

ELASTIC STRESSES IN THE TORISPHERICAL HEAD OF A PRESSURE VESSEL OF UNIFORM THICKNESS

1. NOTATION

D	mean diameter of cylinder	m	in
f_i	stress in head inner surface at centre of crown	N/m ²	lbf/in ²
$f_{i\phi}$	maximum meridional stress in head inner surface remote from centre of crown	N/m ²	lbf/in ²
f_{nom}	nominal hoop stress in cylinder, $\frac{p(D-t)}{2t}$	N/m ²	lbf/in ²
f_o	stress in head outer surface at centre of crown	N/m ²	lbf/in ²
$f_{o\phi}$	minimum meridional stress in head outer surface remote from centre of crown	N/m ²	lbf/in ²
$f_{o\theta}$	minimum hoop stress in head outer surface remote from centre of crown	N/m ²	N/in ²
h	head depth measured from junction plane of knuckle and cylinder to median surface at centre of crown, $R - \sqrt{(R - D/2)(R + D/2 - 2r)}$	m	in
L	length of cylinder	m	in
L_{min}	theoretical minimum length of cylinder	m	in
p	uniform gauge pressure on head and cylinder	N/m ²	N/in ²
R	crown radius of head median surface	m	in
r	knuckle radius of head median surface	m	in
t	thickness of head and cylinder	m	in
Convention:	Tensile stresses are positive Compressive stresses are negative Internal pressure is positive.		

Both SI and British units are quoted but any coherent system of units may be used.

2. NOTES

Curves are given that enable the maximum and minimum (maximum compressive) stresses due to uniform pressure to be determined for an unpierced torispherical head closure to a cylindrical shell of the same thickness.

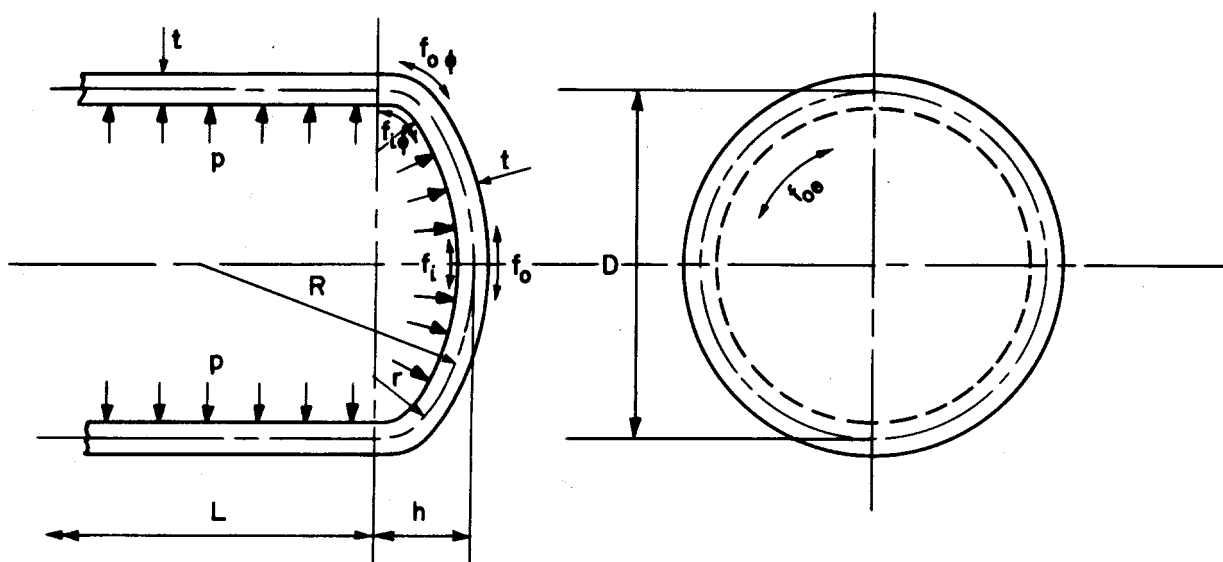
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In Figures 2 to 11 f/f_{nom} is plotted against h/D for various values of t/D . The curves are grouped for common values of r/D . Table 2.1 shows the figure to be used for a particular stress and r/D value.

