



ESDU 68019

Issued December 1968
With Amendments A and B
July 1973
Supersedes part of
Aero W.02.04.01
Aero W.S.02.04.12

The compressible two-dimensional laminar boundary layer, both with and without heat transfer, on a smooth flat plate, with application to wedges, cylinders and cones

Endorsed by
The Royal Aeronautical Society

AVA Documents
Document Sharing Hub

ESDU DATA ITEMS

Data Items provide validated information in engineering design and analysis for use by, or under the supervision of, professionally qualified engineers. The data are founded on an evaluation of all the relevant information, both published and unpublished, and are invariably supported by original work of ESDU staff engineers or consultants. The whole process is subject to independent review for which crucial support is provided by industrial companies, government research laboratories, universities and others from around the world through the participation of some of their leading experts on ESDU Technical Committees. This process ensures that the results of much valuable work (theoretical, experimental and operational), which may not be widely available or in a readily usable form, can be communicated concisely and accurately to the engineering community.

We are constantly striving to develop new work and review data already issued. Any comments arising out of your use of our data, or any suggestions for new topics or information that might lead to improvements, will help us to provide a better service.

THE PREPARATION OF THIS DATA ITEM

The work on this particular Data Item which supersedes ESDU Aero W.02.04.01 & Aero W.S.02.04.12, was monitored and guided by the Aerodynamics Committee. This Committee first met in 1942 and now has the following membership:

Chairman	
Prof. D.W. Holder	— University of Oxford
Vice-Chairman	
Prof. G.M. Lilley	— University of Southampton
Members	
Mr E.C. Carter	— Aircraft Research Association
Dr L.F. Crabtree	— Royal Aircraft Establishment
Mr R.L. Dommatt	— Royal Aircraft Establishment
Mr H.C. Garner	— National Physical Laboratory
Mr M.W. Salisbury	— British Aircraft Corporation (Weybridge) Ltd
Mr J.B. Scott-Wilson	— Hawker Siddeley Aviation Ltd, Woodford
Mr A.C. Southgate	— British Aircraft Corporation (Guided Weapons) Ltd
Mr J.W.H. Thomas	— Hawker Siddeley Aviation Ltd, Hatfield
Mr J. Weir	— University of Salford
Mr W.F. Wiles	— Rolls-Royce Ltd.

The construction and subsequent development of the Data Item was undertaken by

Mr A.J. Ward Smith	— Head of Fluid Mechanics and Thermodynamics Group
Mr P.D. Chappell	— Senior Engineer, Fluid Mechanics and Thermodynamics Group.

The person with overall responsibility for the work in this subject area is Mr R.W. Gilbey, Head of Aircraft Aerodynamics Group.

THE COMPRESSIBLE TWO-DIMENSIONAL LAMINAR BOUNDARY LAYER, BOTH WITH AND WITHOUT HEAT TRANSFER, ON A SMOOTH FLAT PLATE, WITH APPLICATION TO WEDGES, CYLINDERS AND CONES

CONTENTS

	Page
1. NOTATION AND UNITS	1
2. INTRODUCTION	2
3. METHODS USED AND DATA PRESENTATION	3
3.1 Flat Plate Flow	3
3.1.1 Incompressible flow	3
3.1.2 Compressible flow with or without heat transfer	3
3.2 Supersonic Wedge Flow	4
3.3 Axisymmetric Flow	4
3.3.1 Parallel circular cylinder	4
3.3.2 Right circular cone in supersonic flow	4
4. LIMITATIONS	4
5. DERIVATION AND REFERENCES	5
5.1 Derivation	5
5.2 References	5
6. EXAMPLES	6
6.1 Example 1	6
6.2 Example 2	7
FIGURES 1 to 5	10-21

THE COMPRESSIBLE TWO-DIMENSIONAL LAMINAR BOUNDARY LAYER, BOTH WITH AND WITHOUT HEAT TRANSFER, ON A SMOOTH FLAT PLATE, WITH APPLICATION TO WEDGES, CYLINDERS AND CONES

