

ELASTIC STRESSES AND DEFLECTIONS FOR FLAT CIRCULAR PLATES WITH $D/t \geq 4$ UNDER UNIFORM PRESSURE

1. NOTATION

Both SI and British units are quoted but any coherent system of units may be used

D	diameter of plate	m	in
E	Young's modulus	N/m ²	lbf/in ²
f_e	von Mises-Hencky equivalent stress $= \left[\frac{1}{2} \left\{ (f_1 - f_2)^2 + (f_2 - f_3)^2 + (f_3 - f_1)^2 \right\} \right]^{1/2}$	N/m ²	lbf/in ²
f_1, f_2, f_3	principal stresses	N/m ²	lbf/in ²
f_T	maximum total stress due to bending and membrane tension effects at centre	N/m ²	lbf/in ²
f_{Tr}	maximum total radial stress due to bending and membrane tension effects at edge	N/m ²	lbf/in ²
$f_{T\theta}$	maximum total tangential stress due to bending and membrane tension effects at edge	N/m ²	lbf/in ²
k_E	ratio of equivalent stress of thick plate to equivalent stress of thin plate		
$k_e, k_T, k_{Tr}, k_{T\theta}$	ratios of stresses in plate edge elastically restrained against rotation to stresses in plate with edge fixed against rotation		
k_δ	ratio of deflection of plate with edge elastically restrained against rotation to deflection of plate with edge fixed against rotation		
k_Δ	ratio of deflection of thick plate to deflection of thin plate		
p	pressure on plate	N/m ²	lbf/in ²
t	thickness of plate	m	in
δ	maximum deflection of plate	m	in
λ	rotational restraint provided by edge support	N m/m rad	lbf in/in rad
<i>Convention:</i>	Tensile stresses are positive, compressive stresses are negative. A negative sign in the figures denotes a compressive stress (for example, $-f_{T\theta}$).		

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2. INTRODUCTION

Curves are given that enable the maximum stresses and deflection to be determined for initially flat circular plates of uniform thickness under uniform pressure, with various edge conditions.

The curves apply only to stresses within the elastic range of the material.

For thin plates ($D/t \geq 20$), $(f/E)(D/t)^2$ and δ/t are plotted against $(p/E)(D/t)^4$ on Figures 1 and 2, stresses, deflection and pressure being reduced to non-dimensional forms. Any consistent system of units may therefore be used.

For thick plates ($4 \leq D/t < 20$), factors k_Δ and k_E are plotted against t/D on Figures 4 and 5 respectively.

For plates with edges elastically restrained against rotation, factors k on stresses and deflection are plotted against $\lambda D/Et^3$ on Figure 3.

Table 2.1 indicates which figures should be used for a given edge condition.

TABLE 2.1

Edge condition		Number of figure giving				
Rotation	Translation	f_e	f_T	f_{Tr}	$f_{T\theta}$	δ
fixed	fixed or free	1	1	1	1	1 and 4
free	fixed or free	2	2	—	2	2 and 4
elastic	fixed or free	1, 3 and 5	1 and 3	1 and 3	1 and 3	1, 3 and 4

3. USE OF THE CURVES

For thin plates ($D/t \geq 20$), Figures 1 and 2 give values of maximum total stresses and deflection for plates with fixed and simply-supported edges respectively.

When the edge is elastically restrained against rotation, the values of stresses and deflection obtained from Figure 1 must be multiplied by factors k_e , k_T , k_{Tr} , $k_{T\theta}$ and k_δ obtained from 3 for the given value of $\lambda D/Et^3$.

$$(\text{For example, } f_{T(\text{elastic restraint})} = f_{T(\text{fixed})} \times k_T.)$$

The curves of Figure 3 may be used for values of $(p/E)(D/t)^4$ up to 5 when $\lambda = 0$ increasing to 30 when $\lambda = \infty$. The derivation of the curves in Figure 3 assumes that the membrane stresses are negligible in comparison to the bending stresses. Beyond the limiting values of $(p/E)(D/t)^4$ this is no longer true. Reference should then be made to Item No. 65003 for larger deflections of thin plates.

For thick plates ($4 \leq D/t < 20$) values of stress and deflection obtained by the method given must be multiplied by factors dependent upon t/D . Only in the cases of δ and f_e do these factors exceed about five per cent. Figure 4 gives the factor k_Δ so that

$$\delta_{(thick\ plate)} = \delta_{(thin\ plate)} \times k_\Delta.$$

Figure 5 gives the factor k_E for $\lambda D/Et^3 \geq 1.4$ so that

$$f_{e(thick\ plate)} = f_{e(thin\ plate)} \times k_E.$$

For $\lambda D/Et^3 < 1.4$, the factor $k_E = 1.0$.

When plastic deformation sets in at the edge of plates with fixed or elastic rotational edge restraint, the restraint conditions rapidly approach those for a plate with its edge free to rotate.

4. LOCATION OF MAXIMUM TOTAL STRESSES

f_T is a tensile stress on the unloaded face of the plate. f_{Tr} is a tensile stress on the loaded face of the plate. Table 4.1 gives the location of $f_{T\theta}$.

TABLE 4.1

Edge condition		$f_{T\theta}$	
Rotation	Translation	Plate face	Stress
fixed	fixed	loaded	tension
fixed	free	unloaded	compression
free	fixed	unloaded	tension
free	free	loaded	compression

5. DERIVATION

1. TIMOSHENKO, S. *Theory of elasticity*. Second edition. Chapter 13. McGraw-Hill, New York, 1957.
2. TIMOSHENKO, S. *Theory of plates and shells*. Second edition. Chapters 3 and 13. McGraw-Hill, New York, 1959.

6. EXAMPLE

A flat circular plate with its edge elastically restrained against rotation and fixed against translation is subjected to a uniform pressure of 2000 lbf/in².

It is required to find the maximum deflection and total stresses in the plate if $D = 2.0$ in, $t = 0.25$ in. The rotational restraint along the edge is $\lambda = 350\,000$ lbf/in rad. E for the plate material is 30×10^6 lbf/in².

$$\frac{P}{E} \left(\frac{D}{t} \right)^4 = 0.273, \quad E \left(\frac{t}{D} \right)^2 = 469\,000 \text{ lbf/in}^2,$$

$$\frac{\lambda}{E} \frac{D}{t^3} = 1.49, \quad \frac{t}{D} = 0.125.$$

Figures 1 and 3 are used to find the total stresses.

From Figure 1, for a fully fixed edge and $\frac{P}{E} \left(\frac{D}{t} \right)^4 = 0.273$,

$$\frac{f_T}{E} \left(\frac{D}{t} \right)^2 = 0.034.$$

From Figure 3, at $\frac{\lambda}{E} \frac{D}{t^3} = 1.49$,

$$k_T = 1.20.$$

Hence, $f_T = 0.034 \times 1.20 \times 469\,000$
 $= 19\,130 \text{ lbf/in}^2.$

Similarly, $\frac{f_{Tr}}{E} \left(\frac{D}{t} \right)^2 = 0.052$ and $k_{Tr} = 0.86$.

Hence, $f_{Tr} \times k_{Tr} = 0.052 \times 0.86 \times 469\,000$
 $= 21\,000 \text{ lbf/in}^2,$

$$\frac{f_{T\theta}}{E} \left(\frac{D}{t} \right)^2 = 0.016 \quad \text{and} \quad k_{T\theta} = 0.54.$$

Hence, $f_{T\theta} \times k_{T\theta} = 0.016 \times 0.54 \times 469\,000$
 $= 4050 \text{ lbf/in}^2.$