TABLE 2.1

r/D	Number of figure giving:	
	$f_{i\phi}$, f_i & $f_{0\phi}$	f_0 & $f_{0\theta}$
0.04	2	3
0.06	4	5
0.08	6	7
0.12	8	9
0.16	10	11

The curves relate equally to shells under internal or external pressure but in the latter case it should be noted that f_{nom} is compressive and therefore negative. Sketch 2.1 shows the stress systems in the shell under uniform internal pressure.



Sketch 2.1

In most cases the maximum tensile stress in the head is $f_{i\phi}$ which occurs in the knuckle, or in the crown near the knuckle. In a few cases, notably thicker heads with a large knuckle radius but shallow crown (large t/D , large r/D , small h/D) the maximum tensile stress in the head may be f_0 which occurs at the crown centre. Generally the maximum compressive stress is $f_{0\phi}$ or $f_{0\theta}$.

The curves in this Item apply only to stresses within the elastic range of the material. They are based on linear thin shell theory and do not therefore apply above $t/D = 0.1$ or $t/r = 0.2$.

This Item applies only to heads with cylinders of sufficient length that negligible interaction occurs between the discontinuity effects of the two ends of the pressure vessel. The minimum cylinder lengths for which the Item may be used without encountering significant error due to interaction are shown on Figure 1, where L_{min}/D is plotted against t/D .

Head instability is not considered in this Item. For very thin heads under uniform internal pressure ($t/D \approx 0.001$) elastic instability can occur in the hoop direction in the region of the knuckle. This condition is considered in the references quoted.

It is possible for the cylinder under external pressure to collapse before the head stresses quoted here exceed the permissible stress, particularly for long thin cylinders (small t/D) with deep heads (large h/D). Data Sheet 04.09.01 (in the Structures Sub-Series) may be consulted on the collapse pressure of cylinders.

By comparison with experimental results in which the basic assumptions of the present analysis are approximately satisfied it appears that the maximum stress does not vary by more than 10 per cent from the theoretical values shown here.

3. DERIVATION

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4. REFERENCES

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3. THURSTON, G.A. Buckling of cylindrical shell end closures by internal pressure. NASA CR-540, 1966.

5. EXAMPLE

A cylindrical pressure vessel with a torispherical head closure is subjected to a uniform internal pressure of 80 lbf/in² gauge. It is required to find the maximum stresses in the head if the mean diameter of the shell is 50 in, the mean crown radius is 72 in, the mean knuckle radius is 4.5 in, the thickness of the head and cylinder is 0.375 in, and the cylinder length is 24 in.

The parameters required for use of the Item are tabulated.

$\frac{r}{D}$	0.09
$\frac{h}{D}$	0.154
$\frac{t}{D}$	0.0075
$f_{nom} = \frac{p}{2t}(D - t)$	5290 lbf/in ²
$\frac{L}{D}$	0.48

From Figure 1, for $t/D = 0.0075$,

$$\frac{L_{min}}{D} = 0.29.$$

Hence, the remaining curves may be used to analyse the pressure vessel head.

As the values $r/D = 0.09$ and $t/D = 0.0075$ cannot be read directly off the figures it is necessary to interpolate to find the stresses for the value of $h/D = 0.154$ given.

Method of interpolation

The following procedure is carried out for each stress f .

- (1) At each value of r/D , a curve is drawn of f/f_{nom} against t/D for the given value of h/D (0.154).

There are only 4 curves in this Example since the curves for $r/D = 0.16$ do not descend to $h/D = 0.154$.

- (2) The value of f/f_{nom} when $t/D = 0.0075$ is taken from each of the above 4 curves and these values of f/f_{nom} are plotted against r/D .
- (3) The value of f/f_{nom} when $r/D = 0.09$ may then be found.

The values obtained by interpolation are tabulated.

<i>Figures used</i>	<i>Stress ratio</i>		<i>Stress lbf/in²</i>	
2, 4, 6, 8	$\frac{f_{i\phi}}{f_{nom}}$	4.65	$f_{i\phi}$	24 600
2, 4, 6, 8	$\frac{f_i}{f_{nom}}$	1.56	f_i	8 250
2, 4, 6, 8	$\frac{f_{0\phi}}{f_{nom}}$	-2.95	$f_{0\phi}$	-15 600
3, 5, 7, 9	$\frac{f_0}{f_{nom}}$	1.33	f_0	7 050
3, 5, 7, 9	$\frac{f_{0\theta}}{f_{nom}}$	-3.87	$f_{0\theta}$	-20 500

It will be seen from the tabulation that the maximum stress is $f_{i\phi} = 24\,600 \text{ lbf/in}^2$.

