



ESDU 68020

Issued May 1968
With Amendments A and C
March 1988
Supersedes in part
Aero W.02.04.01
Aero W.S.02.04.12

The compressible two-dimensional turbulent boundary layer, both with and without heat transfer, on a smooth flat plate, with application to wedges, cylinders and cones

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THE COMPRESSIBLE TWO-DIMENSIONAL TURBULENT BOUNDARY LAYER, BOTH WITH AND WITHOUT HEAT TRANSFER, ON A SMOOTH FLAT PLATE, WITH APPLICATION TO WEDGES, CYLINDERS AND CONES

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1. NOTATION AND UNITS

Any coherent set of units may be used. Two such sets are given.

		SI	British
C_F	mean skin friction coefficient, $= 2R_0/R_x$		
C_f	local skin friction coefficient, $= \tau_w / \frac{1}{2}\rho_1 u_1^2$		
H	boundary layer shape parameter, δ^*/θ		
M_1	Mach number at edge of boundary layer		
n	u/u_1 at $y = \delta(n)$		
p_1	static pressure at edge of boundary layer	N/m ²	lbf/ft ²
R_x	Reynolds number based on distance measured from virtual origin of boundary layer, $u_1 x / \nu_1$		
$R_{\delta(n)}, R_{\delta^*}, R_\theta$	Reynolds numbers based on boundary layer thickness, displacement thickness and momentum thickness, respectively		
r	temperature recovery factor (taken as 0.89)		
T_w	wall temperature	K	K
T_1	temperature at edge of boundary layer	K	K
u	local velocity in boundary layer	m/s	ft/s
u_1	velocity at edge of boundary layer	m/s	ft/s
x	distance measured along surface from virtual origin of turbulent boundary layer	m	ft
y	distance measured normal to surface	m	ft
γ	ratio of specific heat capacities (taken as 1.4)		
$\delta(n)$	boundary layer thickness, i.e. y at $u/u_1 = n$	m	ft
δ^*	boundary layer displacement thickness, $= \int_0^{\delta(n)} \left(1 - \frac{\rho u}{\rho_1 u_1}\right) dy$	m	ft

Issued May 1968 – 26 pages
With Amendment C, March 1988

θ	boundary layer momentum thickness, $\int_0^{\delta(n)} \frac{\rho u}{\rho_1 u_1} \left(1 - \frac{u}{u_1}\right) dy$		
μ	viscosity	kg/m s	slug/ft s
ν_1	kinematic viscosity at edge of boundary layer	m ² /s	ft ² /s
ρ	local density in boundary layer	kg/m ³	slug/ft ³
ρ_1	density at edge of boundary layer	kg/m ³	slug/ft ³
σ	Prandtl number (taken as 0.72)		
τ_w	wall shear stress	N/m ²	lbf/ft ²

Suffixes

c	relating to cone flow
i	denotes incompressible flow ($M_1 = 0$ and $T_w = T_{wr} = T_1$)
r	denotes recovery (zero heat transfer) conditions
∞	denotes conditions in uniform flow upstream.

2. INTRODUCTION

This Item presents data for the skin friction coefficients (Figures 2, 3, 6 and 7), shape parameter (Figures 5 and 8), momentum thickness (Figures 4 and 7), displacement thickness (Equation (4.1)) and boundary layer thickness (Figure 9) for the turbulent boundary layer on a smooth flat plate with zero pressure gradient. It covers the Reynolds number range $10^5 \leq R_x \leq 10^9$ for Mach numbers up to $M_1 = 5.0$ and ratios of wall temperature to recovery temperature in the range $0.25 \leq T_w/T_{wr} \leq 2.00$.

The data given for compressible flow are for air or other gases having $\gamma = 1.4$ and $\sigma = 0.72$. The data for incompressible flow (*i.e.* $M_1 = 0$ and $T_w = T_{wr} = T_1$) are however applicable to any fluid.

Spalding and Chi in Derivation 2 have employed a large amount of wind-tunnel experimental data to develop a semi-empirical method which, when compared with twenty other methods, produces the best agreement with experimental data for skin friction. The method of Derivation 2 has been used to present data for skin friction and momentum thickness (see Sections 4.1 and 4.3). However, Derivation 5 shows that the predictions for H , R_{δ^*} and $R_{\delta(n)}$, using the method of Derivation 2, are not satisfactory when compared with available experimental data and improved correlations are established.[†] Thus these correlations have been used herein to present the data for shape parameter and boundary layer thickness (see Sections 4.2 and 4.3).

