

CURVES FOR USE IN THE DETERMINATION OF GROSS THRUST AND MASS FLOW IN FLIGHT (AIR BREATHING DUCTED-FLOW ENGINES WITH CONVERGENT NOZZLES)

1. NOTATION AND UNITS

		British	SI
A	cross-sectional area	ft ²	m ²
$F_{G_{std}}$	standard gross thrust	lbf	N
m	mass flow	slugs/s	kg/s
N	engine speed	rev/min	rev/s
p_s	static pressure	lbf/ft ²	N/m ²
p_{s_0}	freestream static pressure	lbf/ft ²	N/m ²
p'_{s_0}	static pressure in discharge region	lbf/ft ²	N/m ²
p_t	total pressure	lbf/ft ²	N/m ²
R	gas constant	lbf ft/slug K	J/kg K
T_s	static temperature	K	K
T_t	total temperature	K	K
W	weight flow	lbf/s	—
γ	ratio of specific heat capacities of exhaust gas		
Δp_s	$p'_{s_0} - p_{s_0}$	lbf/ft ²	N/m ²
ϵ	total pressure factor, p_{t_4}/p_{t_3}		
θ	relative temperature, <i>i.e.</i> ratio of atmospheric absolute temperature to standard sea level value of absolute temperature		
ρ	density	slugs/ft ³	kg/m ³
σ	relative density, <i>i.e.</i> ratio of atmospheric density to standard sea level value of density		

Suffixes

3 and 4 refer to stations in jet pipe and at nozzle throat respectively

Issued April 1969

1

2. INTRODUCTION

This data sheet presents curves for use with both the total head and static pressure sampling methods for measurement of thrust and mass flow in flight given in Item No. 69007.

3. GAS PROPERTIES

Figure 3 presents curves of γ for the products of combustion for various fuel/air mixtures. Typical mean values of γ for engines at full throttle are $\gamma = 1.35$ to 1.38 for turbo-fan engines with mixed exit flows, $\gamma = 1.33$ for turbo-jet engines and $\gamma = 1.25$ for reheated jet flow. It must be emphasised that these values are only given as a guide and that, since the value of γ is dependent on fuel/air ratio, γ will vary to some extent for a given engine with operating conditions, *e.g.* from cruising to maximum power setting.

At high temperatures, above about 1200 K, a small but significant degree of dissociation occurs in the products of combustion. It is then difficult to generalise their thermodynamic properties which, in addition to being dependent on fuel/air ratio, temperature and pressure as at lower temperatures, also depend on the degree of dissociation. However, the expansion in a gas turbine nozzle takes place so rapidly that little change in the degree of dissociation occurs. The conditions are therefore closest to those of frozen flow, *i.e.* a flow in which the degree of dissociation remains constant and the composition is, hence, unchanged throughout any flow process. Thus, the values of γ required to calculate the expansion are those for frozen flow. To plot γ against temperature for frozen conditions would, however, require a different curve for each initial temperature condition. The complexity of such a presentation is avoided on Figure 3 by plotting γ against static temperature from the data of Derivation 2 using, at each temperature, the equilibrium composition appropriate to that temperature but, in the calculation of γ , neglecting any heat input or output produced by dissociation. This presentation is found to give only negligible error in comparison with the use of true frozen conditions.

The temperature which should be used when obtaining γ from Figure 3 is the mean of the static and total temperatures in the exhaust gas expansion (Derivation 1). The static temperature may be obtained from the measured total temperature by means of the following equations, depending on the method of pressure sampling adopted

$$\frac{T_{t_4}}{T_{s_4}} = \left(\frac{\epsilon p_{t_3}}{p_{s_4}} \right)^{(\gamma-1)/\gamma} = \frac{1 - \left(\frac{A_3}{\epsilon A_4} \right)^2 \left(\frac{\epsilon p_{s_3}}{p_{s_4}} \right)^{(\gamma+1)/\gamma}}{1 - \left(\frac{A_3}{\epsilon A_4} \right)^2 \left(\frac{\epsilon p_{s_3}}{p_{s_4}} \right)^{2/\gamma}} \quad (3.1)$$

The use of a mean value of γ is illustrated in the example which also shows that the gross thrust and mass flow obtained by the total head sampling method are relatively insensitive to the value of γ used. However, results obtained using the static pressure sampling method are rather more sensitive to γ variation.

