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Shafts with interference-fit collars Part III: procedure for calculation of nominal elastic interface pressures and stresses in assemblies of three or more elements

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THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item was monitored and guided by the Stress Analysis and Strength of Components Committee which first met in 1964 and now has the following membership:

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SHAFTS WITH INTERFERENCE-FIT COLLARS PART III: PROCEDURE FOR CALCULATION OF NOMINAL ELASTIC INTERFACE PRESSURES AND STRESSES IN ASSEMBLIES OF THREE OR MORE ELEMENTS

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SHAFTS WITH INTERFERENCE-FIT COLLARS

PART III: PROCEDURE FOR CALCULATION OF NOMINAL ELASTIC INTERFACE PRESSURES AND STRESSES IN ASSEMBLIES OF THREE OR MORE ELEMENTS

1. THEORY

1.1 Introduction

This Data Item presents a procedure and a Fortran computer program for calculating the nominal interface pressures and stresses in a set of three or more uniform circular rings, as shown in Sketch 1.1, with axially uniform initial interferences. The effects of a uniform temperature change and of rotation are also considered. The procedure is based on plane-stress elastic theory.

Section 1.3 describes the method of solution of the problem with n elements. This is followed by the solution of the particular case of three elements and $p_0 = p_3 = 0$. The computer program is based on the method presented in Section 1.3.

Section 1.5 presents a worked example to demonstrate the method of solution in Section 1.3.

In Section 1.6, in Table 1.1, expressions for the diametral interference due to a temperature change and due to an angular velocity are presented, along with expressions for the element compliances. In Table 1.2, in Section 1.6, expressions for the circumferential direct stresses are given.

Section 2 describes the use of the program: the input data required and any limitations on their values, and the output produced by the program. Example input and output files corresponding to the problem treated in Section 1.5 are also provided.

1.2 Notation

Both SI and British units are quoted but any coherent system of units may be used.

E	Young's modulus	N/m ²	lbf/in ²
f	circumferential direct stress at bore of element	N/m ²	lbf/in ²
f'	circumferential direct stress at periphery of element	N/m ²	lbf/in ²
k', k'', k'''	element compliances defined in Table 1.1	m ² /N	in ² /lbf
n	number of elements in assembly		
p	interface pressure	N/m ²	lbf/in ²
r	interface radius	m	in
T_s	operating temperature	°C	°F
T_0	manufacturing temperature	°C	°F
α	coefficient of linear thermal expansion	/°C	/°F

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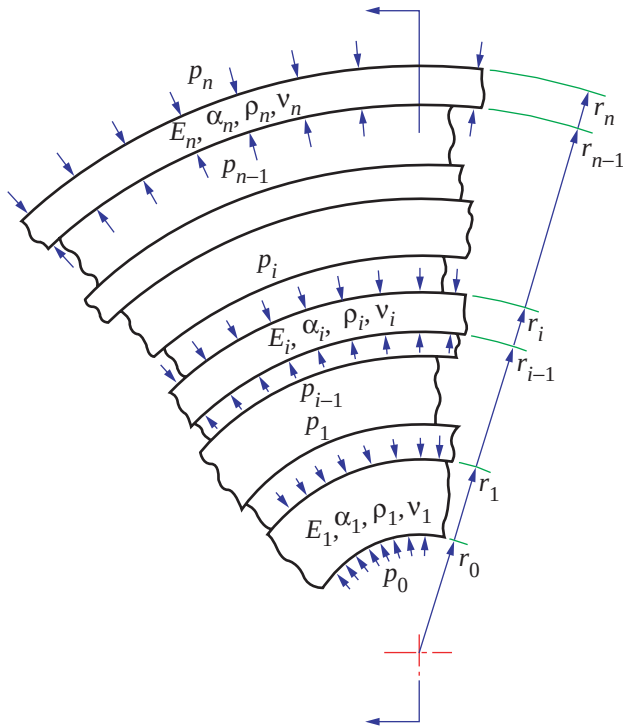
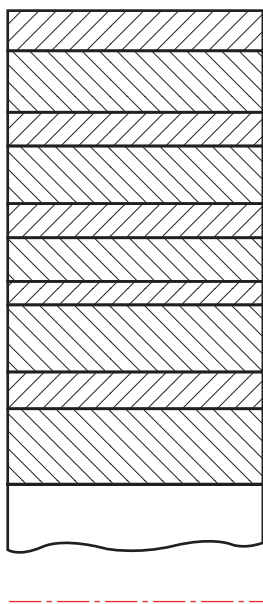
ΔT	temperature difference between manufacturing and operating conditions	$^{\circ}\text{C}$	$^{\circ}\text{F}$
ϵ	diametral interference	m/m	in/in
ν	Poisson's ratio		
ρ	density of material of element	kg/m^3	lbm/in^3^*
Ω	rotational velocity	rpm	rpm
ω	angular velocity	rad/s	rad/s

Suffixes

i	of i th element or at i th interface
$j = 0$	initial value before assembly
$j = 1$	due to temperature change
$j = 2$	due to angular velocity
$j = 3$	due to interface pressures

Convention: tensile stresses are positive, compressive stresses are negative; pressure is positive; temperature increase is positive.