[†] Apart from the inability of the Spalding and Chi method to provide satisfactory predictions for H , R_{δ^*} and $R_{\delta(n)}$ (due primarily to the form of the implied velocity distribution⁵), the correlation procedure from which the Spalding and Chi relations for skin friction and momentum thickness were derived also has certain shortcomings. For example, the position of the virtual origin as estimated by each individual experimenter was accepted. Nevertheless, the method does represent at the present time the best correlation of a wide range of experimental skin friction data including the effects of Mach number and heat transfer, and it is on this basis that it is used here.

The application of the Item is extended to cover the two-dimensional supersonic flow about sharp wedges (see Section 5), the axisymmetric flow along cylinders and the axisymmetric supersonic flow over pointed cones by means of approximate factors applied to the flat plate data (see Section 6). For cones these factors are derived⁴ by a power-law analysis.

Section 7 indicates how the methods presented in this Item compare with available wind-tunnel experimental data, whilst limitations of the methods are discussed in Section 8. One important limitation concerns R_x , and lower boundaries for the applicability of this Item have been calculated. These boundaries are presented in Figure 1 and are also shown hatched on the remaining Figures where this is practicable.

Examples to illustrate the use of this Item are given in Section 10.

3. THE VIRTUAL ORIGIN

Any practical analysis of turbulent boundary layers requires a knowledge of the Reynolds number based on the length of turbulent flow, R_x . In analysing experimental data the conditions and rate of growth of the turbulent boundary layer at a given station are assumed to be equivalent to those of a fully-developed turbulent boundary layer originating from some point aft of the plate leading-edge. This fictitious origin is often referred to as the virtual origin, and R_x is therefore the Reynolds number based on the distance, x , measured from this point.

In practice there are several methods of estimating the position of the virtual origin, the most common of which is an extrapolation to zero of the turbulent boundary layer momentum thickness development. In establishing the method of Derivation 2 the authors accepted the estimation of the virtual origin as given by each individual experimenter, and made no attempt to rationalise the experimental data in this respect. However, it is felt that any resulting errors would be more than offset by differences in experimental technique or variations from one wind-tunnel to another. Any error in estimating the position of the virtual origin will obviously have the greatest effect at low Reynolds numbers (below say 10^6), whilst for high Reynolds numbers (above say 10^8) its effect is usually negligible.

4. FLAT PLATE FLOW[†]

4.1 Skin Friction Coefficients, C_f C_F

Figures 2 and 3 present the local and mean skin friction coefficients for the case of zero heat transfer, C_{fr} and C_{Fr} respectively, for a range of Reynolds numbers from 10^5 to 10^9 and Mach numbers up to 5.0 in steps of 0.5.

Figures 6 and 7 present carpet plots for C_f and C_F respectively for the case of heat transfer. For convenience of presentation the skin friction data are normalised by dividing by the relevant coefficient for zero heat transfer at the same Reynolds number and Mach number. Carpets are given for ratios of wall to recovery temperature from 0.25 to 2.00 in steps of 0.25.

[†] Throughout this Item data are presented with reference to property values at the edge of the boundary layer (denoted by suffix 1). It will be realised that for the flat plate at zero incidence the properties at the edge of the boundary layer can be considered to be identical to those of the uniform flow upstream (denoted by suffix ∞) when the effects of induced pressure gradients are small (see Section 8).

4.2 Shape Parameter, H

The variation of the boundary layer shape parameter with Reynolds number in incompressible flow (*i.e.* $M_1 = 0$ and $T_w = T_{wr} = T_1$) is shown in Figure 5. The effects of Mach number and wall temperature ratio on the shape parameter, normalised with respect to the value for incompressible flow, are presented in Figure 8.

4.3 Thickness Parameters, R_θ , $R_{\delta(n)}$ and R_{δ^*}

Figure 4 gives the Reynolds number based on momentum thickness with zero heat transfer, R_{θ_r} , for Mach numbers in unit steps up to 5.0, and Reynolds numbers from 10^5 to 10^9 .

Figure 7 presents carpet plots of R_θ for the case of heat transfer, normalised with respect to the equivalent value for zero heat transfer. It will be noted that these plots are the same as for mean skin friction, as would be expected due to the relationship $C_F = 2R_\theta/R_x$.

The variation of Reynolds number based on boundary layer thickness $R_{\delta(0.999)}$, with Reynolds number R_x , invariant with Mach number and temperature ratio over the ranges covered by this Item, is given in Figure 9. The inset sketch on Figure 9 indicates the variation of boundary layer thickness with n ($= u/u_1$ at $\delta(n)$), obtained from experimental data for incompressible flow⁵.