The gas constant depends on the fuel/air ratio but the variation is small over the usual ratios involved in jet engine work (a range of about 3091 to 3108 lbf ft/slug K, *i.e.* 287.2 to 288.7 J/kg K, is the maximum for a temperature up to 2000 K) and a value of $R = 3100$ lbf ft/slug K, *i.e.* 288.0 J/kg K, may be assumed.

4. TOTAL PRESSURE SAMPLING METHOD

4.1 Gross Thrust

Below the choking pressure ratio, p_{s_4} is equal to p'_{s_0} and Equation (3.2) of Item No. 69007, when multiplied throughout by p_{s_0}/p'_{s_0} and re-arranged (with ϵp_{t_3} substituted for p_{t_4} as in Equation (3.3) of Item No. 69007) gives the following expression:

$$\frac{F_{G_{std}} - \Delta p_s A_4}{p'_{s_0} A_4} = \frac{2\gamma}{\gamma - 1} \left[\left(\frac{\epsilon p_{t_3}}{p'_{s_0}} \right)^{(\gamma-1)/\gamma} - 1 \right]. \quad (4.1)$$

The left hand side of Equation (4.1) (gross thrust parameter) is plotted in Figure 4a for three values of γ .

At and above the choking pressure ratio, $p_{s_4}/\epsilon p_{t_3}$ is equal to $[2/(\gamma+1)]^{\gamma/(\gamma-1)}$ and the corresponding gross thrust parameter (which is then independent of p'_{s_0}) becomes

$$\frac{F_{G_{std}}}{p_{s_0} A_4} = (\gamma + 1) \left(\frac{2}{\gamma + 1} \right)^{\gamma/(\gamma-1)} \left(\frac{\epsilon p_{t_3}}{p_{s_0}} \right) - 1 \quad (4.2)$$

which is plotted in Figure 4b.

4.2 Mass Flow

Below the choking pressure ratio, p_{s_4} is equal to p'_{s_0} , and Equation (3.5) of Item No. 69007 becomes:

$$\frac{m_4 (RT_{t_4})^{1/2}}{\epsilon p_{t_3} A_4} = \left(\frac{p'_{s_0}}{\epsilon p_{t_3}} \right)^{1/\gamma} \left\{ \frac{2\gamma}{\gamma - 1} \left[1 - \left(\frac{p'_{s_0}}{\epsilon p_{t_3}} \right)^{(\gamma-1)/\gamma} \right] \right\}^{1/2} \quad (4.3)$$

which is shown plotted in Figure 5a for three values of γ . At and above the choking pressure ratio $p_{s_4}/\epsilon p_{t_3}$ is equal to $[2/(\gamma+1)]^{\gamma/(\gamma-1)}$ and the mass flow parameter is:

$$\frac{m_4 (RT_{t_4})^{1/2}}{\epsilon p_{t_3} A_4} = \gamma^{1/2} \left(\frac{2}{\gamma + 1} \right)^{(\gamma+1)/2(\gamma-1)}. \quad (4.4)$$

The choking value of the mass flow parameter is plotted against γ in Figure 5b.

5. STATIC PRESSURE SAMPLING METHOD

5.1 Choking Pressure Ratio

The choking pressure is given by Equation (3.7) of Item No. 69007 and is plotted in Figure 6 as a function of area ratio and total pressure factor ϵ for three values of γ .

