

THE DETERMINATION OF GROSS THRUST AND MASS FLOW IN FLIGHT. (AIR BREATHING DUCTED-FLOW ENGINES WITH CONVERGENT NOZZLES)

1. NOTATION AND UNITS*

		SI	British
A	cross-sectional area	m^2	ft^2
$A_{4_{eff}}$	effective efflux area from calibration (see Section 3.2.1)	m^2	ft^2
a	local speed of sound	m/s	ft/s
$F_{G_{std}}$	standard gross thrust	N	lbf
K_1	thrust ratio (see Section 3.2.1)		
K_2	discharge ratio (see Section 3.2.2)		
M	Mach number = v/a		
m	mass flow	kg/s	slug/s
p_s	static pressure	N/m^2	lbf/ft^2
p_{s_0}	freestream static pressure	N/m^2	lbf/ft^2
p'_{s_0}	static pressure in discharge region	N/m^2	lbf/ft^2
p_t	total pressure	N/m^2	lbf/ft^2
R	gas constant	J/kg K	lbf ft/slug K
T_t	total temperature	K	K
v	velocity	m/s	ft/s
γ	ratio of specific heat capacities of exhaust gas		
ε	total pressure factor = p_{t_4}/p_{t_3}		
ρ	density	kg/m^3	slug/ft^3

Subscripts

3 and 4 refer to stations in jet pipe and at nozzle throat, respectively

* The system of subscripts, 0 to 5, denoting engine stations in this Item and in Item Nos 69006 and 69008 is not widely used elsewhere. A more widely-accepted system is defined in Reference 8 (see Sketch 1.1 in Item No. 76034) and has been applied to the subject of thrust measurement in Reference 9. While this does not affect the information given here, care should be taken to establish which conversion is used in any particular application.

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2. GENERAL

This data sheet is concerned only with the measurement of standard gross thrust and nozzle mass flow and these may be calculated from a knowledge of the total pressure, static pressure and total temperature distributions. The direct method is therefore to traverse the efflux for these quantities as discussed in Section 4. However, the approach most frequently adopted, since there are fewer installation difficulties, is to sample a flow quantity (either total pressure or static pressure) at Station 3 and to determine the gross thrust by reference to test bed calibration.

3. TECHNIQUES INVOLVING MEASUREMENTS IN THE JET PIPE*

3.1 Introduction

The thrust and mass flow are calculated from the measured quantities, assuming one-dimensional isentropic flow (or adiabatic flow if a total pressure factor, ϵ , is defined, see Section 3.2.1) with constant γ , see References 9, 10 and 12. Variations from this ideal (including any deviation of the flow from normal to the exit plane or complex nozzle geometries) may be allowed for by calibration factors determined on a test bed. It is frequently necessary to extrapolate this calibration to higher nozzle pressure ratios than can be achieved on a ground test bed, although an Altitude Test Facility or an ejector shroud may enable the necessary pressure ratios for the flight case to be simulated.

3.2 Total Pressure and Temperature Measurements

For a pure jet engine a single pitot has been found quite satisfactory. The total pressure sampling method has been used successfully on by-pass engines in which the turbine and by-pass gases exhaust through a single nozzle. In such a case a rake is used across the flows and the mean total head and temperature used for the calibration. For by-pass engines in which the by-pass air does not exhaust through the same nozzle as the turbine flow, current experience suggests the use of total head sampling in both flows with model tests, supported by engine tests in an Altitude Test Facility, to relate the total head measurements to thrust (see Reference 14). Engines with reheat involve difficulty because of the high temperature of the efflux although some success has been achieved by means of measurements upstream of the reheat system (Reference 13).

3.2.1 Gross thrust

The standard gross thrust is

$$F_{G_{std}} = \rho_4 v_4^2 A_4 + (p_{s_4} - p_{s_0}) A_4 \quad (3.1)$$

where $\rho_4 v_4^2$ and p_{s_4} are mean values over the area A_4 .