To obtain the Reynolds number based on displacement thickness, R_{δ^*} , use is made of the relation

$$R_{\delta^*} = HR_\theta, \quad (4.1)$$

in which H is obtained from Figures 5 and 8, and R_θ is obtained from Figures 4 and 7.

5. SUPERSONIC WEDGE FLOW

Since the supersonic flow over a sharp wedge is two-dimensional, and since the flow properties between the shock and the boundary layer are constant, all the relationships for skin friction and boundary layer parameters relating to the turbulent flow over a flat plate may be applied directly to this case, provided that the properties of the flow downstream of the shock are used in the calculations.

6. AXISYMMETRIC FLOW

6.1 Parallel Circular Cylinder

Provided that the displacement thickness is of the order of one per cent or less of the radius then flat plate data are expected to apply directly with negligible loss of accuracy.

6.2 Right Circular Cone in Supersonic Flow

Wilson⁴, assuming a 1/7th power law for the velocity profile, gives expressions from which the following relations between flat plate and cone data for identical Reynolds numbers ($R_x \equiv (R_x)_c$)[†], Mach numbers at the edge of the boundary layer ($M_1 \equiv (M_1)_c$) and ratios of wall to recovery temperature ($T_w/T_{wr} \equiv (T_w/T_{wr})_c$) can be deduced:-

[†] For the case of the cone x is measured along the surface.

$$(C_f)_c = 1.18 C_f, \quad (6.1)$$

$$(C_F)_c = 1.05 C_F, \quad (6.2)$$

$$H_c = H, \quad (6.3)$$

$$(R_\theta)_c = 0.523 R_\theta, \quad (6.4)$$

$$(R_{\delta(n)})_c = 0.523 R_{\delta(n)}, \quad (6.5)$$

$$(R_{\delta^*})_c = 0.523 R_{\delta^*}. \quad (6.6)$$

These expressions are valid for thin boundary layers, *i.e.* for displacement thicknesses of the order of one per cent or less of the local radius.

The factor of 1.05 on mean skin friction given in Equation (6.2) has been substantiated by a large number of experimental data³ from a cone for zero heat transfer at Mach numbers of 1.4 and 2.8 over the Reynolds number range $10^6 < (R_x)_c < 10^8$. These data indicate that the factor is a very weak function of $(R_x)_c$, decreasing by less than one per cent over the range $10^6 < (R_x)_c < 10^8$. There is little experimental evidence on which to judge the veracity of the remaining relations given above, but it is thought that they should provide values to the same order of accuracy as those given by the basic flat plate data.

7. COMPARISON WITH EXPERIMENTAL DATA

Available experimental data for skin friction, shape parameter and the thickness parameters (excluding $R_{\delta(n)}$) of the incompressible flat plate turbulent boundary layer^{2, 5} are correlated by the method of the present Item with an r.m.s error of 2 or 3 per cent.

The experimental data collected by Spalding and Chi for compressible flow are correlated with an r.m.s error of 9 per cent by the basic method for predicting flat plate skin friction for the case of zero heat transfer and with an r.m.s. error 13 per cent for the case of heat transfer (see Derivation 2). These values should also give a useful guide to the reliability of the method for predicting momentum thickness.

The available experimental data for shape parameter correlate with the predicted values with an r.m.s. error of 4 per cent for zero heat transfer and 8 per cent for the case of heat transfer. The relevant values for displacement thickness are 5 and 11 per cent respectively (see Derivation 5).

Derivation 5 also shows that the available experimental data for flat plate boundary layer thickness at $n = 0.999$ are correlated with an r.m.s error of 14 per cent by the prediction method.

The comparison of experiment with theory in the case of two-dimensional supersonic flow over sharp wedges and circular cylinders in axial flow should produce r.m.s. error values very similar to those for flat plates, provided that in the case of cylinders the thin boundary layer restriction is applied (*i.e.* displacement thickness of the order of one per cent or less of the radius).

Available experimental data³ for the mean skin friction of a sharp-nosed cone in axial supersonic flow indicate that the factor quoted in Equation (6.2) is accurate to within about two per cent over the range of the given flat plate data, at least for zero heat transfer. For the case of heat transfer and for the other cone relations quoted in Section 6.2 lack of experimental data makes difficult any assessment of the accuracy of the factors involved.

7.1 Ranges of Experimental Data

The ranges of the variables R_x , M_1 and T_w/T_{wr} for which experimental flat plate data exist, for each of the boundary layer parameters presented in this Item, are given in Table 11.1 for incompressible flow and Tables 11.2 and 11.3 for compressible flow, respectively. The experimental data which these ranges represent were used in the derivation of the methods used in this Item and also in the comparisons for the flat plate case discussed in Section 7.

