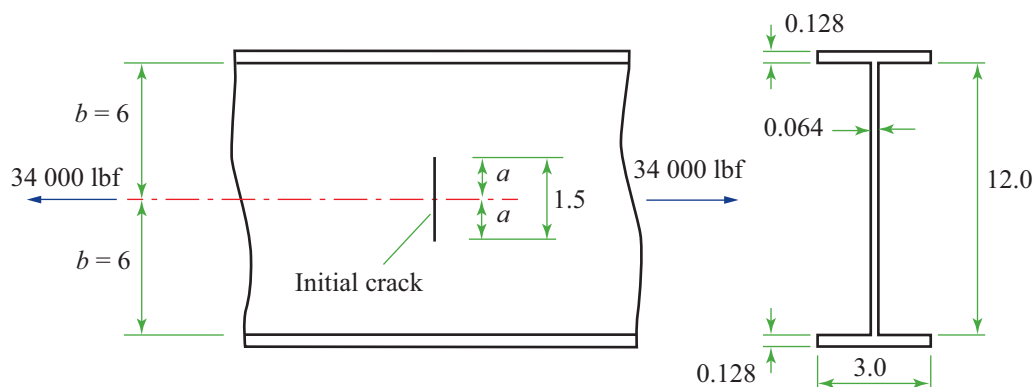
Since this value of net section stress is below the value of proof stress the crack will not be influenced by net section yielding. Further, from Equation (3.2):

$$r_p/t = 0.282/0.064 = 4.41 \geq 1/2$$

so that the conditions of plane stress are assumed to apply.

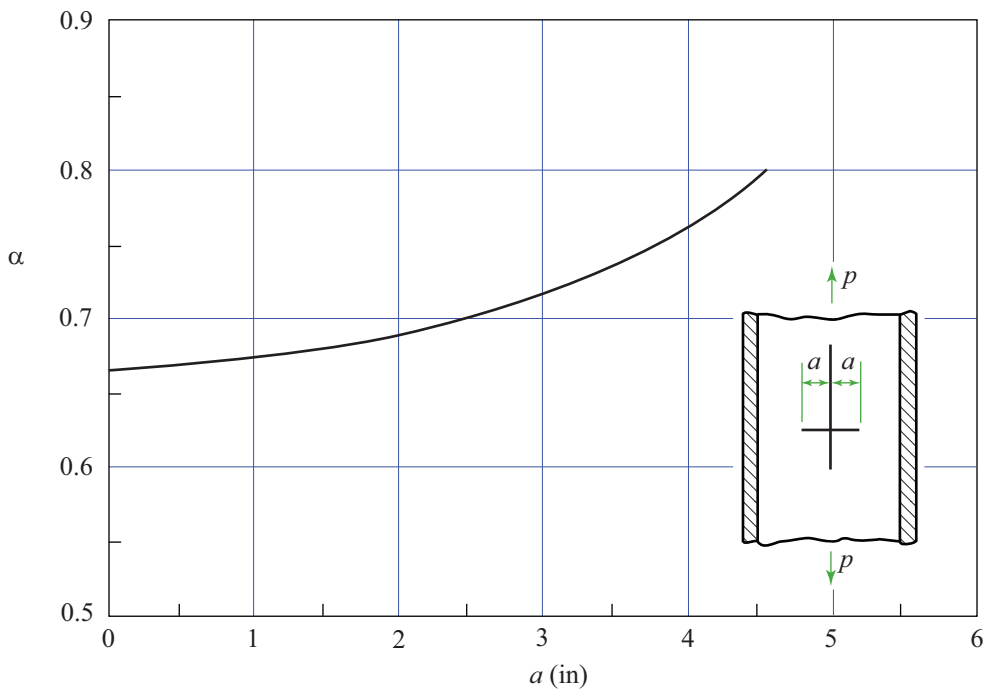
TABLE 6.1

α	a	K_s
	in	$10^3 \text{ lbf/in}^{3/2}$
0.665	0.75	45.2
0.687	2.00	76.2
0.715	3.00	97.2
0.760	4.00	119
0.800	4.50	133

