

ESTIMATION OF THE RMS STRESS IN SKIN PANELS SUBJECTED TO RANDOM ACOUSTIC LOADING

1. INTRODUCTION

The important parameters in determining the response of typical aircraft structures to high intensity acoustic loads have been outlined in Item No. 74037. A complete theory to describe the response has been developed by Powell (Reference 3) but because of its complexity it cannot readily be used for design. Nevertheless, this memorandum outlines the complete theory in order to show the significance of the major parameters and also the limitations, in practice, of any simplified theory.

The simplified theory is then outlined and experimental results are given to show the orders of accuracy which can be achieved on full scale structures in true noise environments. There is a dearth of adequately documented experimental results in the literature but those results which are available show reasonable agreement with the simplified theory. Experimental results are generally within a factor of 2 of the rms stress estimates. In view of the severe simplifications which have been made, this order of accuracy is probably the best that can be expected. It can be seen from an $S_{rms} - N$ curve, as found for example in Item No. 72015, that a factor of 2 on stress corresponds to a factor between 5 and 15 on life.

2. NOTATION

A	surface area
b	shorter dimension of panel
f	frequency
f_n	fundamental natural frequency of skin panel assuming all edges to be fixed
f_r	undamped natural frequency of r^{th} mode of built-up structure
j_{rr}^2	joint acceptance coefficient of response of structure in r^{th} mode to a particular pressure field
$L_{ps}(f)$	spectrum level of acoustic pressure at frequency f
$L_{ss}^2(f)$	spectral density of stresses at frequency f
M_r	generalised mass in r^{th} mode
S_o	stress at point of interest per unit uniform static pressure on panel
S_r	stress at point of interest due to unit displacement in r^{th} mode
S_{rms}	rms stress at point of interest due to acoustic loading
t	thickness of panel

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y_{rms}	rms displacement of centre of panel due to acoustic loading
y_0	static displacement of centre of panel per unit uniform static pressure on panel
δ	damping ratio of fundamental mode of vibration of skin panel
δ_r	damping ratio of r^{th} mode of vibration of skin panel

3. GENERAL THEORY

When a built-up structure made of a series of plates and beams is excited by distributed random forces the spectral density of the stress at position x is given in terms of the normal modes of the structure by

$$L_{ss}^2(f) = \sum_{r=1}^{\infty} \frac{S_r^2(x) A^2 j_{rr}^2 L_{ps}^2(f)}{M_r^2 [(f_r^2 - f^2)^2 + 4\delta_r^2 f_r^2 f^2] 16\pi^4}. \quad (3.1)$$

Then the root mean square value of the stress at position x is given by

$$S_{rms}(x) = \left[\int_0^{\infty} L_{ss}^2(f) df \right]^{1/2}. \quad (3.2)$$

Equation (3.1) has already been simplified by the omission of cross terms arising due to the coupling between the modes of vibration. Such a simplification is reasonable in the case of lightly damped structures such as aircraft fuselages and control surfaces.

Even if the cross terms are small a large number of direct terms will be significant if the excitation forces have a broad frequency bandwidth. Thus in cases of broad band excitation, such as jet noise and boundary layer pressure fluctuations, the computation of response using Equation (3.1) for all the modes having frequencies in the bandwidth of the excitation is prohibitively lengthy. Consequently there has been great interest in developing approximate solutions based on considerable theoretical simplification of the structure and, to a lesser extent, the pressure field. The approximations have been supported by structural tests in true or simulated acoustic environments. As a result of this a semi-empirical design procedure for aircraft skin structure has been built up.

4. SIMPLIFIED THEORY OF RESPONSE FOR STRESSES IN PLATES

4.1 Introduction

The simplified theory is derived on the assumption that the major part of the response is due to the contribution of one dominant mode. Tests on full scale structures have shown that in certain types of structure (primarily large skin plates) the response spectrum has only one major peak resulting from one mode of vibration. In other structures, such as control surface skin plating, there are many peaks in the response spectrum. Even in this case, however, the simple theory gives a reasonable estimate of the overall stress level; here errors due to two or three assumptions appear to have a self-cancelling effect. It is thus most important that experimental evidence be obtained to discover the limitations of the simplified expression. Some test results available to date are discussed in Section 5.

4.2 Plate stresses

The simplified theory is obtained by applying the following assumptions to Equations (3.1) and (3.2).

- (i) Assume that the dominant form of skin vibration is that of the individual plates vibrating in their fundamental mode only.

This eliminates the summation and simplifies the structure to a set of single independent plates responding in one mode only.

- (ii) Assume that the pressure is exactly in phase over the whole plate.

The result of this assumption is that the response coefficient, j_{rr}^2 becomes an integral of the mode shape only. This is reasonable for jet excitation of typical sizes of plates, but may be inadequate for boundary layer excitation.

- (iii) Assume that the spectrum level of acoustic pressures is constant over the range of frequencies close to the natural frequency of the fundamental mode of the panel. This is reasonable for lightly damped structures.

As a result of these simplifications the expression for root mean square stress becomes

$$S_{rms} = \left(\frac{\pi}{4\delta} \right)^{1/2} f_n^{1/2} L_{ps}(f_n) S_o. \quad (4.1)$$

A calculation procedure based on this equation is given in Item No. 72005.

4.3 Effect of change in plate size and thickness

On the assumption that only one mode of vibration is excited by the noise the simplified Equation (4.1) can be used to study the effects of changes in plate dimensions on the rms stress. If the plate aspect ratio is kept constant the natural frequency of the fundamental mode of vibration is proportional to t/b^2 and the static stress parameter is proportional to b^2/t^2 . Then if it is assumed that the spectrum level of acoustic pressure is the same at the new natural frequency, the rms stress varies in the following way:

$$S_{rms} \propto \delta^{-1/2} b t^{-3/2}. \quad (4.2)$$

This gives a very rough guide as to how the rms stress increases with increase in plate width and decreases with increase in plate thickness.

In many practical cases however failure occurs initially in the attachments of the skin to the supporting frames or ribs. The loads coming on to these attachments are due partly to the inertia force from the vibrating skin. The root mean square of the inertia force is proportional to

$$(\text{skin mass}) \times f_n^2 \times y_{rms}.$$

An expression for y_{rms} can be derived in a form similar to Equation (4.1):

$$y_{rms} = \left(\frac{\pi}{4\delta} \right)^{1/2} f_n^{1/2} L_{ps}(f_n) y_o. \quad (4.3)$$