1. NOTATION AND UNITS

		SI	British
C_F	mean skin friction coefficient, $= 2R_\theta/R_x$		
C_f	local skin friction coefficient, $= \tau_w/\frac{1}{2}\rho_1 u_1^2$		
c_p	specific heat capacity at constant pressure	J/kg K	CHU/slug K
H	boundary layer shape parameter, $= \delta^*/\theta$		
M	Mach number		
p	static pressure	N/m ²	lbf/ft ²
R_x	Reynolds number based on distance measured from leading edge of plate, $= u_1 x/v_1$		
$R_\delta, R_{\delta^*}, R_\theta$	Reynolds number based on boundary layer thickness, displacement thickness and momentum thickness, respectively		
r	temperature recovery factor (taken as 0.8485, i.e. $\sqrt{\sigma}$)		
T	temperature	K	K
T_w	wall temperature	K	K
u	local velocity in boundary layer	m/s	ft/s
x	distance measured from leading edge of plate	m	ft
y	distance measured normal to plate surface	m	ft
γ	ratio of specific heat capacities (taken as 1.4)		
δ	boundary layer thickness, taken as y at $u/u_1 = 0.995$	m	ft
δ^*	boundary layer displacement thickness, $= \int_0^\delta \left(1 - \frac{\rho u}{\rho_1 u_1}\right) dy$	m	ft
θ	boundary layer momentum thickness, $= \int_0^\delta \frac{\rho u}{\rho_1 u_1} \left(1 - \frac{u}{u_1}\right) dy$		

μ	viscosity	kg/m s	slug/ft s
μ_w	viscosity at wall	kg/m s	slug/ft s
ν	kinematic viscosity	m ² /s	ft ² /s
ρ	density	kg/m ³	slug/ft ³
σ	Prandtl number (taken as 0.72)		
τ_w	wall shear stress	N/m ²	lbf/ft ²
χ_∞	viscous interaction parameter (see Section 4)		

Suffixes

c	denotes cone flow
i	denotes incompressible flow ($M_1 = 0$, $T_w = T_{wr} = T_1$)
r	denotes recovery (zero heat transfer) conditions
1	denotes conditions at edge of boundary layer
∞	denotes conditions in uniform flow upstream.

2. INTRODUCTION

This Item presents data for the skin friction coefficients (Figures 1 and 3), shape parameter (Figure 4), momentum thickness (Figures 2 and 3), displacement thickness (Equation (2.1)) and boundary layer thickness (Figures 2 and 5) for the laminar boundary layer on a flat plate with zero pressure gradient. It covers the Reynolds number range $10^4 \leq R_x \leq 10^8$ for Mach numbers up to $M_1 = 5.0$, ratios of wall temperature to recovery temperature in the range $0.20 \leq T_w/T_{wr} \leq 2.00$, for ambient temperatures T_1 (K) = 100, 200, 216.5, 300 and 400.

The data given for compressible flow are for air. The data for incompressible flow (*i.e.* $M_1 = 0$, $T_w = T_{wr} = T_1$) are however applicable to any fluid.

Figures 1 and 2 present data based on the Blasius relations for skin friction and various thickness parameters for the case of the flat plate in incompressible flow (*i.e.* $M_1 = 0$, $T_w = T_{wr} = T_1$)[†], (see Section 3.1.1).

For the general case of the flat plate in compressible flow with heat transfer, (see Section 3.1.2), the following data are given:

Figure 3 presents the skin friction coefficients and momentum thickness, normalised with respect to the equivalent values for incompressible flow at the same Reynolds number, R_x . Figure 4 presents shape parameter, and Figure 5 provides data for normalised boundary layer thickness.

[†] Throughout this Item data are presented with reference to property values at the edge of the boundary layer (denoted by suffix 1). It will be realised that for the flat plate at zero incidence the properties at the edge of the boundary layer can be considered to be identical to those of the uniform flow upstream (denoted by suffix ∞) when the effects of induced pressure gradients are small (see Section 4).

Throughout this Item Reynolds number based on displacement thickness is obtained from the universal relation

$$R_{\delta^*} = HR_{\theta}. \quad (2.1)$$

The application of the Item is extended to cover the two-dimensional supersonic flow about sharp wedges, (see Section 3.2), the axisymmetric flow along cylinders and the axisymmetric supersonic flow about sharp-nosed cones by means of theoretical factors applied to the flat plate data, (see Sections 3.3.1 and 3.3.2).

