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Shafts with interference-fit collars Part II: nominal elastic interface pressures and stresses

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Mr R.B. Laycock	— Independent
Prof. J.W.H. Price*	— Monash University, Australia
Mr J.V. Vint	— Independent.

* Corresponding Member

The construction and subsequent development of the Data Item was undertaken by

Dr D.L. Downs	— Engineer.
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The person with overall responsibility for the work in this subject area is Mr A.C. Quilter, Head of Strength Analysis Group.

SHAFTS WITH INTERFERENCE-FIT COLLARS PART II: NOMINAL ELASTIC INTERFACE PRESSURES AND STRESSES

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SHAFTS WITH INTERFERENCE-FIT COLLARS PART II: NOMINAL ELASTIC INTERFACE PRESSURES AND STRESSES

1. THEORY

1.1 Introduction

This Data Item presents figures and a Fortran computer program which give the nominal interface pressures and stresses for rotating shafts with interference-fit collars.

Sections 1.2 and 1.3 describe the theory from which the data in the figures are derived and on which the computer program is based.

Section 1.4 gives two worked examples which demonstrate the use of the figures.

Section 1.5 presents the figures, preceded by the equations for the radial and circumferential direct stresses in the shaft and collar at any radius.

Section 2 describes the use of the program: the input data required and any limitations on their values, and the output produced by the program. Example input and output files corresponding to the problems treated in Section 1.4 are also provided.

Finally, in Appendix A, notes on the effect of significant variation between $\frac{E_c}{\rho_c}$ and $\frac{E_s}{\rho_s}$ on ω_{cr} and f are given.

1.1.1 Notation

Both SI and British units are quoted but any coherent system of units may be used.

D	outside diameter of collar	m	in
d	diameter of shaft	m	in
d_i	internal diameter of hollow shaft	m	in
E_c	Young's modulus of collar material	N/m ²	lbf/in ²
E_s	Young's modulus of shaft material	N/m ²	lbf/in ²
e	interference	m	in
f	circumferential direct stress at bore of collar	N/m ²	lbf/in ²
f_0	circumferential direct stress at bore of collar due to interference alone, <i>i.e.</i> with $\omega = 0$	N/m ²	lbf/in ²
f_r	radial direct stress	N/m ²	lbf/in ²
f_θ	circumferential direct stress	N/m ²	lbf/in ²

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p	interface pressure	N/m ²	lbf/in ²
p_0	interface pressure due to interference alone, <i>i.e.</i> with $\omega = 0$	N/m ²	lbf/in ²
r	radial co-ordinate	m	in
α_c	coefficient of linear thermal expansion of collar material	/°C	/°F
α_s	coefficient of linear thermal expansion of shaft material	/°C	/°F
Δf	change in f due to angular velocity	N/m ²	lbf/in ²
Δp	change in p due to angular velocity	N/m ²	lbf/in ²
ΔT	temperature difference between manufacturing and operating conditions	°C	°F
ε_0	diametral interference at manufacturing temperature ($= e/d$)	m/m	in/in
ε	diametral interference at operating temperature ($= \varepsilon_0 + \Delta T(\alpha_s - \alpha_c)$)	m/m	in/in
ν_c	Poisson's ratio of collar material		
ν_s	Poisson's ratio of shaft material		
ρ_c	density of collar material	kg/m ³	lbm/in ^{3*}
ρ_s	density of shaft material	kg/m ³	lbm/in ^{3*}
Ω	rotational velocity	rpm	rpm
Ω_{cr}	rotational velocity at which $p = p_0 + \Delta p = 0$	rpm	rpm
ω	angular velocity	rad/s	rad/s
ω_{cr}	angular velocity at which $p = p_0 + \Delta p = 0$	rad/s	rad/s

Convention: tensile stresses are positive, compressive stresses are negative; pressure is positive; temperature increase is positive.

* A density value expressed in familiar British units as pounds mass per cubic inch has to be divided by 386 before it can be used in the formulae in this Data Item and in the program that accompanies it. (A force of 1 lbf acting on a mass of 1 lbm produces an acceleration of $12 \times 32.174 = 386 \text{ in/s}^2$.)

1.2 Notes

This Item gives nominal interface pressures and stresses for rotating shafts with interference-fit collars. The data, which are obtained from plane-stress elastic theory, apply to collars of constant thickness assembled with an axially uniform initial interference. The effects of a uniform temperature change and rotation are also considered.

Each of the five pages of figures in Section 1.5 relates to a different value of E_c/E_s . On each page, the first figure gives the interface pressure due to interference alone in the form of curves of $p_0/E_c\epsilon$ against d/D for five values of d_i/d . The second figure gives the corresponding circumferential direct stress at the bore of the collar as $f_0/E_c\epsilon$ against d/D for five values of d_i/d .