5.2 Gross Thrust

Below the choking pressure ratio, p_{s_4} is equal to p'_{s_0} and Equation (3.6) of Item No. 69007, when multiplied throughout by p_{s_0}/p'_{s_0} and re-arranged, gives the following expression

$$\frac{F_{G_{std}} - \Delta p_s A_4}{p'_{s_0} A_4} = \frac{2\gamma}{\gamma - 1} \left\{ \frac{\left(\frac{\epsilon p_{s_3}}{p'_{s_0}} \right)^{(\gamma-1)/\gamma} - 1}{1 - \left(\frac{\epsilon A_4}{A_3} \right)^2 \left(\frac{p'_{s_0}}{\epsilon p_{s_3}} \right)^{2/\gamma}} \right\}. \quad (5.1)$$

This expression is shown plotted in Figures 7a and 7b as a lattice, i.e. the plot for each value of γ is a carpet which has been shifted on a scale of γ . For clarity only one lattice curve for γ is shown, but such curves are nearly straight and the assumption of linearity is usually justified for interpolation between values of γ .

At the choking pressure ratio (as given in Figure 6) the gross thrust parameter is equal to γ for all area ratios, while above the choking condition it becomes

$$\frac{F_{G_{std}}}{p_{s_0} A_4} = (\gamma + 1) \left(\frac{p_{s_4}}{p_{s_0}} \right) - 1 \quad (5.2)$$

which is shown plotted in Figure 7c. The value of p_{s_4} is obtained from a knowledge of the ratio of $\epsilon p_{s_3}/p_{s_4}$ at the choking condition.

5.3 Mass Flow

Below the choking pressure ratio, p_{s_4} is equal to p'_{s_0} and Equation (3.8) of Item No. 69007 becomes

$$\frac{m_3 (RT_{t_3})^{1/2}}{p_{s_3} A_3} = \frac{\left\{ \frac{2\gamma}{\gamma - 1} \left[1 - \left(\frac{p'_{s_0}}{\epsilon p_{s_3}} \right)^{(\gamma-1)/\gamma} \right] \left[\left(\frac{A_3}{\epsilon A_4} \right)^2 \left(\frac{\epsilon p_{s_3}}{p'_{s_0}} \right)^{2/\gamma} - \left(\frac{p'_{s_0}}{\epsilon p_{s_3}} \right)^{(\gamma-1)/\gamma} \right] \right\}^{1/2}}{\left(\frac{A_3}{\epsilon A_4} \right)^2 \left(\frac{\epsilon p_{s_3}}{p'_{s_0}} \right)^{2/\gamma} - 1} \quad (5.3)$$

and is shown plotted in Figures 8a and 8b. The comments regarding the presentation and use of Figures 7a and 7b are equally applicable to these Figures which are also lattice plots.

At the choking pressure ratio the mass flow parameter becomes constant for a particular γ and area ratio, and is plotted against the variables in Figure 8c.

6. DERIVATION

1. FIELDING, D. Thermodynamic data for the calculation of gas turbine performance.
TOPPS, J.E.C. ARC R & M 3099, 1954
2. GENERAL ELECTRIC *Properties of combustion gases system: C_nH_{2n} - Air. Volume I*
COMPANY *Thermodynamic properties. Volume II Chemical composition of*
 equilibrium mixtures. McGraw-Hill Book Company, New York, 1955.

7. EXAMPLE

The calibration curves for thrust and mass flow obtained by means of the total pressure sampling method for an installed engine are presented in Figures 1 and 2*. It is assumed that no total pressure loss occurs between the sampling station and the jet exit, i.e. ϵ is unity. Find the standard net thrust when the aircraft is cruising at 315 knots E.A.S. at 22 000 ft in a standard atmosphere assuming that the static pressure in the discharge region is freestream atmospheric. The jet pipe total pressure ratio is 2.18, the jet pipe total temperature is 640 K and the fuel flow is 2200 lbf/h.

Assume mean value for $\gamma = 1.33$ in the exhaust gas.

At $p_{t_4}/p_{s_0} = 2.18$ and $\gamma = 1.33$, from Figure 4b

$$\frac{F_{G_{std}}}{p_{s_0} A_4} = 1.75$$

and from Figure 5b

$$\frac{m_4 (RT_{t_4})^{1/2}}{p_{t_4} A_4} = 0.673.$$

From Item No. 68046, relative pressure at 22 000 ft is 0.422 32 and hence

$$\begin{aligned} p_{s_0} &= 0.422\ 32 \times 2116.22 \\ &= 893.7\ \text{lbf/ft}^2 \end{aligned}$$

and from Figure 1, for the thrust calibration at $p_{t_4}/p_{s_0} = 2.18$, $A_{4eff} = 2.236\ \text{ft}^2$.

