



# ESDU 67035

Issued June 1967  
Reprinted at Amendment B  
March 2007

## Jet flow parameters

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## JET FLOW PARAMETERS

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## JET FLOW PARAMETERS

### 1. NOTATION AND UNITS

Any coherent set of units may be used. Two such sets are indicated below.

|               |                                                                         | SI              | British                |
|---------------|-------------------------------------------------------------------------|-----------------|------------------------|
| $A$           | cross-sectional area normal to flow                                     | $\text{m}^2$    | $\text{ft}^2$          |
| $A^*$         | cross-sectional area of flow at throat of convergent-divergent nozzle   | $\text{m}^2$    | $\text{ft}^2$          |
| $a$           | speed of sound = $\sqrt{\gamma RT}$                                     | $\text{m/s}$    | $\text{ft/s}$          |
| $F$           | gross thrust                                                            | $\text{N}$      | $\text{lbf}$           |
| $M$           | Mach number = $V/a$                                                     |                 |                        |
| $m$           | mass flow rate                                                          | $\text{kg/s}$   | $\text{slug/s}$        |
| $p$           | static pressure                                                         | $\text{N/m}^2$  | $\text{lbf/ft}^2$      |
| $p_H$         | total pressure                                                          | $\text{N/m}^2$  | $\text{lbf/ft}^2$      |
| $(p_H/p_0)^*$ | critical pressure ratio for convergent nozzle (see Section 3)           |                 |                        |
| $(p_H/p_T)^*$ | critical pressure ratio for convergent-divergent nozzle (see Section 7) |                 |                        |
| $R$           | gas constant                                                            | $\text{J/kg K}$ | $\text{ft lbf/slug K}$ |
| $T$           | static temperature                                                      | $\text{K}$      | $\text{K}$             |
| $T_H$         | total temperature                                                       | $\text{K}$      | $\text{K}$             |
| $V$           | mean jet velocity                                                       | $\text{m/s}$    | $\text{ft/s}$          |
| $X(M)$        | momentum function                                                       |                 |                        |
| $\gamma$      | ratio of specific heat capacities                                       |                 |                        |
| $\Delta p_H$  | pressure difference = $(p_H - p_0)$                                     | $\text{N/m}^2$  | $\text{lbf/ft}^2$      |

### Suffixes

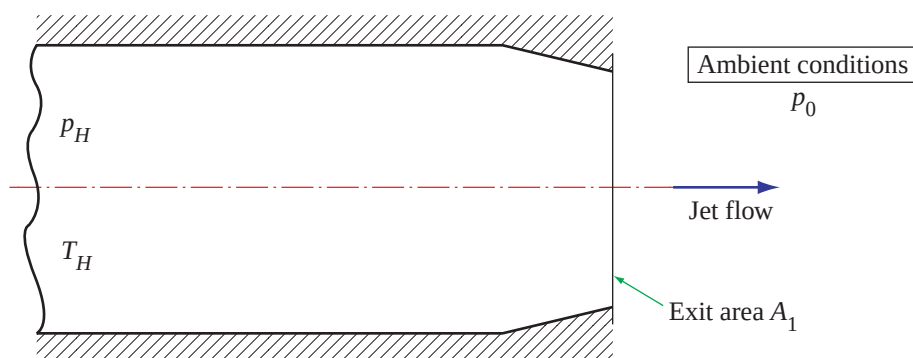
|     |                                                     |
|-----|-----------------------------------------------------|
| 0   | free-stream (ambient) conditions                    |
| 1   | conditions in exhausting jet in nozzle exit plane   |
| $T$ | conditions at throat of convergent-divergent nozzle |

Issued June 1967  
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## 2. INTRODUCTION

A number of parameters are commonly used in work on jets exhausting into the atmosphere. Mass flow and gross thrust are presented here in parametric form as functions of pressure ratio over the range  $1 \leq p_H/p_0 \leq 50$ , except for Figure 6 where the range is extended up to  $p_H/p_0 = 3000$ . The range of values of  $\gamma$  used is  $1 \leq \gamma \leq 1.67$ , the value  $\gamma = 1$  being included in the curves as a limiting value for ease of interpolation.

This Item deals primarily with convergent nozzles exhausting isentropically into an atmosphere at rest. Sketch 2.1 shows the configuration of interest.



**Sketch 2.1 Convergent nozzle configuration**

The effect of external flow can be to alter the effective pressure to which the jet discharges, but by allowing for this the formulae given here can sometimes be used. However, there are circumstances where such a procedure cannot be used so it is necessary to consider each case on its merits.

Some information on convergent-divergent nozzles is given in Section 7

Figures 1 to 6 inclusive have small diagrams showing the nozzle configuration to which the particular graph applies.

## 3. CRITICAL PRESSURE RATIO

For a convergent nozzle, as the ratio  $p_H/p_0$  is increased from a value of unity the Mach number of the exhausting jet increases until  $M_1 = 1$ . The critical pressure ratio, denoted by  $(p_H/p_0)^*$ , is the value of  $p_H/p_0$  for which sonic velocity first occurs at the exit of a convergent nozzle. Increases in  $p_H/p_0$  above the critical value have no effect on the exit Mach number which remains at  $M_1 = 1$ .

