

ELASTIC STRESSES AND DEFLECTIONS FOR FLAT CIRCULAR PLATES WITH $D/t \geq 20$ UNDER UNIFORM PRESSURE

1. NOTATION

Both SI and British units are quoted but any coherent system of units may be used

D	diameter of plate	m	in
E	Young's modulus	N/m ²	lbf/in ²
f_e	von Mises-Hencky equivalent stress $= (f_1^2 + f_2^2 - f_1 f_2)^{1/2}$	N/m ²	lbf/in ²
f_1, f_2	principal stresses	N/m ²	lbf/in ²
f_m	stress in the median plane of the plate at the centre	N/m ²	lbf/in ²
f_{mr}	radial stress in the median plane of the plate at the edge	N/m ²	lbf/in ²
$f_{m\theta}$	tangential stress in the median plane of the plate at the edge	N/m ²	lbf/in ²
f_T	maximum total stress due to bending and membrane tension effects at the centre	N/m ²	lbf/in ²
f_{Tr}	maximum total radial stress due to bending and membrane tension effects at the edge	N/m ²	lbf/in ²
$f_{T\theta}$	maximum total tangential stress due to bending and membrane tension effects at the edge	N/m ²	lbf/in ²
p	pressure on plate	N/m ²	lbf/in ²
t	thickness of plate	m	in
δ	maximum deflection of plate	m	in
λ	rotational restraint provide by edge support	N m/m rad	lbf in/in rad
<i>Convention:</i>	Tensile stresses are positive, compressive stresses are negative. A negative sign in the figures denotes a compressive stress (for example, $-f_{T\theta}$).		

2. INTRODUCTION

Curves are given that enable the maximum deflection and stresses to be determined for initially flat circular plates under uniform pressure, with various edge conditions. The curves apply only to stresses within the elastic range of the material.

In the figures, $(f/E)(D/t)^2$ and δ/t are plotted against $(p/E)(D/t)^4$ for various values of the elastic edge restraint $\lambda D/Et^3$; stresses, deflection and pressure are reduced to non-dimensional form. Any consistent system of units may therefore be used.

Table 2.1 indicates which figures should be used for a given edge condition.

TABLE 2.1

Edge condition		Number of figure giving							
Rotation	Translation	f_e	f_m	f_{mr}	$f_{m\theta}$	f_T	f_{Tr}	$f_{T\theta}$	δ
fixed	fixed	11	4	6	9	4	6	9	2
fixed	free	10	3	—	8	3	5	7	1
free	fixed	11	4	6	9	4	6	9	2
free	free	10	3	—	8	3	5	7	1
elastic	fixed	11	4	6	9	4	6	9	2
elastic	free	10	3	—	8	3	5	7	1

3. USE OF THE CURVES

The maximum deflection in non-dimensional form may be obtained directly from Figures 1 and 2. Figures 3 to 11 give total and median plane stresses in non-dimensional form.

The bending stresses may be calculated by subtracting the stresses in the median plane from the corresponding total stresses.

When the edge is free in translation, the stress in the median plane is zero at the edge so that f_{Tr} arises purely from bending of the plate. The total and median plane stresses in the tangential direction ($f_{T\theta}$ and $f_{m\theta}$ respectively) are compressive for this edge condition.

For results lying in the small deflection linear region of the curves it is preferable to consult Item No. 65002 for accuracy in reading.

When plastic deformation sets in at the edge of a plate with fixed or elastic rotational edge restraint, conditions rapidly approach those for a plate with its edge free to rotate.

4. LOCATION OF TOTAL STRESSES

f_T is a tensile stress on the unloaded face of the plate. f_{Tr} is a tensile stress on the loaded face of the plate. Table 4.1 gives the location of $f_{T\theta}$.

TABLE 4.1

Edge condition		$f_{T\theta}$	
Rotation	Translation	Plate face	Stress
fixed	fixed	loaded	tension
fixed	free	unloaded	compression
free	fixed	unloaded	tension
free	free	loaded	compression

5. DERIVATION

1. WAY, S. Bending of circular plates with large deflections. *Trans Am. Soc. mech. Engrs*, Vol. 56, p. 627, 1934.

6. EXAMPLE

A flat circular plate with its edge elastically restrained against rotation and fixed against translation is subjected to a uniform pressure of 25 lbf/in².

It is required to find the total, median plane, and bending stresses at the edge and centre of the plate and its maximum deflection if $D = 54$ in, $t = 0.5$ in. The rotational restraint along the edge is $\lambda = 75\,000$ lbf in/in rad. E for the plate material is 30×10^6 lbf/in².

The parameters required for use of the Data Sheet are:

$$\frac{P}{E} \left(\frac{D}{t} \right)^4 = 113, \quad E \left(\frac{t}{D} \right)^2 = 2580 \text{ lbf/in}^2$$

and $\frac{\lambda}{E} \frac{D}{t^3} = 1.08.$

Figures 4, 6 and 9 are used to find the total and membrane stresses.