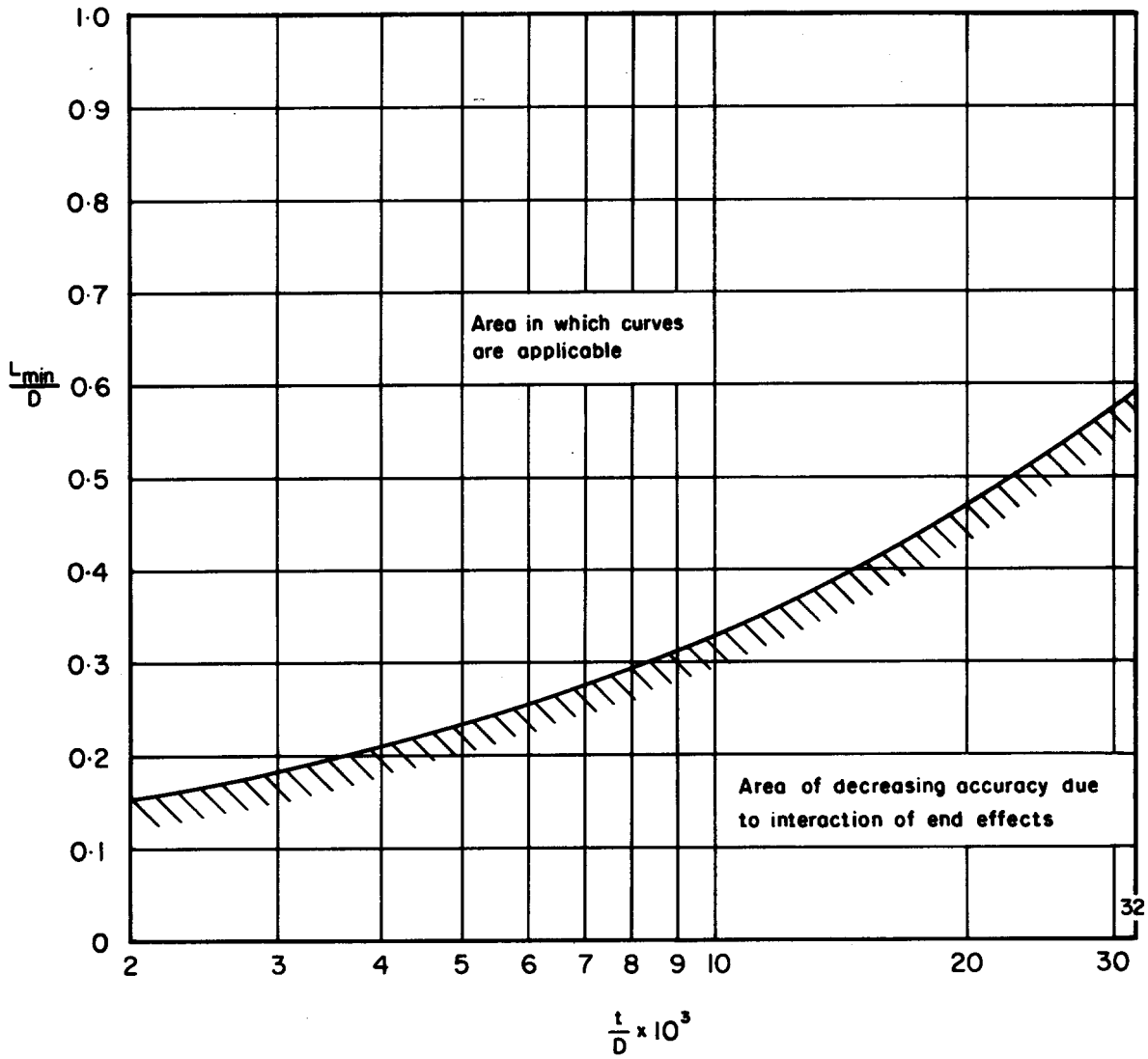


FIGURE 1