Hence

$$\begin{aligned} F_{G_{std}} &= 1.75\ p_{s_0} A_4 \\ &= 1.75 \times 893.7 \times 2.236 \\ &= 3\ 500\ \text{lbf}. \end{aligned}$$

From Figure 2, for the mass flow calibration at $p_{t_4}/p_{s_0} = 2.18$, $A_{4eff} = 2.263\ \text{ft}^2$.

Hence,

$$m_4 = \frac{0.673\ p_{t_4} A_4}{(RT_{t_4})^{1/2}}$$

* In order to reduce the scatter usually found in the experimental data, the curves of Figures 1 and 2 are derived by making some preliminary plottings of the recorded data. To obtain Figure 1, values of $F_{G_{std}}/p_{s_0}$ and p_{t_4}/p_{s_0} are plotted against $N/\sqrt{\theta}$. The best curves for these two graphs are drawn and these curves used to calculate the smooth curve for A_{4eff} plotted in Figure 1. A similar procedure is adopted for Figure 2, the first step being to plot $m_4(T_{t_4})^{1/2}/p_{t_4}$ and p_{t_4}/p_{s_0} against $N/\sqrt{\theta}$.

$$= \frac{0.673 \times 2.18 \times 893.7 \times 2.263}{(3100 \times 640)^{1/2}}$$

$$= 2.11 \text{ slugs/s.}$$

$$\text{Fuel mass flow} = \frac{2200}{32.2} \times \frac{1}{3600}$$

$$= 0.019 \text{ slugs/s.}$$

$$\text{Hence intake air flow} = 2.11 - 0.02 = 2.09 \text{ slugs/s.}$$

From Item No. 68046 at 22 000 ft, $\sigma^{1/2} = 0.70540$ and hence the true airspeed is

$$\frac{315 \times 1.688}{0.70540} = 754 \text{ ft/s.}$$

$$\text{The momentum drag is therefore} \quad 2.09 \times 754 = 1576 \text{ lbf}$$

$$\begin{aligned} \text{and the net thrust} \quad F_{N_{std}} &= 3500 - 1576 \\ &= 1924 \text{ lbf.} \end{aligned}$$

This completes the first step but it is now desirable to check the assumed value of γ .

The fuel/air ratio = $0.019/2.09 = 0.0091$ (i.e. 13.5 per cent of stoichiometric) and at 640 K, from Figure 3, $\gamma = 1.364$. Since the mean temperature will be lower than the total temperature, the value of γ corresponding to the mean temperature will be greater than 1.364. Take the mean value as 1.372. Then

$$\begin{aligned} \frac{T_{t_4}}{T_{s_4}} &= 2.18^{0.372/1.372} \\ &= 1.235. \end{aligned}$$

Hence $T_{s_4} = 640/1.235 = 518 \text{ K}$ and the mean temperature = $(640 + 518)/2 = 579 \text{ K}$. From Figure 3 this gives a mean value of $\gamma = 1.372$ which may now be used to repeat the thrust calculation.

Following the first step but with $\gamma = 1.372$ instead of 1.33

$$\frac{F_{G_{std}}}{p_{s_0} A_4} = 1.76 \quad \text{so} \quad F_{G_{std}} = 3520 \text{ lbf}$$

$$\begin{aligned} \text{and} \quad \frac{m_4 (RT_{t_4})^{1/2}}{p_{t_4} A_4} &= 0.680 \quad \text{so} \quad m_4 = 2.13 \text{ slugs/s.} \end{aligned}$$

$$\text{The momentum drag} = (2.13 - 0.019) \times 754 = 1592 \text{ lbf and the net thrust} = 1930 \text{ lbf.}$$

The fuel/air ratio = $0.019/2.13 = 0.0089$ (i.e. 13.2 per cent of stoichiometric) and no further iteration is necessary.