For a rotating shaft and collar, the change in interface pressure due to angular velocity is given by

$$\frac{\Delta p}{p_0} = -\left(\frac{\omega}{\omega_{cr}}\right)^2, \quad (1.1)$$

where

$$\omega_{cr} = \frac{2.2}{D} \left[\frac{E_c \epsilon}{\rho_c \left[1 - \left(\frac{d_i}{D} \right)^2 \right]} \right]^{1/2} \text{ rad/s.} \quad (1.2)$$

The third figure on each page then gives the change in circumferential direct stress at the bore of the collar due to angular velocity as $(\Delta f/E_c\epsilon)(\omega_{cr}/\omega)^2$ against d/D for five values of d_i/d .

If the assembly is to operate at a different temperature from that at which the diametral interference is measured then $\epsilon = \epsilon_0 + \Delta T(\alpha_s - \alpha_c)$ to allow for differential expansion of the assembly.

In deriving the curves and Equation (1.2) it has been assumed that $\nu_c = \nu_s = 0.3$. In Equation (1.2) and

the figures relating to the effect of rotation it has also been assumed that $\frac{E_c}{\rho_c} = \frac{E_s}{\rho_s}$. Appendix A gives

expressions for the determination of ω_{cr} and f when $\frac{E_c}{\rho_c}$ differs significantly from $\frac{E_s}{\rho_s}$. General expressions

for the stresses at any radius are given in Table 1.1.

The two-dimensional theory on which this Item is based represents quite adequately the average stresses in the shaft and collar. There is, however, some axial variation in the stresses which tend to be somewhat higher near the end of a collar from which the shaft protrudes. The practical significance of this axial stress variation is described in Item No. 68002*.

Provided that ϵ is constant and that d_i/d and d/D do not vary significantly, the data presented here may also be used to estimate interface pressures and stresses for assemblies with slight taper.

* Data Item No. 68002 "Shafts with interference-fit collars. Part I: some factors affecting the design of an assembly".

1.3 Derivation

The expressions:

$$\frac{p_0}{E_c \varepsilon} = \left[\frac{E_c}{E_s} \left(\frac{1 + \left(\frac{d_i}{d}\right)^2}{1 - \left(\frac{d_i}{d}\right)^2} - \nu_s \right) + \frac{1 + \left(\frac{d}{D}\right)^2}{1 - \left(\frac{d}{D}\right)^2} + \nu_c \right]^{-1} ;$$

$$\frac{f_0}{E_c \varepsilon} = \frac{p_0}{E_c \varepsilon} \left[\frac{1 + \left(\frac{d}{D}\right)^2}{1 - \left(\frac{d}{D}\right)^2} \right] ;$$

$$\frac{\Delta f}{E_c \varepsilon} \left(\frac{\omega_{cr}}{\omega} \right)^2 = \frac{1}{1 - \left(\frac{d_i}{D}\right)^2} \left[1 + \frac{1 - \nu_c}{3 + \nu_c} \left(\frac{d}{D}\right)^2 \right] - \frac{f_0}{E_c \varepsilon} .$$

1. WANG, C.T. *Applied elasticity*. Chapter 4. McGraw-Hill, New York, 1953.

1.4 Examples

1.4.1 Example 1

It is required to determine the nominal stresses at the bore of a steel collar shrunk on to a steel shaft. The assembly is to run at a speed of 2000 rpm. The dimensions and material properties are as follows:

$$d = 5 \text{ in}; d_i = 2 \text{ in}; D = 15 \text{ in},$$

$$E_c = E_s = 30 \times 10^6 \text{ lbf/in}^2,$$

$$\varepsilon = \varepsilon_0 = 0.001 \text{ in/in},$$

$$\rho_c = \rho_s = 0.283 \text{ lbm/in}^3, \text{ and}$$

$$\nu_c = \nu_s = 0.28 .$$

The approximation involved in taking $\nu_c = \nu_s = 0.3$ in order to use the figures is considered acceptable. Then, evaluating the first three non-dimensional groups required in reading the figures gives

$$\frac{d}{D} = \frac{5}{15} = 0.333; \frac{d_i}{d} = \frac{2}{5} = 0.4 \text{ and } \frac{E_c}{E_s} = 1.0 .$$

From Figure 4 for $\frac{E_c}{E_s} = 1.0$

$$\frac{p_0}{E_c \varepsilon} = 0.38 \quad \text{and} \quad \frac{f_0}{E_c \varepsilon} = 0.475 .$$

Hence $p_0 = 0.38 \times 0.001 \times 30 \times 10^6 = 11\,400 \text{ lbf/in}^2$.