Assuming γ is constant and that the flow in the pipe is one-dimensional and isentropic Equation (3.1) can be rewritten in the form[†]

$$\left[\frac{F_{G_{std}}}{p_{s_0} A_4} \right]_{ideal} = \frac{2\gamma}{\gamma - 1} \frac{p_{t_4}}{p_{s_0}} \left[\left(\frac{p_{s_4}}{p_{t_4}} \right)^{1/\gamma} - \frac{p_{s_4}}{p_{t_4}} \right] + \left[\frac{p_{s_4}}{p_{s_0}} - 1 \right]. \quad (3.2)$$

* See Addendum A.

† It should be noted that the expression for $F_{G_{std}}$ is equivalent to that for F in Reference 10 and 12: M_4 may be expressed in terms of p_{s_4}/p_{t_4} .

It is not usual to measure the pressure p_{s_4} and its value is obtained from convergent nozzle theory (Reference 1) under the assumptions already made. This gives

$$\frac{p_{s_4}}{p'_{s_0}} = 1$$

below the choking condition and

$$\frac{p_{s_4}}{p_{t_4}} = \left(\frac{2}{\gamma + 1} \right)^{\gamma/(\gamma - 1)}$$

above the choking condition*. For conventional installations it may safely be assumed that p'_{s_0} is equal to p_{s_0} ; in some novel installations this assumption may lead to appreciable error. Item No. 69008 presents curves for Equation (3.2) covering different values of p'_{s_0}/p_{s_0} below the choking condition.

Thus, if a pitot head can be placed in the jet pipe upstream of the exit nozzle, at Station 3, to measure the mean value of p_{t_3} , and the assumption is made that p_{t_3} is equal to p_{t_4} , Equation (3.2) may be used to calculate the standard gross thrust. In practice, however, it may not be possible to place the pitot head in a single position where it will always measure the mean p_{t_3} since the distribution of total pressure will vary with operating conditions and will not be uniform. The pitot head therefore “samples” the total pressure p_{t_3} and the relationship between the measured values and the test bed thrust measurements is established. The procedure is to determine $F_{G_{std}}$ and the sample p_{t_3} and to substitute the values in Equation (3.2) to obtain an “effective area” $A_{4_{eff}}$ which will be a function of p_{t_3}/p_{s_0} . This calibration is then used in obtaining the standard gross thrust from flight measurements. It allows for deviations from the thrust and mass flow predicted by one-dimensional isentropic theory arising both from the lack of flow uniformity and from energy losses (these effects may be treated as thrust and discharge ratios K_1 and K_2 , respectively, defined later, rather than jointly in the calibration factor $A_{4_{eff}}$) and it also allows for any failure to record a mean p_{t_3} arising from both installation and instrument errors.

However, on a simple ground test bed it is not possible to obtain jet pipe pressure ratios very much greater than the choking value (e.g. 1.850 for $\gamma = 1.33$) while considerably higher values will be obtained in flight. The necessary pressure ratios may be obtained by means of an Altitude Test Facility, or ratios up to about 9 may be simulated by means of an ejector shroud around the jet efflux. Since it is found that the effective area becomes constant for pressure ratios greater than about 3, this latter method will enable a more than adequate range of pressure ratios to be covered. However, there is little experience in the application of this method to thrust measurement and difficulties may be experienced since correction may be necessary for the pressure distribution on the tail pipe. If these facilities are not available the calibration curve must be extrapolated and two possible methods are available for doing this.

One method is to extrapolate $A_{4_{eff}}$ above the pressure ratios reached on the test bed on the basis of experience. Another method is by means of model tests, defining a thrust ratio as

$$K_1 = \frac{[F_{G_{std}}/(p_{s_0} A_4)]_{actual}}{[F_{G_{std}}/(p_{s_0} A_4)]_{ideal}}$$