From Figure 4,

$$\text{for } \frac{p}{E} \left(\frac{D}{t} \right)^4 = 113 \quad \text{and} \quad \frac{\lambda}{E} \frac{D}{t^3} = 1.08,$$

$$\frac{f_T}{E} \left(\frac{D}{t} \right)^2 = 12.3.$$

$$\begin{aligned} \text{Hence, } f_T &= 12.3 \times 2580 \\ &= 31\,700 \text{ lbf/in}^2. \end{aligned}$$

Similarly, from Figure 6,

$$\frac{f_{Tr}}{E} \left(\frac{D}{t} \right)^2 = 13.7.$$

$$\begin{aligned} \text{Hence, } f_{Tr} &= 13.7 \times 2580 \\ &= 35\,300 \text{ lbf/in}^2. \end{aligned}$$

From Figure 9,

$$\frac{f_{T\theta}}{E} \left(\frac{D}{t} \right)^2 = 2.2.$$

$$\begin{aligned} \text{Hence, } f_{T\theta} &= 2.2 \times 2580 \\ &= 5670 \text{ lbf/in}^2. \end{aligned}$$

From Figure 4,

$$\frac{f_m}{E} \left(\frac{D}{t} \right)^2 = 4.0.$$

$$\begin{aligned} \text{Hence, } f_m &= 4.0 \times 2580 \\ &= 10\,300 \text{ lbf/in}^2. \end{aligned}$$

From Figure 6,

$$\frac{f_{mr}}{E} \left(\frac{D}{t} \right)^2 = 2.4.$$

$$\begin{aligned} \text{Hence, } f_{mr} &= 2.4 \times 2580 \\ &= 6190 \text{ lbf/in}^2. \end{aligned}$$

From Figure 9,

$$\frac{f_{m\theta}}{E} \left(\frac{D}{t} \right)^2 = 0.70.$$

Hence,

$$\begin{aligned} f_{m\theta} &= 0.70 \times 2580 \\ &= 1800 \text{ lbf/in}^2. \end{aligned}$$

All the above stress are positive and therefore tensile.

To find the bending stresses:

$$\text{Bending stress at centre} = f_T - f_m = 21\,400 \text{ lbf/in}^2.$$

$$\text{Radial bending stress at edge} = f_{Tr} - f_{mr} = 29\,110 \text{ lbf/in}^2.$$

$$\text{Tangential bending stress at edge} = f_{T\theta} - f_{m\theta} = 3870 \text{ lbf/in}^2.$$

Since there is no curve for $\lambda D/Et^3 = 1.08$ on Figure 11, the expression given in the notation will be used to obtain the von Mises-Hencky equivalent stresses at the centre and edge.

At the centre $f_e = f_T$.

Hence,

$$f_e = 31\,700 \text{ lbf/in}^2.$$

At the edge f_{Tr} is a tensile stress on the loaded face. Inspecting Figure 9, the curve for $(f_{T\theta}/E)(D/t)^2$ when $\lambda D/Et^3 = 1.08$ is seen to lie between the minimum value curve $\lambda D/Et^3 = 0.6$ and the curve for $\lambda D/Et^3 = \infty$. Hence, from Table 4.1, $f_{T\theta}$ is a tensile stress on the loaded face.

Therefore, for the loaded face at the edge

$$f_1 = f_{Tr} = 35\,300 \text{ lbf/in}^2$$

and

$$f_2 = f_{T\theta} = 5670 \text{ lbf/in}^2.$$

Hence, using the expression given in the notation,

$$f_e = 32\,800 \text{ lbf/in}^2.$$

For the unloaded face at the edge, the bending component of the radial stress is compressive.

Hence,

$$\begin{aligned} f_1 &= \text{radial bending stress} + f_{mr} \\ &= -29\,110 + 6190 \\ &= -22\,920 \text{ lbf/in}^2. \end{aligned}$$

In this example the bending component of tangential stress is also compressive,

$$\begin{aligned}\text{Hence, } f_2 &= \text{tangential bending stress} + f_{m\theta} \\ &= -3870 + 1800 \\ &= -2070 \text{ lbf/in}^2.\end{aligned}$$

Hence, using the expression given in the notation,

$$f_e = 22\,000 \text{ lbf/in}^2.$$

The maximum value obtained is therefore $f_e = 32\,800 \text{ lbf/in}^2$ at the edge of the loaded face. The note on Figure 11 states that, for the value $\lambda D/Et^3 = 1.08$ used, a greater stress may occur elsewhere in the plate. A high estimate of the stress obtained may be found by using the curve $\lambda D/Et^3 = 10$ on Figure 11.

$$\begin{aligned}\text{Hence, } \frac{f_e}{E} \left(\frac{D}{t} \right)^2 &= 16.4. \\ f_e &= 16.4 \times 2580 \\ &= 42\,300 \text{ lbf/in}^2.\end{aligned}$$

Figure 2 is used to find the maximum deflection.

$$\begin{aligned}\text{For } \frac{p}{E} \left(\frac{D}{t} \right)^4 &= 113 \quad \text{and} \quad \frac{\lambda}{E} \frac{D}{t^3} = 1.08, \\ \frac{\delta}{t} &= 1.04. \\ \text{Hence, } \delta &= 1.04 \times 0.5 \\ &= 0.52 \text{ in.}\end{aligned}$$