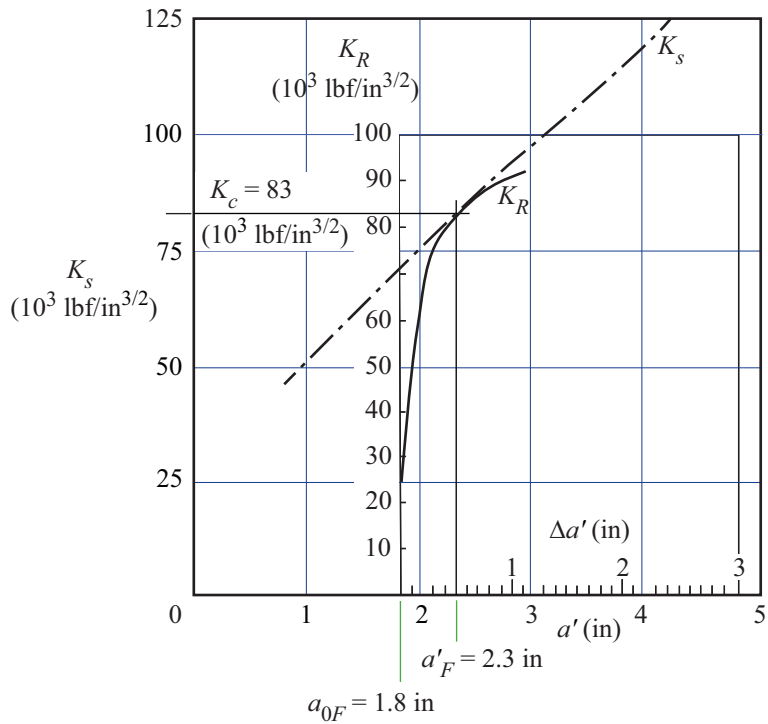


(all dimensions in inches)

Sketch 6.1 Rectangular I-section frame



Sketch 6.2 Variation of α with a for I-section



Sketch 6.3

6.2 Example 2

The channel section frame shown in Sketch 6.4 is found to contain a pair of 10 mm long fatigue cracks growing from the 80 mm diameter hole in the centre of the web. The frame is under tension and it is required to find the load that will cause the existing cracks to fast fracture. The frame is manufactured from 1.6 mm thick BS L71 aluminium alloy sheet.

A solution for α for the cracks is required. Since the frame is in direct tension it can be considered to be equivalent to a flat sheet of width equal to its unfolded dimension of 320 mm, containing a central hole. Sketch 6.5 shows α for this configuration plotted against a .

The frame is manufactured from BS L71. A curve of K_s/σ can be plotted using Equation (4.1). The located cracks have $a = 50$ mm so that the values of a greater than 50 mm will be examined. Table 6.2 sets out the calculation of K_s/σ over the range $50 \leq a \leq 100$ mm.

Sketch 6.6 shows K_s/σ plotted against a and overlaid is the resistance curve for BS L71 with its ordinate at the crack tip position at:

$$a_0 = 40 + 10 = 50 \text{ mm.}$$

The resistance curve is raised up until the point of tangency is located at which $K_R = 90 \text{ MN/m}^{3/2}$ and $\Delta a'_F = 14$ mm and from the 1.0 value of K_s/σ on the K_R scale $\sigma_F = 168 \text{ MN/m}^2$. Thus, the resistance curve indicates that fast fracture will occur at a stress of 168 MN/m^2 from $a_0 = 50$ mm.

It is important to check that gross yielding has not influenced this value of σ_F . At σ_F the value of r_p may be estimated from Equation (3.1). Note that f_p for this material can be found from Appendix B as 463 MN/m^2 .

$$r_p = 1/(2\pi)(90/463)^2 = 0.006 \text{ m.}$$

The physical crack length is then calculated as follows.

$$\begin{aligned} a &= a_s - r_p = a_{0F} + \Delta a'_F - r_p, \\ a &= 50 + 14 - 6 = 58 \text{ mm.} \end{aligned}$$