* As the pressure ratio across a convergent nozzle is increased, the exit Mach number increases. When a Mach number of unity is reached the flow is described as “choking” and the theory defines the associated “critical” nozzle pressure ratio $p_{t_4}/p_{s_4} = [(\gamma + 1)/2]^{\gamma/(\gamma - 1)}$. Below that pressure ratio the flow is wholly subsonic and the exit static pressure, p_{s_4} , adjusts itself to the pressure in the discharge region p'_{s_0} . At critical and super-critical pressure ratios, the conditions in the discharge region cannot affect the flow upstream and p_{t_4}/p_{s_4} remains fixed at the critical value.

where $[F_{G_{std}}/(p_{s_0}A_4)]_{ideal}$ is calculated from Equation (3.2). Then, if the general form of K_1 against pressure ratio p_{t_4}/p_{s_0} is assumed to be the same for the engine as for the model, the actual standard gross thrust is readily calculated for any value of p_{t_4}/p_{s_0} .

In using Equation (3.2) a constant value of γ is assumed appropriate to the gas exit conditions, for example, 1.33 for a pure jet engine. Variations in the value of γ likely to occur over representative engine gas temperature ranges have only a secondary effect on the calculated value of $F_{G_{std}}$. Item No. 69008 discusses this matter in more detail and quotes typical values of γ for various types of jet engine.

It may be more accurate in some installations where there is an appreciable loss between the sampling station and exit to introduce a total pressure factor, ϵ , instead of assuming $p_{t_4} = p_{t_3}$ in Equation (3.2). Thus p_{t_4} is taken as ϵp_{t_3} , where ϵ has been previously established, either on the test bed or by model tests, as the ratio of the mean p_{t_4} to the value of p_{t_3} sampled. Equation (3.2) then becomes

$$\frac{F_{G_{std}}}{p_{s_0}A_4} = \frac{2\gamma}{\gamma-1} \frac{\epsilon p_{t_3}}{p_{s_0}} \left[\left(\frac{p_{s_4}}{\epsilon p_{t_3}} \right)^{1/\gamma} - \frac{p_{s_4}}{\epsilon p_{t_3}} \right] + \left[\frac{p_{s_4}}{p_{s_0}} - 1 \right]. \quad (3.3)$$

In the absence of other information ϵ is usually taken to be unity and the overall calibration factor then includes allowance for any total pressure loss.

3.2.2 Mass flow

The determination of the efflux mass flow requires also the sampling of the total temperature at Station 3, it being assumed that T_{t_4} is equal to T_{t_3} . Then

$$m_4 = \rho_4 v_4 A_4 \quad (3.4)$$

and with the same assumptions as were made to derive Equation (3.2) this yields

$$\left[\frac{m_4 (RT_{t_4})^{1/2}}{p_{t_4} A_4} \right]_{ideal} = \left(\frac{p_{s_4}}{p_{t_4}} \right)^{1/\gamma} \left\{ \frac{2\gamma}{\gamma-1} \left[1 - \left(\frac{p_{s_4}}{p_{t_4}} \right)^{(\gamma-1)/\gamma} \right] \right\}^{1/2}. \quad (3.5)$$

The comments regarding the value of p_{s_4} given with Equation (3.2) hold for Equation (3.5) also. The parameter $[m_4 (RT_{t_4})^{1/2}/(p_{t_4} A_4)]_{ideal}$ is plotted against p_{t_4}/p'_{s_0} below the choking condition in Item No. 69008, or, as for Equation (3.2), the corresponding curves in References 10 or 12 may be used to obtain the values.

As for the thrust measurement, Equation (3.5) may be used to deduce an effective area for mass flow measurement purposes. The use of A_{4eff} determined for thrust purposes will lead to considerable mass flow error below the choking condition. If, however, test bed measurements of mass flow are available Equation (3.5) may be used to obtain a more accurate curve for A_{4eff} . As in the case of thrust a calibration will require extrapolation above the choking pressure ratio. If model tests are used for this purpose a discharge ratio K_2 is defined by

$$K_2 = \frac{\left[m_4 (RT_{t_4})^{1/2}/(p_{t_4} A_4) \right]_{actual}}{\left[m_4 (RT_{t_4})^{1/2}/(p_{t_4} A_4) \right]_{ideal}},$$