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Profile drag at the drag-rise condition of aerofoils having a specified form of upper-surface pressure distribution at this condition

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PROFILE DRAG AT THE DRAG-RISE CONDITION OF AEROFOILS HAVING A SPECIFIED FORM OF UPPER-SURFACE PRESSURE DISTRIBUTION AT THIS CONDITION

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PROFILE DRAG AT THE DRAG-RISE CONDITION OF AEROFOILS HAVING A SPECIFIED FORM OF UPPER-SURFACE PRESSURE DISTRIBUTION AT THIS CONDITION

1. NOTATION AND UNITS

		SI	British
C_{DD}	profile drag coefficient at drag-rise condition (M_D , C_{LD}), $(C_{DD})_u + (C_{DD})_l$		
$(C_{DD})_l$	component of profile drag coefficient due to lower surface		
$(C_{DD})_u$	component of profile drag coefficient due to upper surface		
C_{LD}	lift coefficient when $M_0 = M_D$		
c	aerofoil chord	m	ft
H	stagnation pressure in undisturbed stream	N/m ²	lbf/ft ²
M_0	free-stream Mach number		
M_D	drag-rise Mach number (see Section 3)		
p	local static pressure	N/m ²	lbf/ft ²
$(p/H)_{cr}$	pressure ratio at aerofoil crest		
R_c	Reynolds number based on chord length		
R_θ	Reynolds number based on momentum thickness and local values of velocity and kinematic viscosity		
$(R_\theta)_{tr}$	R_θ at transition		
t	aerofoil maximum thickness	m	ft
x	chordwise distance measured from leading edge	m	ft
x_R	chordwise station defining end of "roof-top"	m	ft
$(x_{tr})_l$	chordwise station identifying transition position on lower surface	m	ft
$(x_{tr})_u$	chordwise station identifying transition position on upper surface	m	ft
θ	boundary layer momentum thickness	m	ft

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2. INTRODUCTION

This Item presents the profile drag coefficient*, C_{DD} , at the drag-rise condition (M_D, C_{LD}), for particular families of aerofoils designed to combine a high drag-rise Mach number with a high lift coefficient for a given thickness/chord ratio (see Section 3). These aerofoils are selected members of the families of aerofoils that are described in detail in Derivation 1 and briefly in Section 3 herein.

Figures 1 to 18 present charts of the separate surface contributions, $(C_{DD})_u$ and $(C_{DD})_l$, to the profile drag coefficient of the above-mentioned families of aerofoils. The charts have been constructed from data computed³ for selected aerofoils. These aerofoils, identified by the designation described in Section 5.1, are listed, together with their values of lift coefficient at the drag-rise condition, in Table 10.1. In each Figure the separate surface contributions are presented in lattice form as functions of thickness/chord ratio, t/c , Reynolds number, R_c , and boundary layer transition position, x_{tr}/c , for a particular combination of drag-rise Mach number, M_D , and chordwise extent of the upper-surface roof-top, x_R/c .

Section 7 draws attention to some limitations of the particular families of aerofoils used in the derivation of this Item. These limitations, although they are of lesser importance in an initial design study, may be sufficiently important to require that modifications be made to members of these particular families, via their specification, in practical applications. To illustrate the principles involved in such a process, Section 9.2 describes how, for one particular design case, some improvement may be achieved in the factor governing the onset of turbulent boundary layer separation.

3. AEROFOIL DESIGN PHILOSOPHY

Derivation 1 presents charts giving the drag-rise characteristics (M_D, C_{LD}) for families of aerofoils specified by the form of the upper-surface pressure distribution at the drag-rise condition and the form of the thickness distribution. Any point on the charts corresponds to a particular camber line and incidence uniquely determinable from the known specifications, and Derivation 1 outlines a design procedure whereby the camber line and incidence and hence complete aerofoil geometry and pressure distribution may be obtained.