The net section stress is therefore:

$$(168 \times 10^6 \times 1.6 \times 10^{-3} \times 320 \times 10^3) / (1.6 \times 10^{-3} \times (320 - 2 \times 58) \times 10^{-3}) = 264 \text{ MN/m}^2.$$

This is well below the proof stress of 463 MN/m^2 so that the value of $\sigma_F = 168 \text{ MN/m}^2$ is valid. Further since

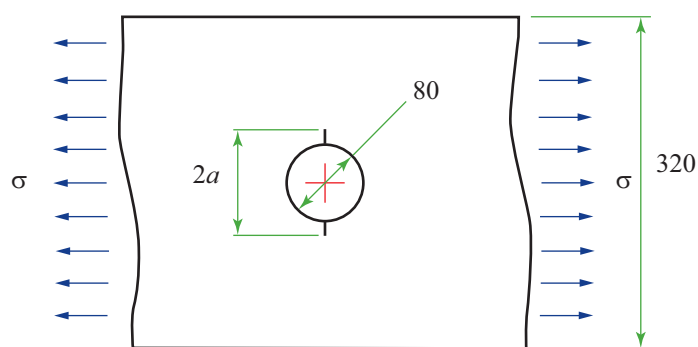
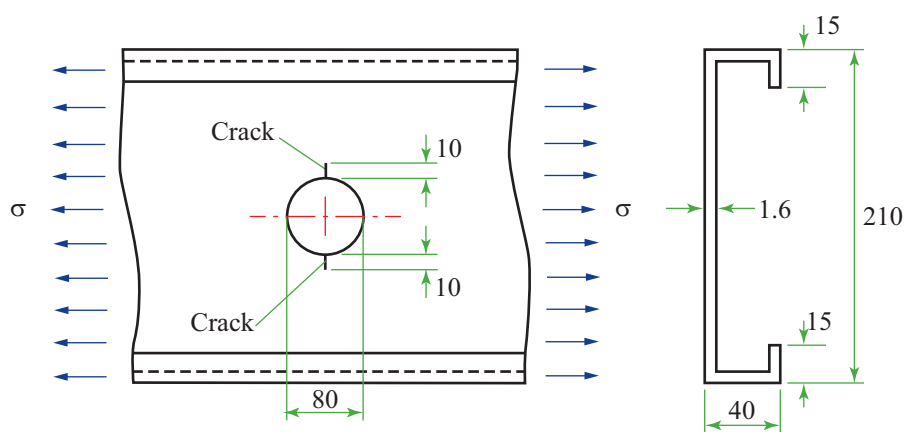
$$r_p / t = 6 / 1.6 \geq 1/2,$$

then from Equation (3.2) conditions of plane stresses are met. The load at fast fracture is therefore

$$168 \times 10^6 \times 320 \times 10^{-3} \times 1.6 \times 10^{-3} = 86 \text{ kN.}$$

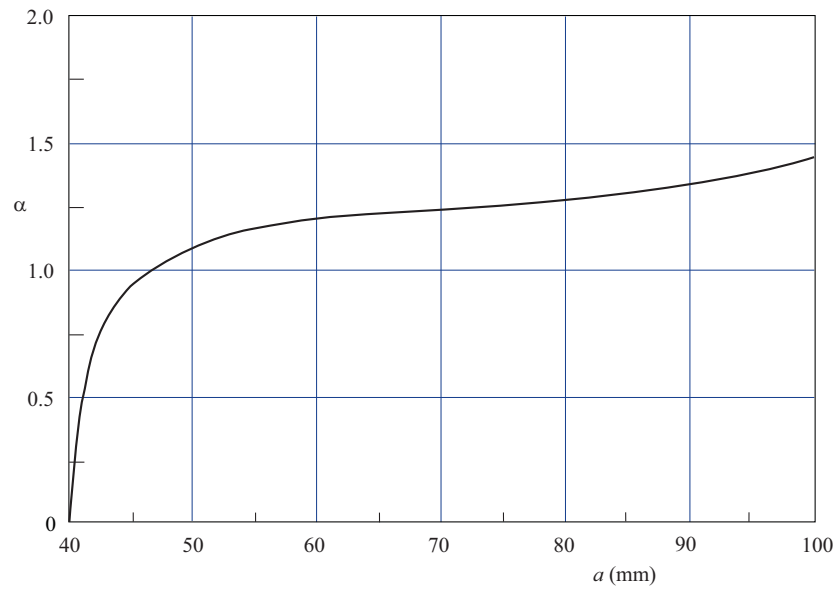
TABLE 6.2

a	α	$K_s/\sigma = Y\sqrt{\pi a}$
m		m ^{1/2}
0.05	1.10	0.436
0.06	1.20	0.521
0.07	1.24	0.581
0.08	1.28	0.642
0.09	1.34	0.713
0.10	1.43	0.802

