

THE BUCKLING OF FLAT PLATES UNDER NON-UNIFORM COMPRESSION SYMMETRIC DISTRIBUTIONS DUE TO INITIAL OR THERMAL STRESS

1. NOTATION

a	plate length	m	in
b	plate width	m	in
E	Young's modulus	N/m ²	lbf/in ²
f_b	elastic buckling stress of plate under uniform compression	N/m ²	lbf/in ²
f_c	value of f_x at plate centre-line	N/m ²	lbf/in ²
f_{cb}	value of f_c at which plate first buckles	N/m ²	lbf/in ²
f_e	value of f_x at plate edge	N/m ²	lbf/in ²
f_x	axial stress on plate at distance y from plate edge, positive in compression	N/m ²	lbf/in ²
K	elastic buckling stress coefficient for uniform loading defined by $f_b = KE\left(\frac{t}{b}\right)^2$		
t	plate thickness	m	in
x, y	reference axes of plate		
ν	Poisson's ratio		

Both SI and British units are quoted but any coherent system of units may be used.

2. NOTES

This Data Item provides a method of checking the elastic stability of rectangular plates subject to stresses that are compressive at the centre-line and distributed parabolically across the width. The data are valid only for distributions, such as those arising from initial or thermal strain, that do not vary along the length of the plate. This Item is not therefore applicable to external load distributions that diffuse rapidly into the plate.

Figure 1 gives values of K plotted against a/b for various edge conditions, assuming $\nu = 0.3$. For values of ν that are appreciably different, K should be multiplied by $0.91/(1-\nu^2)$.

Figure 2 shows the variation of f_{cb}/f_b with f_e/f_c for the same range of edge conditions.

Figure 3 presents the family of parabolas f_x/f_c corresponding to various values of f_e/f_c . The general expression for these distributions is

$$\frac{f_x}{f_c} = \frac{f_e}{f_c} + \frac{4y}{b} \left(1 - \frac{y}{b}\right) \left(1 - \frac{f_e}{f_c}\right).$$

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When calculating f_b allowance should be made for possible variations with temperature in the modulus of the plate material. Where temperatures are non-uniform, the value of E selected should be appropriate to the mean temperature of the middle two thirds of the plate.

The value of f_{cb} can be obtained for problems involving non-parabolic distributions by dividing f_x by f_c and then selecting an appropriate curve from Figure 3. The curve chosen should be a close fit over the middle two thirds of the plate width. The value of f_e/f_c for this curve is used in place of the actual value of f_e/f_c in determining f_{cb} .

A more accurate solution for distributions that are non-parabolic can be obtained from the equations in the Derivation.

3. DERIVATION

Using the Rayleigh-Ritz energy method, and assuming the buckling mode is the same as that of a plate under uniform compression, it can be shown that

$$\frac{f_{cb}}{f_b} = \frac{\int_0^a \int_0^b (\partial w / \partial x)^2 dx dy}{\int_0^a \int_0^b (f_x / f_c) (\partial w / \partial x)^2 dx dy},$$

where w is the normal displacement of the plate.

The accuracy of this expression has been confirmed by finite element analysis. It can be simplified further by assuming wave forms appropriate to the conditions of panel edge restraint, so that:

where the panel sides are simply-supported (Cases (i) and (ii) of Figure 1)

$$\frac{f_{cb}}{f_b} = \frac{b}{2 \int_0^b (f_x / f_c) \sin^2 \frac{\pi y}{b} dy},$$

and where the panel sides are clamped (Cases (iii) and (iv) of Figure 1)

$$\frac{f_{cb}}{f_b} = \frac{3b}{8 \int_0^b (f_x / f_c) \sin^4 \frac{\pi y}{b} dy}.$$

This method of analysis is discussed in the following reference:

BENOY, M.B. An energy solution to the buckling of rectangular plates under non-uniform in-plane loading. *Jl R. aeronaut. Soc.*, Vol. 73, pp. 974-977, November 1969.