The drag-rise condition is that beyond which a rapid increase in shock-wave drag may be anticipated to ensue from further increases in either lift coefficient or free-stream Mach number. The drag-rise Mach number, used to establish the charts referred to, is based on the empirical criterion that the drag-rise is predicted to occur when the local Mach number at the crest of the aerofoil reaches 1.02 or, in terms of pressure ratio, when the shock-free flow crest pressure-ratio, $(p/H)_{cr}$, equals 0.515 (see Section 6.2(a) of Derivation 1 for details). Thus, provided that the empiricism involved can be generalised, it follows that an aerofoil will be free[†] of shock-wave drag and will exhibit a high drag-rise Mach number in combination with a high lift coefficient, for a given thickness/chord ratio, if the constraint is imposed that p equals $0.515H$ over the region in which the crest must lie. A particular form of pressure distribution complying with this constraint is the flat roof-top form of upper-surface pressure distribution in which a pressure plateau extends from the immediate vicinity of the leading edge to a given chordwise station, x_R/c , followed by a linear rise to the trailing edge. It is this type of pressure distribution that is fundamental to the aerofoils considered in this Item and in Derivations 1 and 2.

* Profile drag is sometimes divided into two components, namely, profile drag at zero lift and the increment in profile drag due to lift. The sum of these two components is considered throughout this Item.

† This also excludes the drag-creep, prior to the rapid drag-rise, that occurs on certain aerofoils whose pressure distribution and shape combine in such a way (see first Appendix of Derivation 6) that a local region of supersonic flow ahead of the crest is terminated by a significant drag-producing shock wave. This drag-creep cannot occur at the design incidence for the "roof-top" aerofoils considered here, although it may occur at higher incidences.

The chosen form for the upper-surface pressure distribution has the merit that, not only does it combine high values of drag-rise Mach number and lift coefficient for a given thickness/chord ratio, but it is to be anticipated that at the drag-rise condition it should also lead to relatively low values of profile drag contribution arising from the upper surface, which, especially at high values of C_{LD} , represents the larger of the two separate surface contributions.

3.1 Aerofoil Specification

The essential feature of the aerofoils considered here and in Derivations 1 and 2 is the flat roof-top form of upper-surface pressure distribution referred to earlier, in which a pressure plateau extends from the immediate vicinity of the leading edge to a given chordwise station, x_R/c , followed by a linear rise to the trailing edge. This basic specification for the upper-surface pressure distribution leads to a discontinuity in pressure gradient at x_R . Such a discontinuity is undesirable in practice since it leads to a rapid increase in shock-upstream Mach number in just-supercritical flow, which in turn leads to a rapid onset of shock-induced separation and buffeting. Thus some fairing of the upper-surface pressure distribution in the vicinity of x_R was incorporated into the specification, the details being given in Derivation 1.

To obtain a realistic pressure gradient for the linear pressure recovery it was necessary to choose values of trailing-edge pressure ratio compatible with real flow conditions. The local distortions due to viscosity are most apparent over the final portion of the chord, and in Derivation 1 it was found convenient to locate the trailing-edge pressure by calculating the upper-surface pressure distribution due to thickness in compressible inviscid flow at $M_0 = M_D$ and extrapolating this linearly from $x = 0.885c$ to the trailing edge. Further details of this process, together with the warning that this artifice is not intended to be a method for estimating the real trailing-edge pressure, are given in Derivation 1.

The specification is completed by associating the prescribed upper-surface pressure distribution with a chosen thickness distribution. In Derivation 1, the choice of thickness distribution was rather arbitrary, but for the sake of simplicity an upper-surface pressure distribution having a particular roof-top limit, x_R/c , was associated with a thickness distribution having the same property. Thus, since values of x_R/c varying from 0.3 to 0.6 in steps of 0.1 had been chosen for the upper-surface pressure distributions, the RAE series of thickness distributions were selected (since the sections RAE 101, 102, 103, and 104 are designed to have $x_R/c = 0.3, 0.4, 0.5$ and 0.6 , respectively).

4. AEROFOIL GEOMETRY AND PRESSURE DISTRIBUTION

The aerofoil geometry is specified implicitly by the drag-rise Mach number, the upper surface pressure distribution, the thickness distribution and the thickness/chord ratio. For inviscid flow it can be calculated, as outlined in Derivation 1, by what amounts to an inverse design procedure* that yields the camber line and the associated value of incidence. From the complete aerofoil geometry, the pressure distribution for the lower surface at the drag-rise condition can be calculated, by the method due to Weber⁴ employing the Wilby compressibility factor⁵ in the thickness terms, as in Derivation 1. Typical pressure distributions for aerofoils designed in the foregoing manner are illustrated in Figures 1 to 4 and Figure 6 of Derivation 2.

