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Friction losses for fully-developed flow in straight pipes

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Fluid Mechanics and Thermodynamics Committee of the Institution of Mechanical Engineers, which had at that time the following constitution:

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The Data Item was subsequently incorporated into the Fluid Mechanics, Internal Flow Sub-series, under the guidance of the Internal Flow Panel, which first met in 1979 and had the following membership:

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FRICITION LOSSES FOR FULLY-DEVELOPED FLOW IN STRAIGHT PIPES

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FRICITION LOSSES FOR FULLY-DEVELOPED FLOW IN STRAIGHT PIPES

1. NOTATION AND UNITS

Any coherent set of units may be used. Three such sets are indicated below.

		SI	British	
			<i>fps</i>	<i>technical</i>
A	internal cross-sectional area	m ²	ft ²	ft ²
D	internal diameter of pipe of circular cross section	m	ft	ft
D_E	equivalent diameter (also known as hydraulic diameter) of pipe of non-circular cross section = $4A/P$	m	ft	ft
f	Fanning friction factor	—	—	—
f_c	Fanning friction factor for flow in pipe of circular cross section	—	—	—
g	acceleration due to gravity	m/s ²	ft/s ²	ft/s ²
h	loss of head of flowing fluid due to friction	m	ft	ft
K_2	constant defined by Equation (3.3)	—	—	—
K_3	constant used in Table 9.1 and Figures 2a to 2e to determine D_E	—	—	—
L	length of pipe	m	ft	ft
P	internal perimeter of pipe	m	ft	ft
p	pressure	N/m ²	pdl/ft ²	lbf/ft ²
Δp	pressure drop due to friction	N/m ²	pdl/ft ²	lbf/ft ²
Q	volume flow rate	m ³ /s	ft ³ /s	ft ³ /s
Re	Reynolds number = VD/ν	—	—	—
V	mean velocity = Q/A	m/s	ft/s	ft/s
w	variable defined by Equation (3.1)			
x	distance along pipe	m	ft	ft
y	variable used in Table 9.4	—	—	—

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ϵ	effective roughness height of commercial pipe	m	ft	ft
λ	friction factor ($= 4f$)	—	—	—
μ	dynamic viscosity*	$\left. \begin{matrix} \text{N s/m}^2 \\ \text{or} \\ \text{kg/m s} \end{matrix} \right\}$	$\left. \begin{matrix} \text{pdl s/ft}^2 \\ \text{or} \\ \text{lb/ft s} \end{matrix} \right\}$	$\left. \begin{matrix} \text{lbf s/ft}^2 \\ \text{or} \\ \text{slug/ft s} \end{matrix} \right\}$
ν	kinematic viscosity $= \mu/\rho$	m^2/s	ft^2/s	ft^2/s
ρ	density	kg/m^3	lb/ft^3	slug/ft^3

* The symbol η is often used to denote dynamic viscosity.

2. INTRODUCTION

This Item is concerned with the calculation of frictional pressure losses associated with the flow of fluids in straight pipes running full. Data are given for both laminar flow and turbulent flow for a range of values of surface roughness. Both circular and non-circular cross sections are considered. The information given covers a large proportion of practical cases, but there are limitations to its validity and these are discussed in Section 7.

Two methods of solution of pipe flow problems are outlined. Their distinguishing features are

- I formulae for calculating the friction factor (Section 3.2),
- II a friction factor - Reynolds number chart (Figure 1).

Guidance is given on the most appropriate choice of method for various types of problem. In addition, Appendix A introduces a computer program that calculates any of the following parameters, assuming that the other three are known:

- (i) pipe cross-sectional dimension (or ratio of dimensions),
- (ii) pipe length, L ,
- (iii) volume flow rate, Q ,
- (iv) and pressure loss in duct, Δp .

The program evaluates the friction factor from the more-recently developed Chen equation (Reference 26).

3. BASIC DATA

3.1 Pressure Drop Equations

The pressure drop due to friction, for flow in a pipe of any uniform cross section running full, is given by

$$\Delta p = 4f \frac{L}{D_E} \frac{1}{2} \rho V^2, \quad (3.1)$$

where the mean velocity, V , is given by

$$V = \frac{Q}{A}, \quad (3.2)$$

and the equivalent diameter, D_E , is given by

$$D_E = \frac{4A}{P}. \quad (3.3)$$

For a pipe of circular cross section, D_E is of course equal to D .