Discussions of the limitations of the methods used in the preparation of this Item are given in Section 4 whilst the use of the Item is illustrated by two examples in Section 6.

3. METHODS USED AND DATA PRESENTATION

3.1 Flat Plate Flow

3.1.1 Incompressible flow

The exact solution due to Blasius (see for example Derivation 2) provides the required relations for local skin friction coefficient, mean skin friction coefficient and the Reynolds numbers based on momentum thickness and boundary layer thickness corresponding to $u/u_1 = 0.995$. These data are presented as functions of R_x for $10^4 \leq R_x \leq 10^8$ in Figures 1 and 2. The shape parameter, H , is invariant with Reynolds number, and is equal to 2.59. The Reynolds number based on displacement thickness is obtained using Equation (2.1).

3.1.2 Compressible flow with or without heat transfer

The effects of Mach number, heat transfer and ambient temperature on the laminar boundary layer properties of interest were calculated using the numerical solution of the boundary layer equations, for a perfect gas in equilibrium, of Derivation 3. Sutherland's law was used for the variation of viscosity with temperature and values of 0.72 and 1.4 were taken for the Prandtl number and ratio of specific heat capacities respectively. The computations confirmed the findings reported in Reference 8 that the temperature recovery factor, r , in the relation

$$T_{wr} = T_1(1 + 0.2rM_1^2) \quad (3.1)$$

is given very closely, for Prandtl numbers near unity, by the expression $r = \sqrt{\sigma}$ ($= 0.8485$ for this Item).

Figure 3 presents carpet plots of normalised values of local and mean skin friction coefficient (C_f/C_{fi} and C_F/C_{Fi}) and momentum thickness Reynolds number ($R_{\theta}/R_{\theta i}$) for Mach numbers in the range $0 \leq M_1 \leq 5.0$ and ratios of wall temperatures to recovery temperature in the range $0.20 \leq T_w/T_{wr} \leq 2.00$. Data are given for temperatures at the edge of the boundary layer of 100, 200, 216.5[†], 300 and 400 kelvin. Figures 4 and 5 present shape parameter[‡], H , and normalised boundary layer thickness, ($R_{\delta}/R_{\delta i}$), respectively, as functions of M_1 , T_w/T_{wr} and T_1 , covering the same ranges of these parameters as in

[†] It will be noted that the case of flight in the stratosphere ($T_1 = 216.5$ K for the flat plate case) has been treated as a particular case throughout this Item.

[‡] Since the shape parameter for incompressible flow is invariant with Reynolds number R_x , ($H_i = 2.59$), it is unnecessary to present H in the normalised form used for the other parameters given herein.

For given values of M_1 and T_w/T_{wr} the variation of H with T_1 is relatively small. Because the differences in values of H at $T_1 = 200$ K and 216.5 K are less than one per cent, these two cases have been represented by a single set of curves, Figure 4b.

Figure 3. The Reynolds number based on displacement thickness is again obtained via Equation (2.1).

3.2 Supersonic Wedge Flow

Since the supersonic flow over a sharp wedge is two-dimensional and since the flow properties between the shock and the boundary layer are constant, all the relationships for skin friction and boundary layer parameters relating to laminar flow over a flat plate may be applied directly to this case, provided that the properties of the flow downstream of the shock are used in the calculations.

3.3 Axisymmetric Flow

3.3.1 Parallel circular cylinder

Use of Reference 6, in which the effects of transverse curvature are considered for the case of unit Prandtl number, indicates that provided the displacement thickness is less than about two per cent of the radius then the flat plate data of this Item may be applied directly with a loss of accuracy less than about two per cent.