* It should be noted that the design procedure outlined in Derivation 1 takes no account of the effective displacement of the aerofoil surface due to the boundary layer. To achieve in viscous flow exactly the particular form of upper-surface pressure distribution specified would require the boundary layer to be taken into account. In this context the work of Powell¹⁰ would be of value (see Addendum A).

5. THE SELECTED AEROFOILS

Figures 1 to 18 have been based on calculations of profile drag at the drag-rise condition for only a limited number of the aerofoils implicit in the derivation of the drag-rise Mach number charts in Derivation 1. These aerofoils have been chosen such that their lift coefficients at the drag-rise Mach number fall roughly within the range $0.2 < C_{LD} < 1.0$, the upper limit of this range being considered to represent the extreme for which the method given in Derivation 1 is justified by available experimental data. The calculations were made for t/c varying from 0.06 to 0.18 in steps of 0.04 and M_D varying from 0.6 to 0.8 in steps of 0.05 for each of $x_R/c = 0.3, 0.4, 0.5$ and 0.6 . Within the limits stated for C_{LD} the chosen aerofoils cover the full range of the drag-rise Mach number charts of Derivation 1.

Table 10.1 lists the selected aerofoils (using the designation of Section 5.1), the lift coefficient at the drag-rise condition, C_{LD} , and the number of the Figure providing the relevant profile drag data. Details of t/c , x_R/c and M_D for each aerofoil are incorporated into the designation.

Details of the ordinates, as well as incidences and pressure distributions at the drag-rise condition in inviscid flow, are tabulated in Derivation 2.

5.1 Aerofoil Designation

The aerofoil designation is best described by means of an example:—

102–12–40–75,

where

102 denotes RAE series number

12 = 100 t/c ,

40 = 100 x_R/c ,

75 = 100 M_D .

6. BASIS OF FIGURES PRESENTING PROFILE DRAG

The separate surface components, $(C_{DD})_u$ and $(C_{DD})_l$, of profile drag coefficient, C_{DD} , at the drag-rise condition, were estimated for each aerofoil using the method due to Nash, Osborne and Macdonald³, with some modifications from later work⁷ at the NPL. Individual surface contributions to profile drag are presented to afford the designer the facility of accounting for differential transition positions on upper and lower surfaces.

The broken lines on Figures 1 to 18 indicate those regions in which a criterion developed by Stratford, and modified by Wilkinson to account for compressibility (see Appendix B of Derivation 1), predicts that the pressure rise from the end of the roof-top will induce boundary layer separation upstream of the trailing edge on the upper surface. The boundary layer calculations implicit in the method of Derivation 3 indicate that the lower surface remains free of separation for all the aerofoils presented here with the exception of the 104-18-60-70 aerofoil for a Reynolds number of 10^6 with transition positions forward of $0.6c$ (see Figure 16). Data for this region are omitted. It should be noted that the method of Derivation 3 is formulated for attached flow conditions and hence does not properly calculate the drag in separated flow, so that the drag data given in those areas for which separation is predicted are incorrect. Also, owing to a computer-imposed limitation preventing a smooth continuity of the calculated profile drag coefficient variation with Reynolds number (at fixed transition) well into the predicted separation regions, some extrapolation of the curves in these regions was necessary to complete the Figures.

Preston⁸ has shown that, for incompressible flow at least, a fully-turbulent boundary layer cannot exist until the local Reynolds number based on momentum thickness, R_θ , has reached a value of about 320. Nash *et al*³ have assumed this criterion to hold true for compressible flow by incorporating it in their method for estimating aerofoil profile drag in such a way that if transition is called for at a chordwise position at which $R_\theta < 320$, then the value of R_θ is stepped up to 320 for the start of the turbulent boundary layer. It was hoped in this way to take some account of the drag of the external triggering agent (*e.g.* local roughness, waviness or surface discontinuities) that would have to be present to induce transition at such a position. It has been shown⁹, however, that such a treatment can lead to improbable turbulent boundary layer developments for those cases in which step increments in $(R_\theta)_{tr}$ approaching the full 320 are called for. However, in view of the fact that the method of Derivation 3 provides estimates of profile drag coefficient in good agreement with experimental data for aerofoils tested at low Reynolds numbers with transition fixed near the leading edge by local roughness, the $(R_\theta)_{tr} \geq 320$ criterion has been retained for the present calculations.