The friction factor, f , used in Equation (3.1) is dependent on Reynolds number, Re , the cross-sectional shape of the pipe and, in the turbulent flow regime, the relative roughness of the pipe surface, ε/D_E . For a circular pipe the friction factor is denoted by f_c . The Reynolds number is here defined as

$$Re = \frac{\rho V D_E}{\mu} = \frac{V D_E}{\nu}. \quad (3.4)$$

The pressure drop may be expressed as a loss of head of the flowing fluid, where

$$h = \frac{\Delta p}{\rho g}, \quad (3.5)$$

so Equation (3.1) can be written in the alternative form

$$h = 4f \frac{L}{D_E} \frac{V^2}{2g}. \quad (3.6)$$

Other common forms of Equation (3.1) and (3.6) use the friction factor, λ , in place of f where λ and f are related by the expression

$$\lambda = 4f. \quad (3.7)$$

3.2 Evaluation of Friction Factor for Flow in Pipes of Circular Cross Section

Any commercial pipe will have surface irregularities or roughnesses which can be represented by an effective roughness height, ε . Approximate values of this quantity for a number of commercially available pipes are given in Table 9.2.

Figure 1 presents f_c as a function of Re and ε/D for flow in commercial pipes. This diagram is based on the work of Colebrook and White (References 3 and 4) and Moody (Reference 7) and the curves are represented by the equations set out in this section.

The friction factor for laminar flow in a pipe of circular cross section is independent of roughness and is given by

$$f_c = \frac{16}{Re}. \quad (3.8)$$

This relation is valid for $Re < 2000$.

In the critical zone, in which transition from laminar to turbulent flow takes place, the friction factor is uncertain and there is therefore corresponding uncertainty in pressure drop estimates if the Reynolds number falls in this range. However, as confirmed by the measurements reviewed in ESDU TN 08008²⁷ and TN 08009²⁸, the friction factor is unlikely to fall outside limits given by extrapolations of the laminar line and the turbulent line corresponding to the relative roughness of the pipe surface.

The frictional relationships for turbulent flow may be summarised by the following formulae:

- (i) in the hydraulically smooth regime (see Figure 1) the Prandtl law for smooth pipes¹ is

$$\frac{1}{\sqrt{f_c}} = 4 \log_{10} (Re \sqrt{f_c}) - 0.4, \quad (3.9)$$

- (ii) in the hydraulically rough regime (see Figure 1) the von Kármán rough pipe formula² is

$$\frac{1}{\sqrt{f_c}} = 4 \log_{10} \left(3.7 \frac{D}{\varepsilon} \right) \text{ and} \quad (3.10)$$

- (iii) in the intermediate* zone, that is the zone between the hydraulically smooth and rough zones, the relation for the friction factor, due to Colebrook and White, is

$$\frac{1}{\sqrt{f_c}} = -4 \log_{10} \left\{ \frac{1.25}{Re \sqrt{f_c}} + \frac{\varepsilon}{3.7D} \right\}. \quad (3.11)$$

The mechanism of surface friction in turbulent flow is described in various text books, *e.g.* Reference 13. A review of the different measurements and correlations in the literature for friction factor in laminar, transitional and turbulent flows is provided in ESDU TN 08008²⁷ and TN 08009²⁸.

3.2.1 Approximations for the Friction Factor

In some circumstances it may be required to evaluate the friction factor without reference to Figure 1. In such cases the forms of Equations (3.9) and (3.11) are often inconvenient for engineering calculations, since f_c is given implicitly.

* This zone is frequently referred to in the technical literature as the transition zone. In order to avoid confusion with the process of transition from laminar to turbulent flow the use of the term has been avoided here.

An explicit form²⁴ which represents Equation (3.9) to within 1 per cent in the range $10^4 < Re < 2.5 \times 10^8$ is

$$f_c = [4 \log_{10} (Re / \{4.53 \log_{10} (Re) - 3.82\})]^{-2}. \quad (3.1a)$$

A simpler expression⁴ which represents Equation (3.9) to within 1 per cent in the range $10^4 < Re < 10^7$ is

$$f_c = [3.6 \log_{10} (Re / 7)]^{-2} = [1.56 \log_e (Re / 7)]^{-2}. \quad (3.1b)$$

Equation (3.11) cannot be simply replaced by an equation giving f_c explicitly. However, Equation (3.11) can be evaluated more conveniently using the following procedure:

- (i) Calculate

$$w = \frac{\varepsilon}{D} \frac{Re}{18.57} + \log_{10} \frac{Re}{5.02}. \quad (3.1)$$

- (ii) Evaluate y from Table 9.4.

- (iii) Calculate

$$f_c = \frac{1}{16 \left(y + \log_{10} \frac{Re}{5.02} - \log_{10} w \right)^2}. \quad (3.2)$$

Alternative explicit expressions for the friction factor can be found in ESDU TN 08008²⁷.

3.3 Corrections for Non-Circular Cross Section

For flow in pipes of non-circular cross section a further non-dimensional parameter, K_2 , is introduced.

The friction factor, f , for a non-circular pipe is generally not equal to the factor f_c for an equivalent circular pipe. Their ratio is defined as K_2 and is given by

$$K_2 = \frac{f}{f_c}, \quad (3.3)$$

where f and f_c are evaluated at the same Reynolds number.