3.3.2 Right circular cone in supersonic flow

Using the Mangler transformation (see for example Derivation 1) for axisymmetric flow, the following relationships between flat plate and cone data for identical Reynolds numbers $(R_x \equiv (R_x)_c)^\dagger$, Mach numbers at the edge of the boundary layer $(M_1 \equiv (M_1)_c)$ and ratios of wall temperature to recovery temperature $(T_w/T_{wr} \equiv (T_w/T_{wr})_c)$ can be deduced:-

$$(C_f)_c = \sqrt{3} C_f \quad (3.2)$$

$$(C_F)_c = (2/3) \sqrt{3} C_F \quad (3.3)$$

$$H_c = H \quad (3.4)$$

$$(R_\theta)_c = (1/3) \sqrt{3} R_\theta \quad (3.5)$$

$$(R_{\delta^*})_c = (1/3) \sqrt{3} R_{\delta^*} \quad (3.6)$$

$$(R_\delta)_c = (1/3) \sqrt{3} R_\delta \quad (3.7)$$

Reference 6 shows that, for a given Reynolds number, displacement thickness/radius, and identical flow conditions at the edge of the boundary layer, the effects of transverse curvature on the laminar boundary layer over a cone are less than for a cylinder (see Section 3.3.1).

4. LIMITATIONS

This Item applies only to the quoted cases of two-dimensional and axisymmetric attached flow over smooth surfaces with zero pressure gradient. In Derivation 3 the effects of induced pressure gradients have been ignored. These effects can be significant for high values of the parameter $\chi_\infty = M_\infty^3 \sqrt{(C_\infty/R_\infty)}$, in which $C_\infty (= \mu_w T_\infty / \mu_\infty T_w)$ is the coefficient in the linear formula[‡] for viscosity, and $R_\infty (= u_\infty x / \nu_\infty)$ is the Reynolds number based on upstream values. The criterion $\chi_\infty < 0.1$ can be taken as a very rough guide to those conditions for which induced pressure gradient effects are small. For Mach numbers less than about

[†] For the case of the cone x is measured along the surface.

[‡] Also known as the Chapman-Rubesin formula.

three these effects are negligible at flight Reynolds numbers and, though they are more significant under wind-tunnel conditions, errors arising from this source are generally well within the accuracy of most supersonic wind-tunnel data.

The range of Reynolds numbers, R_x , for which data are presented in this Item is that considered most likely to be useful in practice. The practical upper limit is, for example, dictated by the onset of transition. For incompressible flow this is not likely to be much above 10^6 , whilst for compressible flow, particular combinations of M_1 and T_w/T_{wr} (<1) can, in theory (see for example Reference 5), lead to a completely stable boundary layer and, in practice, transition Reynolds numbers as high as 10^8 have been achieved (Reference 5).

Use of the data for compressible flow in this Item is restricted to the case of air. The data for incompressible flow are applicable to any fluid.

The data given in this Item should be applied with caution to cases of high wall temperature for a number of reasons. There is evidence (see References 4 and 7) to show that, if thermal variations of γ , c_p and σ are taken into account in the calculation procedure, whilst the effect on skin friction coefficients and momentum thickness is small (less than about two per cent in the worst case over the range of data presented in this Item) significant deviations (up to about ten per cent) from the perfect gas values for shape parameter, displacement thickness and boundary layer thickness can ensue for combinations of high M_1 , T_w/T_{wr} and T_1 . The magnitudes and trends of such deviations are however very dependent on the thermal variation of Prandtl number assumed in the calculation procedure, and values for this quantity are uncertain at high temperatures (above about 600 K) due to lack of corroborative experimental data. Further, elevation of the temperature above about 2000 K, depending on the pressure, results in dissociation.

The methods suggested in this Item for wedges and cones do not apply to subsonic flow. Also, for the cases of cylinders and cones, the use of these data is restricted to thin boundary layers, *i.e.* displacement thickness less than about two per cent of the radius.

5. DERIVATION AND REFERENCES

5.1 Derivation

The Derivation lists selected sources of information that have assisted in the preparation of this Item.

1. HOWARTH, L. *Modern Developments in Fluid Dynamics. High Speed Flow.* Vol. 1, Ch. X, pp. 382-386. Oxford University Press, 1953.
2. CURLE, N. *The Laminar Boundary Layer Equations.* Ch. 2, Section 1. Oxford University Press, 1962.
3. SELLS, C.C.L. Two-dimensional laminar compressible boundary layer programme for a perfect gas. ARC R & M 3533, 1966.

5.2 References

The References list sources of information supplementary to that in this Item.

4. YOUNG, G.B.W. JANSSEN, E. The compressible boundary layer. *J. aeronaut. Sci.*, Vol. 19, pp. 229-236, April 1952.