In the light of the foregoing discussion it was thought helpful to indicate the boundaries at which $(R_\theta)_{tr} = 320$. These are shown in Figures 19a and 19b as plots of $\log_{10} R_c$ versus the appropriate transition position for which $(R_\theta)_{tr} = 320$. The boundaries are principally functions of drag-rise Mach number, the data spread indicated on the curves being due to the remaining aerofoil variables, namely t/c and x_R/c .

7. LIMITATIONS AND ACCURACY

For the given values of x_R/c and M_D , interpolation and extrapolation for t/c to obtain profile drag coefficient is a simple matter in most cases by virtue of the fact that the data have been plotted in lattice form in Figures 1 to 18. However, some cross-plotting, linear interpolation or linear extrapolation will be necessary if drag data are required for values of x_R/c and M_D other than those given. The corresponding values of C_{LD} can be obtained from Derivation 1.

It is estimated that the method of Derivation 3 (with the modifications of Derivation 7) is accurate to within about 3 or 4 per cent. Thus, a profile drag coefficient obtained from this Item for an aerofoil specified in Table 10.1 is also roughly within this accuracy, provided the flow remains fully attached. Interpolation or extrapolation for M_D , x_R/c or t/c might be expected to increase this error. Also, any attempt to use data in those regions for which flow separation is predicted would result in an increase in the error. It should be noted that in Derivation 3 the implication is that the stagnation point is at the leading edge of the aerofoil. This is not strictly true, of course, and leads to small errors in the separate surface profile drag values. However, the total aerofoil profile drag should not be significantly affected because the errors in upper- and lower-surface contributions are of opposite sign and similar magnitude.

Figures 19a and 19b show the boundaries for which $(R_\theta)_{tr} = 320$. Natural transition would not be expected to occur forward of positions indicated by these boundaries. It can be seen from Figures 19a and 19b that at flight Reynolds numbers these boundaries approach the leading edge, whereas for the Reynolds number range achievable in most wind-tunnel tests the boundaries are well aft and some form of external triggering agent (*e.g.* a band of distributed roughness) would be needed to provoke transition forward of the boundaries. An allowance for the drag of such triggering agents has been made in the data presented in Figures 1 to 18 (see Section 6).

It will be seen from Figures 1 to 18 that, for attached flow at least, the upper-surface contribution to profile drag coefficient, $(C_{DD})_u$, is not very sensitive to variations in t/c , other conditions being constant. This is because the upper-surface pressure distribution, except for the value of the trailing-edge pressure, is specified independently of t/c . In addition, t/c has only a small effect on trailing-edge pressure calculated by the method given in Section 3.1. The lower surface can therefore be optimised to some extent without significantly affecting the upper-surface contribution to the profile drag coefficient. Indeed, some form of

optimisation of both surfaces will usually be desirable in practice and, because of this, use of the data presented in this Item should be considered appropriate only to the early stages in such an optimisation process. In this respect the RAE series of thickness distributions are not considered completely satisfactory as they stand, primarily because of the discontinuity in surface curvature at $x = x_R$. The aerofoils used to derive the present Item, however, do not have such discontinuities in surface curvature at the rear of the roof-top on the upper surface, because the pressure distribution has been rounded off to avoid this (see Section 3.1). Any discontinuity arising from the use of the RAE series of thickness distributions would therefore appear on the lower surface only, but its magnitude would be governed by the discontinuities in surface curvature occurring on each surface of the thickness distribution.

Another feature of the RAE series of thickness distributions, which may be undesirable in practice both on aerodynamic and structural grounds, is that of the small leading-edge radii. In these respects small leading-edge radii can lead to laminar separation in the region of the leading-edge, and can also make the incorporation of leading-edge high-lift devices difficult.

A further characteristic of the aerofoils used to derive this Item that may require modification in practical application concerns the slope of the camber lines over their rearward portions. To facilitate the incorporation of trailing-edge controls or high-lift devices it may be desirable for an aerofoil to have a straight profile on upper and lower surfaces immediately forward of the trailing edge. This feature is not present in the aerofoils used to derive this Item owing to the continuously changing slope of the camber lines, so that it may be necessary at the design stage to achieve some sort of compromise between aerodynamic and engineering design.