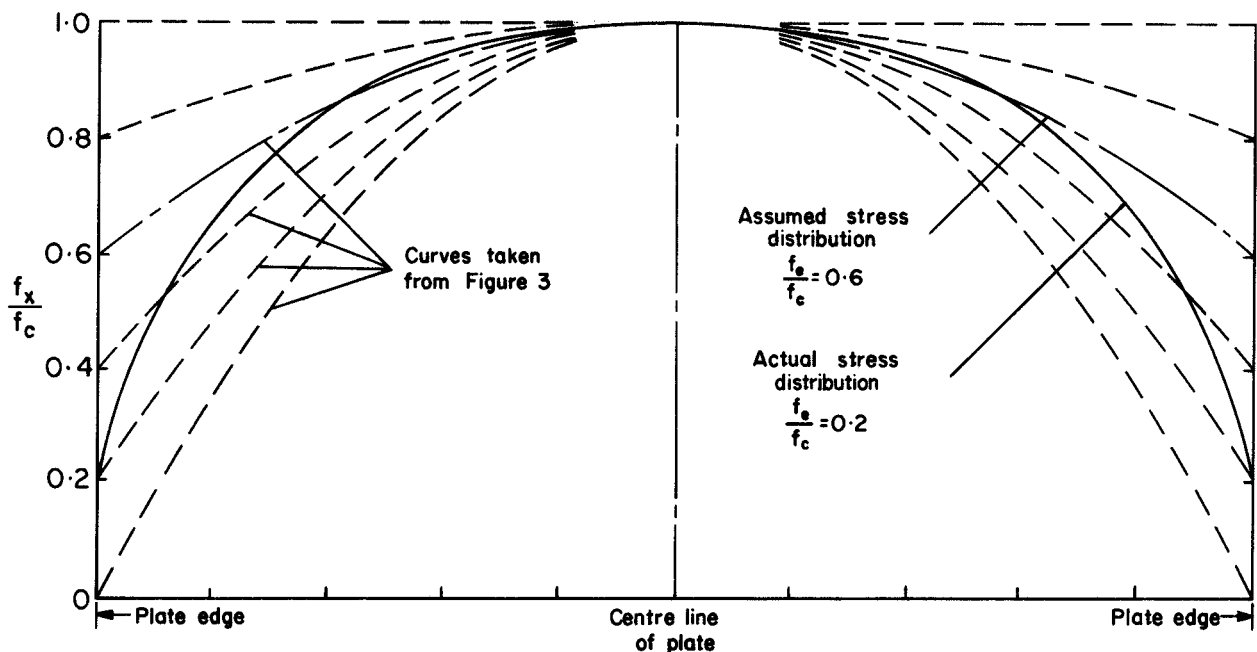
4. EXAMPLE

During deceleration from supersonic flight, transient thermal compressive stresses are set up in an aircraft panel. It is required to determine whether or not, at any time during the deceleration, buckling occurs.

The panel is made from DTD 5070 material, it has ends clamped and sides simply-supported, and the principal dimensions are

$$a = 12 \text{ in}, \quad b = 6.5 \text{ in}, \quad t = 0.080 \text{ in}.$$

It is necessary to check the stability of the panel under a number of temperature and stress distributions, corresponding to various points in time. One of these stress distributions is illustrated below. It has $f_c = 7000 \text{ lbf/in}^2$, $f_e/f_c = 0.2$, and the mean temperature in the middle of the plate is 100°C .



For the purpose of analysis this stress distribution is compared with the curves of Figure 3 (as shown above), and an equivalent parabola is assumed having $f_e/f_c = 0.6$.

Reference to material properties data indicates that at 100°C the elastic modulus of DTD 5070 falls to 97 per cent of its room temperature value, given as $10.6 \times 10^6 \text{ lbf/in}^2$.

$$\text{Hence,} \quad E = 0.97 \times 10.6 \times 10^6 = 10.3 \times 10^6 \text{ lbf/in}^2.$$

From Figure 1, at $a/b = 12/6.5 = 1.85$,

$$K = 4.57,$$

$$\text{so that} \quad f_b = 4.57 \times 10.3 \times 10^6 \left(\frac{0.080}{6.5} \right)^2 = 7130 \text{ lbf/in}^2$$