It has been shown theoretically and experimentally that K_2 is independent of Reynolds number in laminar flows. Furthermore, experimental work has shown that K_2 is approximately independent of Reynolds number for turbulent flow in hydraulically smooth pipes. It is here assumed, in the absence of better information, that the same values can be applied in the intermediate and rough zones.

Data for K_2 for several common non-circular shapes are given in Table 9.1 and Figures 2a to 2e. For pipes of circular cross section, $K_2 = 1$.

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4. THE SOLUTION OF FLOW PROBLEMS

For both laminar and turbulent flows there are two basic types of problem. The first may be called the direct problem. In this case it is required to calculate Δp (or h), values for all the other variables which affect the problem being known. In the second type of problem, the inverse problem, Δp (or h) is known and it is required to calculate one of the other variables, such as Q , V or D_E , in terms of the given quantities.

This Item presents two methods of solution, in addition to the computer program introduced in Appendix A:

- (i) using the appropriate equations of Section 3.2 in conjunction with the equations of Section 3.1 and, for non-circular cross sections, Section 3.3,
- (ii) using Figure 1 in conjunction with the equations of Section 3.1 and, for non-circular cross sections, Section 3.3.

For hand calculation of the direct problem Method (ii) is the most convenient. It is applicable to laminar and turbulent flows, in circular or non-circular pipes (see Examples in Section 6). However, inverse problems are not easily solved using Methods (i) and (ii) and the program should be used.

If it is known for certain which flow regime prevails (*i.e.* laminar or turbulent), then Method (i) can be conveniently used. On the other hand, if there is any doubt it is usually quicker to use Method (ii).

The outline procedure for the two Methods* is now given.

(Given Q , L , ν and cross-sectional geometry; to find Δp or h)

- (i) Calculate V from Equation (3.2).
- (ii) If the duct is of non-circular cross section, calculate D_E from Equation (3.3) or use Table 9.1 and Figures 2a to 2e.
- (iii) If the duct is of non-circular cross section, determine K_2 from Table 9.1 or Figures 2a to 2e.
- (iv) Determine ε from Table 9.2.
- (v) Calculate ε/D_E and VD_E/ν .
- (vi) Select the appropriate relation from Equations (3.8) to (3.2) according to the ranges of those parameters and hence evaluate f_c (Method (i)) or read f_c from Figure 1 (Method (ii)).
- (vii) Calculate f using Equation (3.3).
- (viii) Hence calculate Δp or h using Equation (3.1) or (3.6) respectively.

* The procedure for the solution of the direct problem is outlined here. An iterative approach based on these procedures would be required for the solution of inverse problems using these Methods.

5. RAPID SOLUTIONS FOR SPECIFIED FLUIDS

If a large number of calculations of the types outlined in Section 4 have to be made for one particular fluid, it is frequently more convenient to make use of charts or tables which apply specifically to the one fluid. A list of references giving charts and tables of this kind is contained in Table 9.3, where the range of variables covered by each reference is indicated.

6. EXAMPLES

(1) Methods (i) and (ii), worked in British technical units

A gas of density $\rho = 0.00238$ slug/ft³ and kinematic viscosity $\nu = 14.86$ centistoke flows in a rectangular pipe, 4 in $\times$ 2 in, made from sheet metal. Determine the pressure drop over a length of 10 ft if the velocity is 120 ft/s using a coherent set of British units.

$$\nu = 14.86 \text{ centistoke} = 1.6 \times 10^{-4} \text{ ft}^2/\text{s}.$$

Using the notation of Table 9.1 and Figure 2a of the present Data Item,

$$\frac{a}{b} = \frac{2}{4} = 0.5.$$

From Figure 2a,

$$K_3 = 1.333.$$

$$D_E = a \times K_3 = 2 \times 1.333 = 2.67 \text{ in} = 0.223 \text{ ft}.$$

$$Re = \frac{VD_E}{\nu} = \frac{120 \times 0.223}{1.6 \times 10^{-4}} = 1.67 \times 10^5.$$

From Table 9.2, and taking an average value for sheet metals,

$$\epsilon = 6 \times 10^{-5} \text{ in},$$

$$\frac{\epsilon}{D_E} = \frac{6 \times 10^{-5}}{2.67} = 2.25 \times 10^{-5}.$$

f_c can be evaluated either using Method (i) or Method (ii).

Method (i)

Using Equation (3.1),

$$w = \frac{2.25 \times 1.67 \times 10^5}{10^5 \times 18.57} + \log_{10} \frac{1.67 \times 10^5}{5.02} = 4.724.$$

From Table 9.4, it is found that $w = 4.724$ lies between the tabulated values of $w = 2$ and $w = 10$. To calculate f_c to an accuracy of 1 per cent, the value of y corresponding to either $w = 2$ or $w = 10$ can be taken.

For $w = 2$, $y = 0.057$.