The trailing-edge pressure, derived by the simple method described in Section 3.1 may not correspond to the true value in practice. In the longer term it could well be profitable to consider the finer detail of the pressure distribution near the trailing edge with a view to ascertaining whether in practice some control over the trailing-edge pressure, and hence the drag, can be achieved by detailed design. It is relevant to note here that unpublished work at the NPL has shown that, for the aerofoils considered in this Item, a change in upper-surface rear pressure gradient of about 5 per cent (via an appropriate change in the specified trailing-edge pressure) produces a change in the profile drag of the aerofoil of about 3 per cent in the worst case. It is stressed that the full method given in Derivation 3 (with the modifications of Derivation 7) should be used to obtain the profile drag coefficient in at least the later stages of the optimisation process. For these calculations revised estimates of the pressure distribution^{4, 5}, including in particular the effect of the boundary layer¹⁰, will probably be required.

When the design conditions have been met a check should be made to ensure that the off-design requirements* are satisfied.

* See Derivation 1 and Addendum A.

8. DERIVATION

1. ESDU Drag-rise Mach number of aerofoils having a specified form of upper-surface pressure distribution: charts and comments on design. ESDU Item No. 71019, December 1971. (Supersedes Item No. 67009.)
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OSBORNE, J.
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4. WEBER, J. The calculation of the pressure distribution on the surface of thick cambered wings and the design of wings with given pressure distribution. ARC R & M 3026, 1957.
5. WILBY, P.G. The calculation of sub-critical pressure distributions on symmetrical aerofoils at zero incidence. NPL Aero Rep. 1208, March 1967.
6. ESDU A method of estimating drag-rise Mach number for two-dimensional aerofoil sections. ESDU Transonic Data Memor. 6407, July 1964.
7. THOMPSON, B.G.J. Unpublished work. National Physical Laboratory, 1967.
8. PRESTON, J.H. The minimum Reynolds number for a turbulent boundary layer and the selection of a transition device. *J. fluid Mech.* Vol. 3, Part 4, pp. 373-384, January 1958.
9. CHAPPELL, P.D. Unpublished work. Royal Aeronautical Society, 1967.
10. POWELL, B.J. The calculation of the pressure distribution on a thick cambered aerofoil at subsonic speeds including the effect of the boundary layer. NPL Aero Rep. 1238, June 1967.

9. EXAMPLE

The following example illustrates in Section 9.1 how this Item may be used, in the initial design stages of an aerofoil, to choose an upper-surface pressure distribution and thickness distribution to meet specified conditions.

An indication of the various ways in which an aerofoil satisfying these conditions could be refined (via its specification) using Derivation 1 is given in Section 9.2.

9.1 Initial Design Study

Aerofoil Design Specification

- (i) The sectional lift coefficient should be 0.5 at a drag-rise Mach number of 0.70.
- (ii) The thickness/chord ratio should be at least 0.12, and preferably larger.
- (iii) For a Reynolds number of 2.5×10^7 , and with the assumption that the upper- and lower-surface transition positions are 0.2 and 0.3, respectively, the aerofoil should be free of separated flow.

Find the profile drag coefficient of an aerofoil satisfying the above specification.

By cross-plotting $C_{LD} \sim t/c$ data with $M_D = 0.70$ from Table 10.1 (or Figures 2 to 5 of Derivation 1), it is found that the first design condition can be met by the following combinations of x_R/c and t/c , or interpolation* between them:

x_R/c	0.3	0.4	0.5	0.6
t/c	0.105	0.123	0.140	0.156.

The second design condition is met by all except the first of these combinations.

From Table 10.1 it is found that the profile drag data for the aerofoils implied in the last three combinations (i.e. 102-12.3-40-70, 103-14-50-70 and 104-15.6-60-70, respectively) can be obtained from Figures 8, 12 and 16. Referring to these Figures, it can be seen that, of these particular alternatives, the first two are indicated as being free of separated flow at the specified design conditions, namely $R_c = 2.5 \times 10^7$, $(x_{tr}/c)_u = 0.2$ and $(x_{tr}/c)_l = 0.3$, thus satisfying the third design condition. Of the remaining two aerofoils the 103-14-50-70 will be chosen, for the purposes of this example, for an initial design study, because it satisfies the rider to the second design condition.