For values of  $p_H/p_0 \leq (p_H/p_0)^*$ , the exit static pressure  $p_1$  is equal to  $p_0$ . However for supercritical pressure ratios,  $p_1$  is greater than  $p_0$  and  $p_H/p_1 = (p_H/p_0)^*$ . For one-dimensional isentropic flow,

$$\left(\frac{p_H}{p_0}\right)^* = \left(\frac{\gamma + 1}{2}\right)^{\frac{\gamma}{\gamma - 1}}. \quad (3.1)$$

Values of  $(p_H/p_0)^*$  for various values of  $\gamma$  are given in Table 3.1.

**TABLE 3.1**

| $\gamma$      | 1.0    | 1.1    | 1.2    | 1.3    | 4/3    | 1.4    | 1.67   |
|---------------|--------|--------|--------|--------|--------|--------|--------|
| $(p_H/p_0)^*$ | 1.6488 | 1.7104 | 1.7715 | 1.8324 | 1.8526 | 1.8929 | 2.0549 |

For given upstream conditions, the maximum mass flow rate obtainable occurs when  $p_H/p_1 = (p_H/p_0)^*$ .

## 4. MASS FLOW RATE

The mass flow rate,  $m$ , can be expressed in the non-dimensional form

$$\frac{m}{A_1} \left( \frac{RT_{H1}}{\Delta p_H p_0} \right)^{1/2} = \left\{ \frac{2\gamma}{(\gamma-1)} \frac{(p_H/p_0)^{\frac{\gamma-1}{\gamma}}}{(\Delta p_H/p_0)} \left[ \left( \frac{p_H}{p_0} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \right\}^{1/2}, \quad (4.1)$$

or alternatively

$$\frac{m}{A_1 p_H} (RT_{H1})^{1/2} = \left\{ \frac{2\gamma}{(\gamma-1)} \frac{1}{(p_H/p_0)^{\frac{\gamma+1}{\gamma}}} \left[ \left( \frac{p_H}{p_0} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \right\}^{1/2}. \quad (4.2)$$

Equations (4.1) and (4.2) are exact for one-dimensional isentropic flow for pressure ratios up to the critical value. Equation (4.1) is plotted in Figure 1 and Equation (4.2) is plotted in Figure 2.

### 4.1 Low Pressure Ratio

As  $p_H/p_0 \rightarrow 1$ , Equation (4.1) reduces to the limiting form

$$\frac{m}{A_1} \left( \frac{RT_{H1}}{\Delta p_H p_0} \right)^{1/2} = \sqrt{2} \left( 1 - \frac{(3-2\gamma)\Delta p_H}{4\gamma p_0} \right). \quad (4.3)$$

Equation (4.3), which is derived in Reference 3, shows the dependence of the mass flow rate on the pressure ratio  $\Delta p_H/p_0$  at low values of pressure ratio. Equation (4.3) is within 1 per cent of the exact form over the pressure-ratio range  $1 \leq p_H/p_0 \leq 1.3$  for all values of  $\gamma$ .

Values of the mass flow rate for low values of pressure ratio can be obtained from Figure 1, where Equation (4.1) is plotted.

## 4.2 Intermediate Pressure Ratio

Equation (4.1) is particularly useful for graphical presentation in the intermediate pressure-ratio range. Figure 1 presents Equation (4.1) plotted for various values of  $\gamma$  over the pressure-ratio range  $1 \leq p_H/p_0 \leq (p_H/p_0)^*$ . Values of  $(p_H/p_0)^*$  are given in Table 3.1.

## 4.3 High Pressure Ratio

Equation (4.2) is plotted as a function of  $p_H/p_0$  and  $\gamma$  in Figure 2 for the pressure-ratio range  $1.1 \leq p_H/p_0 \leq (p_H/p_0)^*$ . For values of  $p_H/p_0 \geq (p_H/p_0)^*$ , Equation (4.2) becomes

$$\frac{m}{A_1 p_H} (RT_{H1})^{1/2} = \left[ \gamma \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma + 1}{\gamma - 1}} \right]^{1/2}. \quad (4.4)$$

It will be noted that the right-hand side of Equation (4.4) is a function of  $\gamma$  only and is independent of pressure ratio. Values of  $m(RT_{H1})^{1/2}/(A_1 p_H)$  for  $p_H/p_0 \geq (p_H/p_0)^*$  are given for various values of  $\gamma$  in Table 4.1.

TABLE 4.1

| $\gamma$                            | 1.0    | 1.1    | 1.2    | 1.3    | 4/3    | 1.4    | 1.67   |
|-------------------------------------|--------|--------|--------|--------|--------|--------|--------|
| $\frac{m}{A_1 p_H} (RT_{H1})^{1/2}$ | 0.6065 | 0.6284 | 0.6486 | 0.6673 | 0.6732 | 0.6847 | 0.7267 |

## 5. GROSS THRUST

In a flow that is subsonic throughout the duct, the gross thrust,  $F$ , and the momentum flux,  $mV_1$ , are the same, i.e.

$$F = mV_1. \quad (5.1)$$

In supercritical flow, for a convergent nozzle the pressure difference at the exit,  $(p_1 - p_0)$ , must be taken into account, giving

$$F = mV_1 + (p_1 - p_0)A_1. \quad (5.2)$$

In subcritical flow, the gross thrust can be written in the non-dimensional form

$$\frac{F}{A_1 \Delta p_H} = \frac{mV_1}{A_1 \Delta p_H} = \frac{2\gamma}{\gamma - 1} \left[ \left( \frac{p_H}{p_0} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] \frac{1}{\Delta p_H / p_0}. \quad (5.3)$$