

# **Inertia cross-coupling during a rapid rolling manoeuvre. Theoretical background and discussion of simplified stability boundaries**

Endorsed by  
The Royal Aeronautical Society

## ESDU DATA ITEMS

Data Items provide validated information in engineering design and analysis for use by, or under the supervision of, professionally qualified engineers. The data are founded on an evaluation of all the relevant information, both published and unpublished, and are invariably supported by original work of ESDU staff engineers or consultants. The whole process is subject to independent review for which crucial support is provided by industrial companies, government research laboratories, universities and others from around the world through the participation of some of their leading experts on ESDU Technical Committees. This process ensures that the results of much valuable work (theoretical, experimental and operational), which may not be widely available or in a readily usable form, can be communicated concisely and accurately to the engineering community.

We are constantly striving to develop new work and review data already issued. Any comments arising out of your use of our data, or any suggestions for new topics or information that might lead to improvements, will help us to provide a better service.

## THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Dynamics Committee. This Committee first met in 1962 and has the following membership:

### Chairman

Mr R. Melling — Hawker Siddeley Aviation Ltd, Brough

### Members

Mr J. Arnall — Hawker Siddeley Aviation Ltd, Kingston-upon-Thames

Mr J.R. Collingbourne — Ministry of Aviation

Dr G.J. Hancock — Queen Mary College, London University

Miss L.K. Harrison — Ministry of Aviation

Mr G.F. Langdon — Aeroplane and Armament Experimental Establishment

Mr D.K. Mendela — Hawker Siddeley Aviation Ltd, Hatfield

Mr H.H.B.M. Thomas — Royal Aircraft Establishment, Farnborough

Mr C.F. Toms — Hawker Siddeley Aviation Ltd, Hatfield.

The members of staff of the Engineering Sciences Data Unit were:

Mr A.J. Ward Smith — Head of Fluid Mechanics and Thermodynamics Group

Mr J.A. Shaw — Assistant Director.

# **INERTIA CROSS-COUPLING DURING A RAPID ROLLING MANOEUVRE. THEORETICAL BACKGROUND AND DISCUSSION OF SIMPLIFIED STABILITY BOUNDARIES**

## **CONTENTS**

|                                                         | <b>Page</b> |
|---------------------------------------------------------|-------------|
| <b>1. NOTATION</b>                                      | <b>1</b>    |
| <b>2. INTRODUCTION</b>                                  | <b>1</b>    |
| <b>3. ANALYSIS</b>                                      | <b>1</b>    |
| <b>3.1 Equations of Motion</b>                          | <b>1</b>    |
| <b>3.2 Simplifications</b>                              | <b>2</b>    |
| <b>3.3 Conditions for Stability</b>                     | <b>3</b>    |
| <b>4. STABILITY BOUNDARIES</b>                          | <b>4</b>    |
| <b>5. THE INFLUENCE OF VARIOUS FACTORS ON STABILITY</b> | <b>5</b>    |
| <b>6. REFERENCES</b>                                    | <b>6</b>    |
| <b>FIGURE 1 Stability Boundaries with Zero Damping</b>  | <b>7</b>    |

## INERTIA CROSS-COUPPLING DURING A RAPID ROLLING MANOEUVRE. THEORETICAL BACKGROUND AND DISCUSSION OF SIMPLIFIED STABILITY BOUNDARIES

### 1. NOTATION

$G_4$  to  $G_0$  coefficients of the characteristic equation (see Section 3.3)

$-n_v^*$  “effective” yaw stiffness =  $-\left(n_v - l_v \frac{n_\xi}{l_\xi}\right)$

$p_0$  assumed constant roll rate

For other notation see Item No. 67002<sup>[1]</sup>.

### 2. INTRODUCTION

Item No. 67005<sup>[2]</sup> gives in broad terms a physical description of the phenomenon known as inertia cross-coupling in roll as an introduction to the subject matter of the present series of Data Items. The purpose of this Data Item is to present the equations of motion in the form usually employed for rolling manoeuvres when inertia coupling effects become important and, in particular, to discuss in detail the simplification that results from imposing the constraint that rate of roll is constant. The conditions for stability of the resulting motion are deduced. Further approximations are then introduced that lead to the conditions for stability as presented by Phillips<sup>[3]</sup> and the case of an aircraft with zero rotary damping terms is considered in more detail. The elegantly simple results for this case permit a ready discussion of the main factors affecting the stability and thereby supplement the qualitative discussion of Item No. 67005<sup>[2]</sup>. A stability boundary diagram is a convenient method of presenting such results and the form it takes for the case with zero damping is discussed.

### 3. ANALYSIS

#### 3.1 Equations of Motion

In the study of rapid rolling manoeuvres it is convenient to use equations of motion referred to principal body axes and these are readily obtained from Section 7 of Item No. 67003<sup>[4]</sup>. The following assumptions are made.

- (a) A datum condition is taken for which  $v_e = p_e = q_e = r_e = \Phi_e = \Psi_e = 0$ .
- (b) The engine rotors rotate about the x-axis.
- (c) Perturbations are small except for  $p'$ , the disturbance in rolling angular velocity, so that the only terms retained involving products of disturbance quantities are  $w'p'$ ,  $v'p'$ ,  $r'p'$  and  $q'p'$ .

Issued October 1966 - 7 pages  
 Reprinted with Amendment A, January 2004 - 7 pages  
 This page Amendment A