Upper surface

From the upper chart in Figure 12, with $t/c = 0.14$, $\log_{10} R_c = 7.4$ and $(x_{tr}/c)_u = 0.2$,

$$(C_{DD})_u = 0.00426.$$

Lower surface

From the lower chart in Figure 12, with $t/c = 0.14$, $\log_{10} R_c = 7.4$ and $(x_{tr}/c)_l = 0.3$,

$$(C_{DD})_l = 0.00220.$$

* If a strict adherence to the aerofoil specification (Section 3.1) on which this Item and Derivations 1 and 2 are based (namely, that an upper-surface thickness distribution having a particular roof-top limit is associated with a thickness distribution having the same property) is maintained, interpolation between the quoted combinations of x_R/c and t/c would involve the development of a new thickness distribution, because the RAE series of thickness distributions have been developed for only a limited range of integer values of $10x_R/c$. In practice, however, there is considerable freedom of choice in the thickness distribution that may be associated with any particular roof-top extent.

Thus the profile drag coefficient of the aerofoil designated 103-14-50-70 at the drag-rise condition is given by

$$\begin{aligned} C_{DD} &= (C_{DD})_u + (C_{DD})_l \\ &= 0.00426 + 0.00220 \\ &= 0.00646 \\ &\approx 0.0065. \end{aligned}$$

The ordinates, together with the incidence and inviscid flow pressure distribution at the drag-rise condition for the chosen aerofoil can be obtained from Table 6.5 of Derivation 2. For the more general case in which these data are required for an aerofoil other than those given in Table 10.1, approximate estimates can be obtained by interpolation in the data given in Tables 6.3 to 6.6 of Derivation 2, but for more accurate estimates the full procedure outlined in Derivation 1 should be used.

9.2 Refinement of the Design Study

Scrutiny of Figure 12, for the aerofoil designated 103-14-50-70 (*i.e.* the aerofoil selected for the initial design study of Section 9.1) at $R_c = 2.5 \times 10^7$, $(x_{tr}/c)_u = 0.2$ and $(x_{tr}/c)_l = 0.3$, reveals that this aerofoil is only just on the attached-flow side of the separation threshold. In a practical design it would be necessary to interpret this condition in terms of the Mach number and lift coefficient margins between the design point and the onset of separation effects. If the margins were found to be grossly inadequate it would be necessary to incorporate some substantial modification and, for the purpose of this example, reversion to the 102-12.3-40-70 aerofoil might be considered. If, however, the margins were only slightly inadequate, minor modifications could be made with the basic aim of reducing the upper-surface pressure gradient over the rear of the aerofoil. There are a number of ways in which this could be effected.

- (i) To employ an aerofoil having an $(x_R/c, t/c)$ combination in which the values of both parameters are less than the ones chosen for the initial design study of Section 9.1 but without necessarily reducing the value of x_R/c to 0.4. The reduction in x_R/c reduces the rear pressure gradient directly by increasing the chordwise extent over which the pressure recovery takes place. The reduction in t/c reduces the rear pressure gradient by lowering the trailing-edge pressure.
- (ii) To employ an aerofoil having a shorter length of roof-top only, but retaining the same thickness form.
- (iii) To employ an aerofoil with a lower thickness/chord ratio only, but retaining the same roof-top extent.
- (iv) To employ an aerofoil with a thickness distribution having a shorter length of roof-top, but to retain the same roof-top extent.

It should be noted that, of the above methods, only the first allows retention of the specified value of C_{LD} . Use of the other methods will result in changes to the value of C_{LD} , the magnitude of the change depending on the individual method.

For the purposes of this discussion, method (iv) will be employed to modify the design study of Section 9.1. The alternative thickness distributions that could be used are the RAE 101 or RAE 102 series. Since the use of either of these would imply lower-surface suctions of a greater magnitude over the forward part of the aerofoil, a check would have to be made that the pressure distribution on the lower surface was wholly subcritical (*i.e.* $(p/H)_{cr}$ not greater than 0.515) at the design condition. Boundaries for $(p/H)_{cr} = 0.515$ on the lower surface, for the alternative thickness distributions having a chordwise station corresponding