

DESIGN FOR MINIMUM WEIGHT. STRUTS OF UNIFORM SECTION. I-SECTIONS

1. NOTATION

$2d$	width of flanges of strut	m	in
$2d_0$	width of flanges of elastic strut of least weight	m	in
E	Young's modulus of strut material	N/m ²	lbf/in ²
E_t	tangent modulus at stress f	N/m ²	lbf/in ²
f	average stress in strut at failure	N/m ²	lbf/in ²
f_n	stress at which $E_t = \frac{1}{2}E$	N/m ²	lbf/in ²
f_0	average stress at failure in elastic strut of least weight	N/m ²	lbf/in ²
h	distance between centroids of flanges of strut	m	in
h_0	distance between centroids of flanges of elastic strut of least weight	m	in
l	actual length of strut	m	in
l'	equivalent length of strut (see Data Sheet Struct. 01.01.01)	m	in
m	material characteristic (see Item No. 76016)		
P	end load in strut at failure	N	lbf
t_d	thickness of flanges of strut	m	in
t_{d0}	thickness of flanges of elastic strut of least weight	m	in
t_h	thickness of web of strut	m	in
t_{h0}	thickness of web of elastic strut of least weight	m	in
W	weight of strut	N	lbf
W_{min}	minimum weight of strut	N	lbf

Both SI and British units are quoted but any coherent system of units may be used.

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2. INTRODUCTION

This Data Sheet gives the cross-sectional dimensions for minimum weight of struts of uniform I-section and given length. Information is also presented on the weight penalty incurred by restricting any one of the cross-sectional dimensions.

Figure 1 gives values of d_0/l' , h_0/l' , t_{d0}/l' and t_{h0}/l' against P/EI'^2 . Figures 2, 3 and 4 give values of d/d_0 and h/h_0 , t_d/t_{d0} and t_h/t_{h0} against f_0/f_n for various values of m . Figures 5, 6, 7 and 8 give values of W/W_{min} and the three unspecified dimensional ratios against d/d_0 , h/h_0 , t_d/t_{d0} and t_h/t_{h0} respectively for elastic struts with a dimensional restriction.

It has been assumed throughout that the value of l' is the same for overall buckling with respect to either principal axis of the section.

In deriving Figure 1, the effect of shear deflection on overall instability of the strut in the plane of the web has been included. The proportions of the strut of least weight are such that interaction between the two local modes of buckling is negligible. No allowance was made for eccentricity or overall curvature of the strut. The effect of a practical degree of local irregularity is negligible.

In deriving Figures 2, 3 and 4 the appropriate modulus effective in each mode of plastic buckling has been used (Derivation 1). These figures are strictly valid only in the region where there is no shear deflection of the strut. When P/EI'^2 exceeds 10^{-6} , the curves are approximately valid, the degree of approximation increasing with f_0/f_n .

Figures 5 to 8 are strictly valid for elastic struts in which there is no shear deflection. In materials with low to moderate proof strength, the shear deflection of an I-section strut is negligible within the elastic range.

Item No. 66008 gives information on the minimum weights of struts of uniform section in various materials.

3. DIMENSIONS OF STRUT OF LEAST WEIGHT

To determine the dimensions of a strut of least weight, the material is first assumed to be perfectly elastic. The cross-sectional dimensions d_0 , h_0 , t_{d0} and t_{h0} are obtained from Figure 1 for the appropriate value of P/EI'^2 . The stress f_0 is then obtained from the relation

$$f_0 = \frac{P}{4d_0t_{d0} + h_0t_{h0}}.$$

If f_0 is above the elastic limit, the values of f_n and m appropriate to the strut material are used to obtain d/d_0 , h/h_0 , t_d/t_{d0} and t_h/t_{h0} from Figures 2, 3 and 4. The actual dimensions d , h , t_d and t_h may then be evaluated.

4. WEIGHT PENALTY FOR STRUTS WITH DIMENSIONAL RESTRICTION

In particular cases, one of the cross-sectional dimensions may be dictated by considerations other than design for minimum weight. Figures 5, 6, 7 and 8 show the least value of W/W_{min} that can be achieved when one dimension is initially specified. Each figure also gives the value of the remaining section dimensions that must be used and indicates the modes of buckling that occur.

If d is specified, d/d_0 is evaluated. Then W/W_{min} , h/h_0 , t_d/t_{d0} and t_h/t_{h0} are obtained from Figure 5.