* A density value expressed in familiar British units as pounds mass per cubic inch has to be divided by 386 before it can be used in the formulae in this Data Item and in the program that accompanies it. (A force of 1 lbf acting on a mass of 1 lbm produces an acceleration of $12 \times 32.174 = 386 \text{ in/s}^2$.)



Sketch 1.1

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1.3.2 Solution of Problem with Three Elements and $p_0 = p_3 = 0$

For this particular case, Equations (1.3) reduce to

$$\left. \begin{aligned} k_1'' p_1 + k_1''' p_2 + \sum_{j=0}^2 \varepsilon_{1j} &= 0 \\ k_2' p_1 + k_2'' p_2 + \sum_{j=0}^2 \varepsilon_{2j} &= 0 \end{aligned} \right\} \quad (1.4)$$

of which the solution is

$$\left. \begin{aligned} p_1 &= - \frac{\sum_{j=0}^2 \varepsilon_{1j} - \left(\frac{k_1'''}{k_2''} \right) \sum_{j=0}^2 \varepsilon_{2j}}{k_1'' - \left(\frac{k_1'''}{k_2''} \right) k_2'} \\ p_2 &= - \frac{\sum_{j=0}^2 \varepsilon_{2j} + k_2' p_1}{k_2''} \end{aligned} \right\} \quad (1.5)$$

and

1.4 Derivation

1. WANG, C.T. *Applied elasticity*, Chapter 4. McGraw-Hill, New York, 1953.

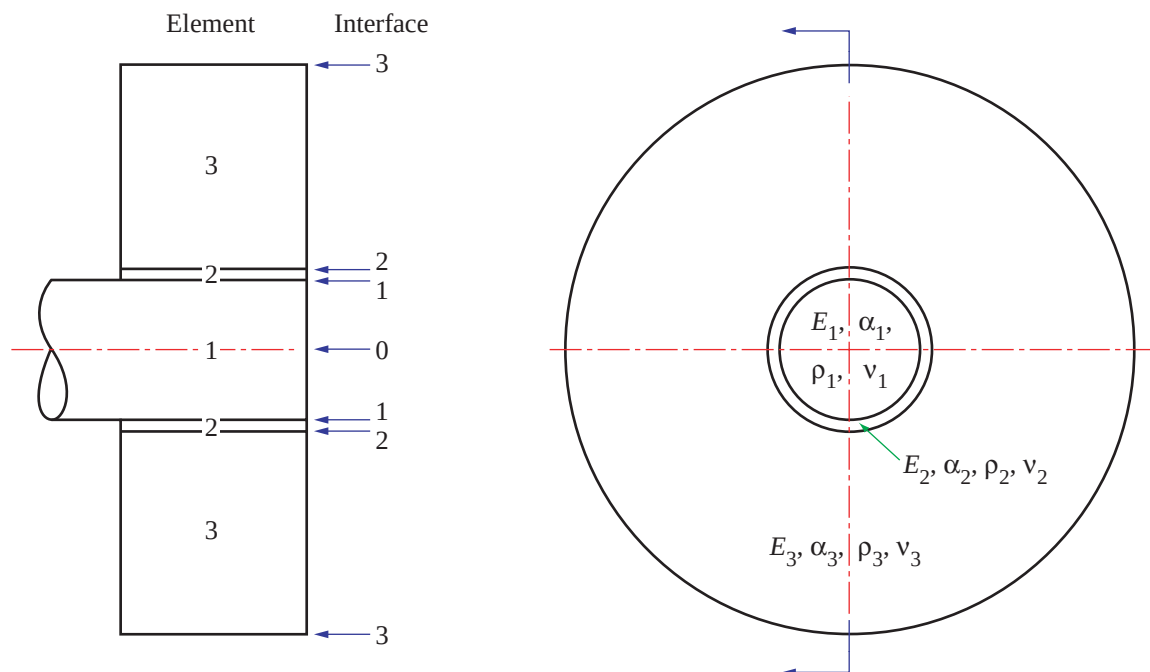
1.5 Example

It is required to determine the nominal interface pressures and stresses in an assembly comprising a solid shaft and a bushed collar. The three elements were measured at 59°F and the assembly will run at 4000 rpm and a steady temperature of 230°F. The dimensions and material properties are tabulated below.

	Interface			
	0	1	2	3
r_i in	0	2.0	2.3	8.0
ε_{i0}	—	0	0.001	—

	Element		
	1	2	3
E_i lbf/in ²	30×10^6	13×10^6	30×10^6
α_i /°F	0.000 0065	0.000 0104	0 000 0065
ρ_i^*	0.000 733	0.000 785	0.000 733
v_i	0.26	0.33	0.26

* Values in lbm/in³ divided by 386. (See Section 1.2.)



Sketch 1.2 Illustration of the assembly for Example 1

Noting that $\Delta T = 230 - 59 = 171^\circ\text{F}$, the necessary terms for substitution in Equations (1.4) are now evaluated.

	Interface	
	1	2
ϵ_{i0}	0	0.001
ϵ_{i1}	-0.000 665	0.000 665
ϵ_{i2}	-0.000 051	-0.000 183
$\sum_{j=0}^2 \epsilon_{ij}$	-0.000 716	0.001 482
k'_i in ² /lbf	Not used	477×10^{-9}
k''_i in ² /lbf	-604×10^{-9}	-577×10^{-9}
k'''_i in ² /lbf	631×10^{-9}	Not used