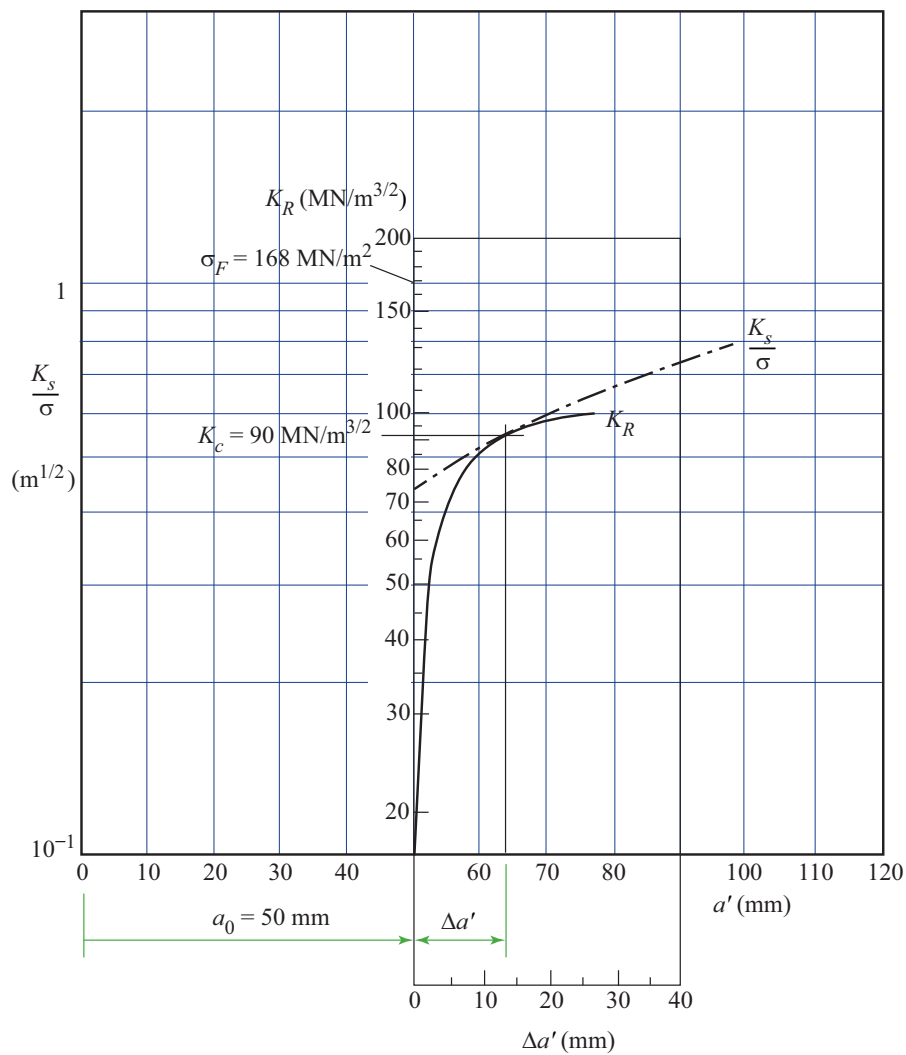


(all dimensions in millimetres)