8. LIMITATIONS

This Item applies only to the quoted cases of two-dimensional and axisymmetric attached flow over aerodynamically smooth surfaces with zero pressure gradient.

Caution should be exercised in using those data in this Item which lie outside the ranges of the experimental data from which the methods used herein were developed (see Tables 11.1 to 11.3).

In developing the methods used in this Item the effects of self-induced pressure gradients have been ignored. Errors from this source are negligible when compared with the overall accuracy of the methods.

Use of the data for compressible flow is restricted to air or other gases having $\gamma = 1.4$ and $\sigma = 0.72$. The data for incompressible flow are applicable to any fluid. The data should be applied with caution to conditions of high wall temperature because of the likely existence of real gas effects such as vibrational excitation and dissociation.

Lack of sufficient experimental data, other than those for mean skin friction under conditions of zero heat transfer, prevents thorough assessment of the method suggested for the case of sharp-nosed cones. The method should therefore be considered somewhat tentative.

It should be noted that the methods suggested in this Item for wedges and cones are not applicable to subsonic Mach numbers. Also for the cases of cylinders and cones the use of this Item is restricted to thin boundary layers, *i.e.* displacement thickness of the order of one per cent or less of the radius.

Although the data have been presented for Reynolds numbers down to $R_x = 10^5$ it should be pointed out that there is some evidence¹ to suggest that a turbulent boundary layer cannot exist at Reynolds numbers based on momentum thickness less than a value of about 320, for incompressible flow at least. This corresponds to a Reynolds number $R_x \approx 10^5$ at $M_1 = 0$ and $T_w/T_{wr} = 1.0$, and application of the Spalding and Chi transformation produces the boundaries shown in Figure 1. These boundaries are also shown hatched on Figures 2, 3, 4, 6 and 7. Extreme caution should be exercised in using data for values of R_x less than, or in the immediate vicinity of these boundaries. Data for R_x less than the boundary values are shown as broken lines on the Figures.

9. DERIVATION

The Derivation lists sources of information that have assisted in the preparation of this Item.

1. PRESTON, J.H. The minimum Reynolds number for a turbulent boundary layer and the selection of a transition device. *J. fluid Mech.*, Vol. 3, Part 4, pp. 373-384, January 1958.
2. SPALDING, D.B.
CHI, S.W. The drag of a compressible turbulent boundary layer on a smooth flat plate with and without heat transfer. *J. fluid Mech.*, Vol. 18, Part 1, pp. 117-143, January 1964.
3. WINTER, K.G. Unpublished work at RAE (Bedford), 1964.
4. WILSON, R.E. Handbook of supersonic aerodynamics. Sections 13 and 14. Viscosity and heat transfer effects. NAVORD Rep. 1488 (Vol. 5), August 1966
5. CHAPPELL, P.D. Some correlations for the turbulent boundary layer on a flat plate. *Aeronaut. J.*, Vol. 74, pp. 393-396, May 1970.

10. EXAMPLES

10.1 Example 1

A stream of air at a Mach number of 2.5 flows over a perfectly insulated smooth flat plate set at zero incidence. Find the local skin friction coefficient, the momentum thickness and shape parameter at a station corresponding to a Reynolds number based on the length of turbulent flow of 10^8 . Assume that the free-stream values of kinematic viscosity and speed of sound are $1.224 \times 10^{-4} \text{ m}^2/\text{s}$ and 295 m/s respectively.

From Figure 2, for $M_1 (= M_\infty) = 2.5$ and $R_x = 10^8$,

$$C_{fr} = 0.00123 .$$

From Figure 4, for $M_1 = 2.5$ and $R_x = 10^8$,

$$R_{\theta r} = 7.20 \times 10^4 .$$

Thus,

$$\theta_r = R_{\theta r} \frac{v_1}{u_1} = 7.20 \times 10^4 \times \frac{1.224 \times 10^{-4}}{295 \times 2.5}$$

$$= 0.01195 \text{ m} .$$

From Figure 5, for $R_x = 10^8$,

$$H_i = 1.255 .$$

Also, from Figure 8, for $M_1 = 2.5$ and $\frac{T_w}{T_{wr}} = 1$,

$$\frac{H_r}{H_i} = 3.12 .$$

Thus,

$$H_r = \frac{H_r}{H_i} H_i = 3.12 \times 1.255 = 3.92 .$$