

Geometric and kinematic relationships for various axis systems

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The Dynamics Committee, which first met in January 1962, has the co-operation of many engineers and scientists in industry, research establishments and universities from whom much assistance and unpublished information is being received.

GEOMETRIC AND KINEMATIC RELATIONSHIPS FOR VARIOUS AXIS SYSTEMS

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GEOMETRIC AND KINEMATIC RELATIONSHIPS FOR VARIOUS AXIS SYSTEMS

1. SUMMARY

In the solution of certain problems of aircraft and missile dynamics it is helpful and often necessary to use more than one system of axes and to do so requires that a number of geometric and kinematic relationships be established.

This Item presents geometric relationships between the components of any vector in one orthogonal axis system and the components of the same vector in any other orthogonal system. Firstly the general relationship is given and then the particular forms this takes for body and earth axes (see Item 67003^[1]) and for the special axis systems (path axes) defined here are deduced.

The angular velocity is sometimes represented in terms of the rate of change of attitude angles. Accordingly additional relationships between these non-orthogonal components and the more usual orthogonal set are given. Again a general relationship is given and particular forms are deduced.

2. GENERAL RELATIONSHIPS

2.1 Notation

Q_{bc}	for any axes "b" relative to "c", matrix relating rates of change of attitude angles to components of angular velocity
$R; P; Y$	rotation matrices corresponding to rotations about x-axis; y-axis; z-axis
S_{bc}	matrix relating components of vector in axis system "b" to those in system "c"
$\vec{V}$	velocity of origin relative to air
$\vec{V}^K$	velocity of origin relative to earth
$\Phi_{bc}, \Theta_{bc}, \Psi_{bc}$	angles defining attitude of axis system "b" relative to system "c"

Dressings on Velocity Symbols

Subscripts and/or superscripts are attached to velocity symbols to distinguish the axis systems involved or to indicate the nature of the velocity, as follows.

Subscripts

b (or c)	components of linear or angular velocity are taken along axes of system "b" (or "c") - attached to u, v, w, p, q, r
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Superscripts

bc	angular velocity of system "b" relative to system "c" is considered - attached to p, q, r
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Dressings on Matrix Symbols

Superscripts

-1	inverse of matrix
T	transpose of matrix

Some additional notation required for the applications is given in Section 4.1. For other notation see Item 67002^[2].

2.2 Relationships Between Vector Components in Two Axis Systems

For any vector, the components in one axis system can be related to those in another in terms of quantities defining the relative orientation of the two systems of axes.

Matrix notation[†] provides a compact means of expressing these relationships. For example the components of the velocity in axis system “b” in terms of those in axis system “c” are given by

$$\begin{bmatrix} u_b \\ v_b \\ w_b \end{bmatrix} = \mathbf{S}_{bc} \begin{bmatrix} u_c \\ v_c \\ w_c \end{bmatrix}.$$

In this section only right-handed, orthogonal Cartesian axis systems are considered. Each row and column of any matrix is then a set of direction cosines. The matrix is orthogonal and the inverse, $\mathbf{S}_{bc}^{-1}$, is equal to the transpose $\mathbf{S}_{bc}^T$. Accordingly

$$\begin{bmatrix} u_c \\ v_c \\ w_c \end{bmatrix} = \mathbf{S}_{bc}^T \begin{bmatrix} u_b \\ v_b \\ w_b \end{bmatrix}.$$

The elements of the transformation matrix $\mathbf{S}_{bc}$ are often expressed in terms of attitude angles. In Item 67003^[1] the orientation of a body axis system relative to a system of normal earth axes has been defined by three attitude angles Φ , Θ and Ψ . Similarly the orientation of an arbitrary axis system “b” relative to another system “c” can be defined in an analogous manner by *relative attitude angles* Φ_{bc} , Θ_{bc} and Ψ_{bc} .

The transformation matrix for this sequence can be written

$$\mathbf{S}_{bc} = \begin{bmatrix} \cos \Theta_{bc} \cos \Psi_{bc} & \cos \Theta_{bc} \sin \Psi_{bc} & -\sin \Theta_{bc} \\ \sin \Phi_{bc} \sin \Theta_{bc} \cos \Psi_{bc} - \cos \Phi_{bc} \sin \Psi_{bc} & \sin \Phi_{bc} \sin \Theta_{bc} \sin \Psi_{bc} + \cos \Phi_{bc} \cos \Psi_{bc} & \sin \Phi_{bc} \cos \Theta_{bc} \\ \cos \Phi_{bc} \sin \Theta_{bc} \cos \Psi_{bc} + \sin \Phi_{bc} \sin \Psi_{bc} & \cos \Phi_{bc} \sin \Theta_{bc} \sin \Psi_{bc} - \sin \Phi_{bc} \cos \Psi_{bc} & \cos \Phi_{bc} \cos \Theta_{bc} \end{bmatrix}.$$

The transpose is obtained by interchange of rows and columns.

[†] A knowledge of the basic rules of matrix algebra is assumed (see for example Aitken^[3]).