Sketch 6.4 Chanel section frame



Sketch 6.5 Variation of α with a for a flat sheet with a circular hole

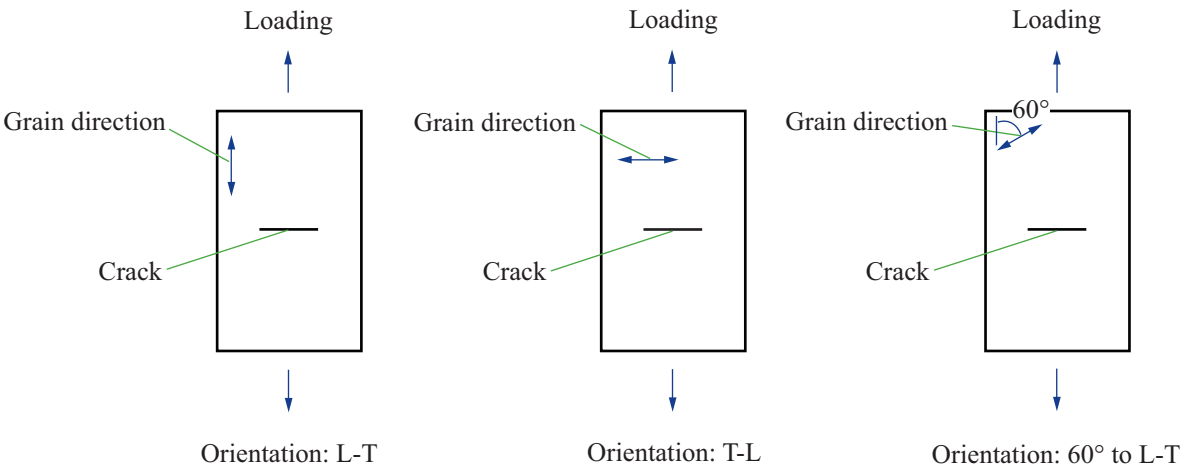


Sketch 6.6

APPENDIX A GRAIN ORIENTATION AND CRACK DIRECTION

A1. Definition

In this Data Item, the orientation of the grain direction with respect to the applied loading, and the orientation of the crack in the coupon are shown in Sketch A1.1.



Orientation:

L-T	Grain direction longitudinal (parallel) to loading Crack orientation transverse (perpendicular) to grain direction
T-L	Grain direction transverse to loading Crack orientation longitudinal to grain direction
60° to L-T	Grain direction 60° to loading Crack orientation transverse

Sketch A1.1 System used to indicate crack orientation with respect to grain flow

APPENDIX B CRACK RESISTANCE CURVE DATA

This Appendix presents resistance curves for each alloy specification given in Table 4.1. A test summary is provided for each material giving information about the method of testing used. This summary also includes details of the material specification and, where available, of the static properties of the batch from which the test specimen was cut. The curves are terminated at either the point where the specimen boundary begins to influence the curve and thus invalidating further values, or when testing was stopped before any boundary influence could occur.

B1. Aluminium Alloys

B1.1 BS L70-T3 aluminium alloy

TABLE B1.1 Index to Data for BS L70-T3 Aluminium Alloy

Derivation	1	
Specification properties (minima)		
f_p , MN/m ² (10 ³ lbf/in ²)	longitudinal	-
	transverse	260 (37.7)
f_t , MN/m ² (10 ³ lbf/in ²)	longitudinal	-
	transverse	400 (58)
e , per cent	longitudinal	-
	transverse	14
Specimen details		
Thickness, m (in)		0.0016 (0.063)
Orientation		L-T
Width, m (in)		0.275 (10.8)
Length, m (in)		0.440 (17.3)
Type		Compact tension
Measured properties of test material	f_p , MN/m ² (10 ³ lbf/in ²)	319 (46.4)
	f_t , MN/m ² (10 ³ lbf/in ²)	471 (68.3)
	E , MN/m ² (10 ³ lbf/in ²)	75 000 (10 900)
	e , per cent	19
Test details		
Test environment		Air, room temperature
Loading rate, m/min (in/min)		0.0005 (0.02) No crack stabilization allowed
Crack measurement method		Compliance method
Plastic zone correction		Included in compliance method
Buckling restraint		Yes
Variations in the measured value of K_R		The curve was provided by the derivation as a smooth line which was stated to be the average for two tests. Derivations from the line were stated as being less than ± 3 per cent.
Notes		-