Similarly $f_0 = 14\,250 \text{ lbf/in}^2$.

For substitution into Equation (1.2)

$$1 - \left(\frac{d_i}{D}\right)^2 = 1 - \left(\frac{2}{15}\right)^2 = 0.982 ,$$

so that
$$\omega_{cr} = \frac{2.2}{15} \left[\frac{30 \times 10^6 \times 0.001}{\frac{0.283}{386} \times 0.982} \right]^{\frac{1}{2}}$$

$$= 947 \text{ rad/s (or } \Omega_{cr} = 9040 \text{ rpm).}$$

Hence
$$\left(\frac{\omega}{\omega_{cr}}\right)^2 = \left(\frac{\Omega}{\Omega_{cr}}\right)^2 = \left(\frac{2000}{9040}\right)^2 = 0.049 .$$

From Equation (1.1) therefore,

$$\Delta p = -0.049 \times 11\,400 = -560 \text{ lbf/in}^2$$

and finally $p = 11\,400 - 560 = 10\,840 \text{ lbf/in}^2$.

Entering Figure 4 for $\frac{E_c}{E_s} = 1.0$ with $d/D = 0.333$ and $d_i/d = 0.4$ gives

$$\left(\frac{\Delta f}{E_c \varepsilon}\right) \left(\frac{\omega_{cr}}{\omega}\right)^2 = 0.57 .$$

Hence
$$\begin{aligned} \Delta f &= 0.57 \times 0.049 \times 0.001 \times 30 \times 10^6 \\ &= 840 \text{ lbf/in}^2 , \end{aligned}$$

so the tangential stress at 2000 rpm is

$$f = 14\,250 + 840 = 15\,090 \text{ lbf/in}^2 .$$

1.4.2 Example 2

Suppose now that the collar in the previous example is to be made not of steel but of cast iron with properties:

$$\begin{aligned} E_c &= 14 \times 10^6 \text{ lbf/in}^2, \\ \rho_c &= 0.261 \text{ lbm/in}^3, \text{ and} \\ \nu_c &= 0.26. \end{aligned}$$

The dimensions of the collar and the shaft remain the same. The assembly is to operate at a temperature of 100°C, having been manufactured at a temperature of 23°C. The diametral interference is therefore given by the expression

$$\varepsilon = \varepsilon_0 + \Delta T(\alpha_s - \alpha_c),$$

where $\Delta T = 100 - 23 = 77^\circ\text{C}$, $\alpha_s = 0.000\ 012 / ^\circ\text{C}$ and $\alpha_c = 0.000\ 0117 / ^\circ\text{C}$

giving $\varepsilon = 0.001 + 77 \times (0.000\ 012 - 0.000\ 0117) = 0.001\ 023$.

As before, $\frac{d}{D} = 0.333$; $\frac{d_i}{d} = 0.4$ but $\frac{E_c}{E_s} = \frac{14}{30} = 0.466$.

Again, the approximation involved in taking $\nu_c = \nu_s = 0.3$ in order to use the figures is acceptable. By interpolating between Figures 1 and 2 for $\frac{E_c}{E_s} = \frac{1}{3}$ and $\frac{E_c}{E_s} = \frac{1}{2}$ respectively, the following values are obtained

$$\frac{p_0}{E_c \varepsilon} = 0.494 \quad \text{and} \quad \frac{f_0}{E_c \varepsilon} = 0.618.$$

Hence $p_0 = 0.494 \times 14 \times 10^6 \times 0.001\ 023 = 7075 \text{ lbf/in}^2$

and similarly $f_0 = 8850 \text{ lbf/in}^2$.

Since it is clear that $\frac{E_c}{\rho_c}$ is not even approximately equal to $\frac{E_s}{\rho_s}$, the expressions in Appendix A are used to estimate ω_{cr} and f . Substituting into Equation (A1.1),

$$\begin{aligned} \omega_{cr} &= \frac{4}{15} \left[\frac{14 \times 10^6 \times 0.001\ 023}{\frac{0.261}{386}} \right]^{1/2} \left[3.26 + 0.74 \times 0.333^2 - \frac{0.283 \times 14}{0.261 \times 30} \times 0.333^2 (0.72 + 3.28 \times 0.4^2) \right]^{-1/2} \\ &= 675 \text{ rad/s (or } \Omega_{cr} = 6450 \text{ rpm)}. \end{aligned}$$

$$\text{Hence} \quad \left(\frac{\omega}{\omega_{cr}} \right)^2 = \left(\frac{\Omega}{\Omega_{cr}} \right)^2 = \left(\frac{2000}{6450} \right)^2 = 0.096.$$

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