Similarly, Figure 6 is used if h is specified, Figure 7 if t_d is specified and Figure 8 if t_h is specified, noting the scale changes that occur at values of unity on the abscissae.

Figures 5, 6, 7 and 8 apply only to elastic struts. In particular, the value of W_{min} is that appropriate to an elastic strut of least weight. If the dimensionally restricted strut is indeed elastic but the strut of least weight is inelastic, the quoted value of W/W_{min} is misleading. In this case the unspecified cross-sectional dimensions are obtained from the appropriate Figure 5, 6, 7 or 8. The average stress in the dimensionally restricted strut can then be calculated and so can the stress in the strut of least weight. The realistic weight penalty for the dimensionally restricted strut is then given by

$$\frac{W}{W_{min}} = \frac{\text{stress in strut of least weight}}{\text{stress in dimensionally restricted strut}}.$$

5. DERIVATION

1. GERARD, G. BECKER, H. Handbook of structural stability. Part I: Buckling of flat plates. NACA TN 3781, 1955.
2. BECKER, H. Handbook of structural stability. Part II: Buckling of composite elements. NACA TN 3782, 1955.
3. COX, H.L. *The design of structures of least weight*. Pergamon, Oxford, 1965.

6. EXAMPLES

6.1 Example 1

It is required to determine the cross-sectional dimensions of an I-section strut to carry an end load of 200 000 lbf over a distance of 30 in. One end of the strut is fixed and the other is free. The material properties are

$$E = 11.1 \times 10^6 \text{ lbf/in}^2; \quad m = 18; \quad f_n = 62\,500 \text{ lbf/in}^2.$$

From Data Sheet 01.01.01, for one fixed and one free end

$$l' = 2l = 60 \text{ in.}$$

Hence,

$$\frac{P}{El'^2} = \frac{200\,000}{11.1 \times 10^6 \times 60^2} = 5.05 \times 10^{-6}.$$

From Figure 1,

$$\frac{d_0}{l'} = 0.042, \quad \frac{h_0}{l'} = 0.052,$$

$$10^3 \left(\frac{t_{d0}}{l'} \right) = 5.0, \quad 10^3 \left(\frac{t_{h0}}{l'} \right) = 1.52.$$

Hence,

$$\begin{aligned}
 d_0 &= 0.042 \times 60 &= 2.52 \text{ in ,} \\
 h_0 &= 0.052 \times 60 &= 3.12 \text{ in ,} \\
 t_{d0} &= 5.0 \times 10^{-3} \times 60 &= 0.30 \text{ in ,} \\
 t_{h0} &= 1.52 \times 10^{-3} \times 60 &= 0.091 \text{ in ,} \\
 f_0 &= \frac{200\ 000}{4 \times 2.52 \times 0.30 + 3.12 \times 0.091} &= 60\ 500 \text{ lbf/in}^2
 \end{aligned}$$

and

$$\frac{f_0}{f_n} = \frac{60\ 500}{62\ 500} = 0.969 .$$

From Figures 2, 3 and 4, interpolating for $m = 18$,

$$\begin{aligned}
 \frac{d}{d_0} &= \frac{h}{h_0} = 1.053 , \\
 \frac{t_d}{t_{d0}} &= 1.021, & \frac{t_h}{t_{h0}} &= 1.042 .
 \end{aligned}$$

Hence,

$$\begin{aligned}
 d &= 1.053 \times 2.52 &= 2.66 \text{ in ,} \\
 h &= 1.053 \times 3.12 &= 3.29 \text{ in ,} \\
 t_d &= 1.021 \times 0.30 &= 0.306 \text{ in ,} \\
 t_h &= 1.042 \times 0.091 &= 0.095 \text{ in ,} \\
 f &= \frac{200\ 000}{4 \times 2.66 \times 0.306 + 3.29 \times 0.095} &= 56\ 000 \text{ lbf/in}^2 .
 \end{aligned}$$

6.2 Example 2

It is required to determine the dimensions of the strut and the weight penalty associated with the restriction if the condition that d shall not exceed 1.8 in is added to the specification of the preceding problem.

Since d is specified, Figure 5 is used.

$$\frac{d}{d_0} = \frac{1.8}{2.52} = 0.715 .$$

Hence, from Figure 5,

$$\begin{aligned}
 \frac{W}{W_{min}} &= 1.825 , & \frac{h}{h_0} &= 0.93 , \\
 \frac{t_d}{t_{d0}} &= 2.72 , & \frac{t_h}{t_{h0}} &= 0.68 .
 \end{aligned}$$